LINEAR PROGRAMMING PROBLEM USED IN TRANSPORTATION PROBLEM

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Abstract: Mathematics is the queen of science in our daily life, planning is required on various occasions, especially when the resources are limited. Any planning is meant for attaining certain objectives the best strategy is one that gives a maximum output from a minimum input. Linear programming problem used in transportation problem . The Transportation is the movement of people and goods, from one place to another the term is derived from the Latin trans ("across") and portare ("to carry") industries. which have the business of providing equipment, actual transport goods and services used in transport of goods or people make up a large broad and important sector of most national economic and are collectively referred to as transport industries.

Keywords: *Lpp, Simplex Method, graphical method, transportation problem.*

I. Introduction

Operation Research is a relatively new discipline. The contents and the boundaries of the OR are not yet fixed. Therefore, to give a formal definition of the term Operations Research is a difficult task. The OR starts when mathematical and quantitative techniques are used to substantiate the decision being taken. The main activity of a manager is the decision making. In our daily life we make the decision seven without noticing them. The decisions are taken simply by common sense, judgment and expertise without using any mathematical or any other model in simple situations. But the decision we are concerned here with are complex and heavily responsible. Examples are public transportation network planning in a city having its own layout of factories, residential blocks or finding the appropriate product mix when there exists a large number of products with different profit contributions and production requirement etc.

Operations Research tools are not from any one discipline. Operations Research takes tools from different discipline such as mathematics, statistics, economics, psychology, engineering etc and combines these tools to make a new set of knowledge for decision making. Today, O.R. became a professional discipline which deals with the application of scientific methods for making decision, and especially to the allocation of scarce resources. The main purpose of O.R. is to provide a rational basis for decisions making in the absence of complete information, because the systems composed of human, machine, and procedures may do not have complete information.

Operations Research can also be treated as science in the sense it describing, understanding and predicting the systems behaviour, especially man-machine system. Thus O.R. specialists are involved in three classical aspect of science, they are as follows: i) Determining the systems behaviour. ii) Analyzing the systems behaviour by developing appropriate models, iii) Predict the future behaviour using these models. The emphasis on analysis of operations as a whole distinguishes the O.R. from other research and engineering. O.R. is an interdisciplinary discipline which provided solutions to problems of military operations during World War II, and also successful in other operations. Today business applications are primarily concerned with O.R. analysis for the possible alternative actions. The business and industry befitted from O.R. in the areas of inventory, reorder policies, optimum location and size of warehouses, advertising policies, etc.

1.1 Definition of operation research:

- Morse and Kimball have stressed O.R. is a quantitative approach and described it as "a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control".
- Saaty considers O.R. as tool of improving quality of answers. He says, "O.R. is the art of giving bad answers to problems which otherwise have worse answers".
- Miller and Starr state, "O.R. is applied decision theory, which uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough-going rationality in dealing with his decision problem".

II. Linear Programming: Linear programming is a powerful quantitative technique (or operational research technique) designs to solve allocation problem. The term 'linear programming' consists of the two words 'Linear' and 'Programming'. The word 'Linear' is used to describe the relationship between decision variables, which are directly proportional. For example, if doubling (or tripling) the production of a product will exactly double (or triple) the profit and required resources, then it is linear relationship. The word 'programming' means planning of activities in a manner that achieves some 'optimal' result with available resources. A programme is

'optimal' if it maximises or minimises some measure or criterion of effectiveness such as profit, contribution (i.e. sales-variable cost), sales, and cost. Thus, 'Linear Programming' indicates the planning of decision variables, which are directly proportional, to achieve the 'optimal' result considering the limitations within which the problem is to be solved.

2.1: Step Involved in the Formulation of LP Problem

The steps involved in the formation of linear programming problem are as follows:

Step 1: Identify the Decision Variables of interest to the decision maker and express them as X_1, X_2, X_3

Step 2: Ascertain the Objective of the decision maker whether the wants to minimize or to maximize.

Step 3: Ascertain the cost (in case of minimization problem) or the profit (in case of maximization problem) per unit of each of the decision variables.

Step 4: Ascertain the constraints representing the maximum vailability or minimum commitment or equality and represent them as less than or equal to (<) type inequality or greater than or equal to (>) type inequality or 'equal to' (=) type equality respectively.

