SR - MAGIC SQUARE (SRMS) OF FOURTH ORDER ON SOME SPECIAL NUMBERS

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Abstract:
In this paper, a generic definition for the Fourth order magic squares SR- magic square (SRMS) and some examples for the fourth order SR - magic square on some special numbers are given.

Keywords: - Magic Square, Magic Constant, SR magic square, SRMS.

INTRODUCTION

Magic squares have been known in India from very early times. The renowned mathematician Ramanujan had immense contributions in the field of Magic Squares. A magic square is an n×n matrix filled with the integers in such a way that the sum of the numbers in each row, each column or diagonally remain the same, in which one integer is used once only. The constant sum is called magic constant or magic number. Along with the conditions of normal magic squares, we define a new magic squares of order 4 called SR - magic square (SRMS), having some new property.

2. MATHEMATICAL PRELIMINARIES

2.1 Magic Square

A magic square of order n is an n^{th} order matrix $[a_{i,j}]$ such that

$\sum_{j=1}^{n} a_{i,j} = \propto$ for i=1, 2, 3, 4, ………n --------------- (1)

$\sum_{j=1}^{n} a_{j,i} = \propto$ for i=1, 2, 3, 4, ………n --------------- (2)

$\sum_{i=1}^{n} (a_{i,i}) = \propto$ --------------- (3)

$\sum_{i=1}^{n} (a_{i,n+1-i}) = \propto$ --------------- (4)

Equation (1) represents the row sum, equation (2) represents the column sum, equation (3) represents the diagonal and co-diagonal sum and symbol $\propto$ represents the magic constant.

2.2 Magic Constant

The constant $\propto$ in the above definition is known as the magic constant or magic number. The magic constant of the magic square A is denoted as $\propto [A]$

2.4 GAPFUL NUMBER

L. Colucci calls a number N of at least 3 digits a *Gap ful* number if N is divisible by the number formed by the first and last digit of N.

For example, 583 is gap ful because it is divisible by 53.

2.5 HARSHAD NUMBER

A number N is a *Harshad* (also called Niven) number if it is divisible by the sum of its digits.

For example, 666 is divisible by 6+6+6 so it is a Harshad number.

2.6 D-POWERFUL

An integer N is called *Digitally powerful* (here D-powerful) if it can be expressed as a sum of positive powers of its digits.
For example, 3459872 = $3^1 + 4^6 + 5^5 + 9^6 + 8^3 + 7^7 + 2^{21}$ is d-powerful.

2.7 PERNICIOUS
A number N is called P*ernicious* if it contains a prime number of ones in its binary representation.
For example, 21 = $(10101)_2$ is pernicious since it contains 3 ones and 3 is a prime number.

2.8 ODIOUS AND EVIL
A number N is called *Odious* if the sum of its binary digits is *odd* and *Evil* if the sum of its binary digits is even.

For example, 21 = $(10101)_2$ is odious since the sum of its binary digits is 3.

2.9 CYCLIC NUMBER
A number N is a *Cyclic number* if N and $\emptyset(N)$ have no common prime factors.

2.10 STRAIGHT-LINE NUMBER
A number N > 99 is said to be *straight-line* if its digits form an arithmetic progression.
For example, 123456 (step is 1), 666 (step is 0), or 7531 (step is -2).

3. SR MAGIC SQUARE (SRMS) GENERIC DEFINITION :
A SR - magic square over a field R is a matrix $A = [a_{i,j}]$ of order 4 × 4 with entries in R such that, the following conditions holds.
Let $A = [a_{i,j}]$ be a matrix of order 4 × 4, such that

$\sum_{j=1}^{4} a_{i,j} = \propto [A]$ for i=1, 2, 3, 4 --------------- (1)

$\sum_{j=1}^{4} a_{j,i} = \propto [A]$ for i=1, 2, 3, 4 --------------- (2)

$\sum_{i=1}^{4} (a_{i,i}) = \propto [A]$ --------------- (3)

$\sum_{i=1}^{4} (a_{i,5-i}) = \propto [A]$ --------------- (4)

$\sum_{i=1}^{2} (a_{i,1} + a_{i+2,3}) = \propto [A]$ --------------- (5)

$\sum_{i=1}^{2} (a_{i,2} + a_{i+2,4}) = \propto [A]$ --------------- (6)

$\sum_{i=1}^{2} (a_{i,3} + a_{i+2,1}) = \propto [A]$ --------------- (7)