Step 5: Put non-negativity restriction as under: $X_j > 0$; j = 1, 2... n (non-negativity restriction)

Step 6: Now formulate the LP problem as under: Subject Maximize (or Minimize)

$$
\begin{array}{lllllllll}
\max & c_1x_1 & + & c_2x_2 & + & \dots & + & c_nx_n & & \text{Objective function} \\
\text{subject to} & a_{11}x_1 & + & a_{12}x_2 & + & \dots & + & a_{1n}x_n & \leq & b_1 \\
& a_{21}x_1 & + & a_{22}x_2 & + & \dots & + & a_{2n}x_n & \leq & b_2 \\
& \vdots & + & \vdots & & & & \vdots & & \vdots \\
& a_{m1}x_1 & + & a_{m2}x_2 & + & \dots & + & a_{mn}x_n & \leq & b_m \\
& & & x_1, x_2, \dots, x_n & & & & \geq & 0 & \text{Sign restrictions}
\end{array}
$$

Constraints

Figure 2.1: linear program in a standard form

- **Objective Function:** is a linear function of the decision variables representing the objective of the manager/ decision maker.
- **Constraints:** are the linear equations or inequalities arising out of practical limitations.
- **Decision Variables:** are some physical quantities whose values indicate the solution.
- **Feasible Solution:** is a solution which satisfies all the constraints (including the non-negative) presents in the problem.
- **Feasible Region**: is the collection of feasible solutions.
- **Multiple Solutions**: are solutions each of which maximize or minimize the objective function.
- **Unbounded Solution**: is a solution whose objective function is infinite.
- **Infeasible Solution**: means no feasible solution.

Example 2.1: A toy company manufactures two types of doll, a basic version-doll A and a deluxe version-doll B. Each doll of type B takes twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 per day. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs. 3.00 and Rs. 5.00 per doll, respectively on doll A and B, then how many of each dolls Should be produced per day in order to maximize the total profit. Formulate this problem.

Formulation: Let X_1 and X_2 be the number of dolls produced per day of type *A* and *B,* respectively. Let the doll *A* require *t* hrs so that the doll *B* require *2t* hrs. So the total time to manufacture X1 and X2 dolls should not exceed 2000t hrs. Therefore, $tX_1 + 2tX_2 <$ 2000t, other constraints are simple. Then the linear programming problem become

$$\text{Maximise} \qquad p = 3x_1 + 5x_2$$

Subject to the restriction	$X_1 + 2x_2 \leq 2000$	(time constraint)
	$X_1 + X_2 \leq 1500$	(plastics constraints)
	$X_2 \leq 600$	(dress constraints)
And non negative restrictions	$X_1 \geq 0, X_2 \geq 0$	

2.12. Graphical method:

Example2.2: solve the given problem by using graphical methods.

$$\text{Max } Z_x = 3x_1 + 2x_2$$
$$x_1 + x_2 \leq 80$$
$$2x_1 + x_2 \leq 100$$
$$x_1 \leq 40$$
$$x_1, x_2 \geq 0$$

Find the feasible region.

• Plot each constraint as an equation _ line in the plane

• Feasible points on one side of the line – plug in (0,0) to find out which
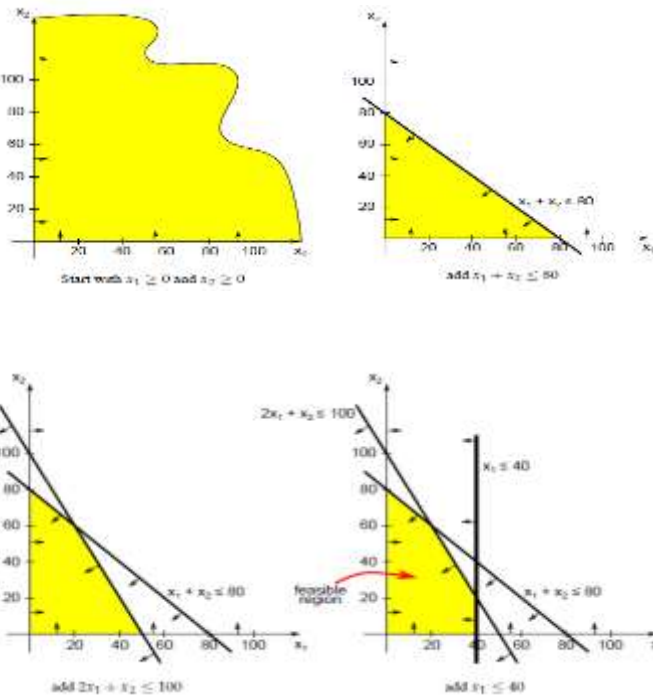
Figure.2.12 graphical representation

A corner (extreme) point X of the region R= every line through X intersects R in a segments whose one endpoint is X. solving a linear program amounts to finding a best corner point by the following theorem.

III. Simplex method

The process consists of two steps
1. Find a feasible solution (or determine that none exists).
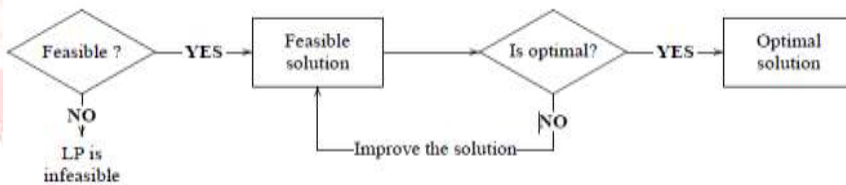2. Improve the feasible solution to an optimal solution.

Figure 3.1 Simplex method variable following

3.1 Canonical form:

Linear program (LP) is in a canonical form if
• all constraints are equations.
• all variables are non-negative.

$$
\begin{aligned}
\max \quad & c_1 x_1 + c_2 x_2 + \ldots + c_n x_n \\
\text{subject to} \quad & a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n = b_1 \\
& a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n = b_2 \\
& \vdots \quad + \quad \vdots \quad\quad \vdots \\
& a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n = b_m \\
& x_1, x_2, \ldots, x_n \geq 0
\end{aligned}
$$

3.2 Slack variables: To change a inequality to an equation, we add a new non-negative variable called a slack variable.

$$x_1 + x_2 \leq 80 \quad\quad \rightarrow \quad\quad x_1 + x_2 + s_1 = 80$$
$$x_1 + x_2 \geq 80 \quad\quad \rightarrow \quad\quad x_1 + x_2 - e_1 = 80$$

Notes:
- the variable e_1 is sometimes called an **excess** variable.
- We can use x_i for slack variables (where i is a new index)

3.3 Simplex method
Example:**3.31** Consider the 2.2 example toyshop example above .convert to equalities by adding slack variables

$$\max 3x_1 + 2x_2$$
$$x_1 + x_2 \leq 80$$
$$2x_1 + x_2 \leq 100 \qquad \longrightarrow$$
$$x_1 \leq 40$$
$$x_1, x_2 \geq 0$$

$$\max 3x_1 + 2x_2$$
$$x_1 + x_2 + x_3 = 80$$
$$2x_1 + x_2 + x_4 = 100$$
$$x_1 + x_5 = 40$$
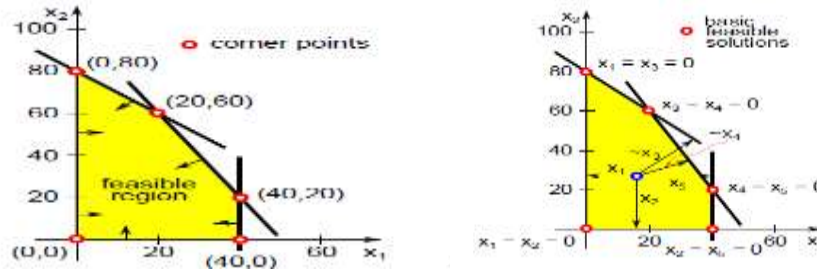$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Figure 3.31 graphical representation of simplex method

Step Starting feasible solution
- Set variables x_1, x_2 to zero and set slack variables to the values on the right-hand side yields a feasible solution $x_1 = x_2 = 0$, $x_3 = 80$, $x_4 = 100$, $x_5 = 40$
- Recall that the solution is feasible because all variables are non-negative and satisfy all equations.(we get a feasible solution right away because the right-hand side is non-negative; this may not always work)
- Note something interesting: in this feasible solution two variables (namely x_1, x_2) are zero. Such a solution is called a basic solution of this problem, because the value of at least two variables is zero.
- In a problem with n variables and m constraints, a solution where at least $(n - m)$ variables are zero is a basic solution. A basic solution that is also feasible is called a basic feasible solution (BFS). The importance of basic solutions is revealed by the following observation.
- Basic solutions are precisely the corner points of the feasible region. Recall that we have discussed that to find an optimal solution to an LP, it suffices to find a best solution among all corner points. The above tells us how to compute them. they are the basic feasible solutions.
- A variable in a basic solution is called a non-basic variable if it is chosen to be zero. Otherwise, the variable is basic. The basic variables we collectively call a basis