$\sum_{i=1}^{2} (a_{i,4} + a_{i+2,2}) = \propto [A]$ --------------- (8)

$\sum_{i=1}^{2} (a_{3,i} + a_{1,i+2}) = \propto [A]$ --------------- (9)

$\sum_{i=1}^{2} (a_{4,i} + a_{2,i+2}) = \propto [A]$ --------------- (10)

$\sum_{i=1}^{2} (a_{1,i} + a_{3,i+2}) = \propto [A]$ --------------- (11)

$\sum_{i=1}^{2} (a_{2,i} + a_{4,i+2}) = \propto [A]$ --------------- (12)

$\sum_{i=1}^{2} (a_{3,i*i} + a_{4,i*i}) = \propto [A]$ --------------- (13)

$\sum_{i=1}^{2} (a_{1,i*i} + a_{2,i*i}) = \propto [A]$ --------------- (14)

$\sum_{i=1}^{2} (a_{2,i*i} + a_{3,i*i}) = \propto [A]$ --------------- (15)

$\sum_{i=1}^{2} (a_{1,i+1} + a_{4,i+1}) = \propto [A]$ --------------- (16)

$\sum_{i=1}^{2} (a_{1,i} + a_{4,i}) = \propto [A]$ --------------- (17)

$\sum_{i=1}^{2} (a_{1,i+2} + a_{4,i+2}) = \propto [A]$ --------------- (18)

$\sum_{i=1}^{2} (a_{i,2} + a_{i,3}) = \propto [A]$ --------------- (19)

$\sum_{i=1}^{2} (a_{3,i+1} + a_{4,i+1}) = \propto [A]$ --------------- (20)

$\sum_{i=1}^{2} (a_{2,i} + a_{3,i}) = \propto [A]$ --------------- (21)

$\sum_{i=1}^{2} (a_{2,i+2} + a_{3,i+2}) = \propto [A]$ --------------- (22)

$\sum_{i=1}^{2} (a_{2,i+1} + a_{3,i+1}) = \propto [A]$ --------------- (23)

$\sum_{i=1}^{2} (a_{1,2i-1} + a_{3,2i-1}) = \propto [A]$ --------------- (24)

$\sum_{i=1}^{2} (a_{2,2i-1} + a_{4,2i-1}) = \propto [A]$ --------------- (25)

$\sum_{i=1}^{2}(a_{1,2i} + a_{3,2i}) = \propto [A]$ --------------- (26)

$\sum_{i=1}^{2}(a_{2,2i} + a_{4,2i}) = \propto [A]$ --------------- (27)

$\sum_{i=1}^{2}(a_{1,i*i} + a_{4,i*i}) = \propto [A]$ --------------- (28)

$\sum_{i=1}^{2}(a_{i,i+2} + a_{i+2,i}) = \propto [A]$ --------------- (29)

$\sum_{i=1}^{2}(a_{2i,2i-1} + a_{2i-1,2i}) = \propto [A]$ --------------- (30)

$\sum_{i=1}^{2}(a_{1,i} + a_{2,i}) = \propto [A]$ --------------- (31)

$\sum_{i=1}^{2}(a_{i,3} + a_{i,4}) = \propto [A]$ --------------- (32)

$\sum_{i=1}^{2}(a_{3,i} + a_{4,i}) = \propto [A]$ --------------- (33)

$\sum_{i=1}^{2}(a_{3,i+2} + a_{4,i+2}) = \propto [A]$ --------------- (34)

Where $\propto [A]$ is a magic constant of SR magic square.

3.1 Magic Constant (SRMS)

Given A = $[a_{i,j}]$ be a SR - magic square of order 4. Then its magic constant or magic number is defined as $\propto [A] = \frac{1}{4}\sum_{i=1}\sum_{j=1} a_{i,j}$

3.2 Example:

18	25	14	13
16	11	20	23
21	22	17	10
15	12	19	24

Whose Magic Constant $\propto [A]$ is 70.

18	25	14	13
16	11	20	23
21	22	17	10
15	12	19	24

Eqn - (1)

18	25	14	13
16	11	20	23
21	22	17	10
15	12	19	24

Eqn - (2)

18			
	11		
		17	
			24

Eqn - (3)

			13
		20	
	22		
15			

Eqn - (4)

18			
16			
		17	
		19	

Eqn - (5)

	25		
	11		
			10
			24

Eqn - (6)

		14	
		20	
21			
15			

Eqn - (7)

			13
			23
	22		
	12		

Eqn - (8)

		14	13
21	22		

Eqn - (9)

		20	23
15	12		

Eqn - (10)

18	25		
		17	10

Eqn - (11)

16	11		
		19	24

Eqn - (12)

21			10
15			24

Eqn - (13)

18			13
16			23

Eqn - (14)

16			23
21			10

Eqn - (15)

	25	14	
	12	19	

Eqn - (16)

18	25		
15	12		

Eqn - (17)

		14	13
		19	24

Eqn - (18)

	25	14	
	11	20	

Eqn - (19)

	22	17	
	12	19	

Eqn - (20)

16	11		
21	22		

Eqn - (21)

		20	23
		17	10

Eqn - (22)

	11	20	
	22	17	

Eqn - (23)

18		14	
21		17	

Eqn - (24)

16		20	
15		19	

Eqn - (25)

	25		13
	22		10

Eqn - (26)

	11		23
	12		24

Eqn - (27)

18			13
15			24

Eqn - (28)

		14	
			23
21			
	12		

Eqn - (29)

	25		
16			
			10
		19	

Eqn - (30)

18	25		
16	11		

Eqn - (31)

		14	13
		20	23

Eqn - (32)

21	22		
15	12		

Eqn - (33)

		17	10
		19	24

Eqn - (34)

***Note:**

1. The equations (1), (2), (3) and (4) gives the normal magic square of order 4.
2. The equations (15), (16), (21), (22), (23), (28), (29), (30), (31), (32), (33) and (34) is similar to that of the magic Square (Birth Day Magic square) given by the great Indian mathematician Srinivasa Ramanujan.

4. PROPOSITIONS AND THEOREMS:

4.1 Proposition

If A and B are two SRMS's of order 4 with $\propto [A] = a$ and $\propto [B] = b$, then A +B is an SRMS with $\propto [A+B] = a + b = \propto [A] + \propto [B]$

4.2 Proposition

If A is a SRMS of order 4 with $\propto [A] = a$, then $B = \upsilon A$, $for\ \upsilon \in R$ is also a SRMS with $\propto [B] = \upsilon \cdot \propto [A]$

4.3 Proposition

If A is a SRMS of order 4 with $\propto [A] = a$, then $-A$ *is* also a SRMS with $\propto [-A] = -a$

4.4 Proposition

Let A be a SRMS of order 4 with $\propto [A] = a$, then A^{-} is also a SRMS with magic constant $\propto | A^{-} | = a$.

4.5 Proposition

Let A be a SRMS of order 4 with $\propto [A] = a$, then $\frac{A+A^{-}}{2}$ is also a SRMS with magic constant a. and $\frac{A-A^{-}}{2}$ is also a SRMS with magic constant 0.

4.6 Proposition

Let A be a SRMS of order 4 with $\propto [A] = a$, then A is also a strongly magic square of order 4 with magic constant $\propto [A] = a$

4.65 Proposition

Let A be a SRMS of order 4, then A^{-} does not exists, since $|A| = 0$.

5. SR - MAGIC SQUARE (SRMS) OF FOURTH ORDER ON SOME SPECIAL NUMBERS

5.1 SRMS on Gapful Numbers

220	297	165	154
187	132	242	275
253	264	198	121
176	143	231	286

5.2 SRMS on Harshad Numbers

1998	2040	222	216
234	204	2010	2028
2016	2022	240	198
228	210	2004	2034

5.3 SRMS on Cyclic Numbers

53	95	29	23
41	11	65	83
71	77	47	5
35	17	59	89

5.4 SRMS on Straight line Numbers

840	987	222	147
246	123	864	963
876	951	258	111
234	135	852	975

5.5 SRMS on D-Powerful Numbers

2131	2138	375	374
377	372	2133	2136
2134	2135	378	371
376	373	2132	2137

5.6 SRMS on Prenicious Numbers

173	194	121	118
127	112	179	188
182	185	130	109
124	115	176	191

5.7 SRMS on Evil Numbers

189	210	57	54
63	48	195	204
198	201	66	45
60	51	192	207

5.8 SRMS on Odious Numbers

173	194	73	70
79	64	179	188
182	185	82	61
76	67	176	191

6. CONCLUSION

In this paper, the concept of SR magic square of order 4 has been introduced. As it is a new one, much more work could be done and many properties could be derived.

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