3.4. Dictionary
To conveniently deal with basic solutions, we use the so-called dictionary. A dictionary lists values of basic variables as a function of non-basic variables. The correspondence is obtained by expressing the basic variables from the initial set of equations. (We shall come back to this later; for now, have a look below.).Express the slack variables from the individual equations

$$\max 3x_1 + 2x_2$$
$$x_1 + x_2 + x_3 = 80$$
$$2x_1 + x_2 + x_4 = 100 \qquad \longrightarrow$$
$$x_1 + x_5 = 40$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

$$x_3 = 80 - x_1 - x_2$$
$$x_4 = 100 - 2x_1 - x_2$$
$$x_5 = 40 - x_1$$
$$z = 0 + 3x_1 + 2x_2$$

x_1, x_2 independent (non-basic) variables

x_3, x_4, x_5 dependent (basic) variables

$\{x_3, x_4, x_5\}$ is a basis set $x_1 = x_2 = 0$ the corresponding (feasible) solution is $x_3 = 80$, $x_4 = 100$, $x_5 = 40$ with value $z = 0$

Improving the solution:
Try to increase x_1 from its current value 0 in hopes of improving the value of   try $x_1=20$,$x_2=0$ and substitute into the dictionary to obtain the values of x_3, x_4, x_5 and Z→$x_3=60$,$x_4=60$,$x_5=20$ with the value of $z=60$ → feasible

Try again $x_1=40$, $x_2=0$→$x_3=40$, $x_4=20$,$x_5=0$ with the value $z=120$→feasible.

Now try $x_1=50$, $x_2=0 \to$ $x_3=30, x_4=0, x_5=10 \to$ not feasible.

If $x_1 = t$ and $x_2 = 0$, then the solution is feasible if

$$\left. \begin{array}{ll} x_3 = 80 - t - 0 \geq 0 & t \leq 80 \\ x_4 = 100 - 2t - 0 \geq 0 \quad \Rightarrow \quad t \leq 50 \\ x_5 = 40 - t \quad \geq 0 & t \leq 40 \end{array} \right\} \Rightarrow t \leq 40$$

Maximal value is $x_1 = 40$ at which point the variable x_5 becomes zero x_1 is *incoming* variable and x_5 is *outgoing* variable (we say that x_1 enters the dictionary/basis, and x_5 leaves the dictionary/basis)

Ratio test

The above analysis can be streamlined into the following simple "ratio" test

$$x_3 : \frac{80}{1} = 80$$

$$x_4 : \frac{100}{2} = 50$$

$$x_5 : \frac{40}{1} = 40$$

$$\begin{array}{rrrrr} x_3 = & 80 & x_1 & - & x_2 \\ x_4 = & 100 & - 2x_1 & - & x_2 \\ x_5 = & 40 & - & x_1 & \\ \hline z = & 0 & + 3x_1 & + & 2x_2 \end{array}$$

ratio for x_4:

$$\frac{100}{2} = 50$$

(**watch-out:** we only consider this ratio because the coefficient of x_1 is negative (-2). . .more on that in the later steps)

Minimum achieved with $x_5 =$) outgoing variable

Express x_1 from the equation for x_5

$x_5 = 40 - x_1 \qquad \longrightarrow \qquad x_1 = 40 - x_5$

Substitute $x1$ to all other equations new feasible dictionary

$$\begin{array}{rll} x_1 = & (40 - x_5) \\ x_3 = & 80 - (40 - x_5) - x_2 \\ x_4 = & 100 - 2(40 - x_5) - x_2 \\ \hline z = & 0 + 3(40 - x_5) + 2x_2 \end{array} \qquad \longrightarrow \qquad \begin{array}{rlll} x_1 = & 40 & & - x_5 \\ x_3 = & 40 & - x_2 & + x_5 \\ x_4 = & 20 & - x_2 & + 2x_5 \\ \hline z = & 120 & + 2x_2 & - 3x_5 \end{array}$$

now x_2, x_5 are independent variables and x_1, x_3, x_4 are dependent $\to \{x_1, x_3, x_4\}$ is a basis.

we repeat: we increase $x_2 \to$ incoming variable, ratio test:

x_1 : does not contain $x_2 \to$ no constraint

$$x_2 : \frac{40}{1} = 40$$

$$x_4 : \frac{20}{1} = 20$$

Minimum achieved for $x4 \to$ outgoing variable

$x_4 = 20 - x_2 + 2x_5 \qquad \longrightarrow \qquad x_2 = 20 - x_4 + 2x_5$

$$\begin{array}{rll} x_1 = & 40 & - x_5 \\ x_2 = & (20 - x_4 + 2x_5) \\ x_3 = & 40 - (20 - x_4 + 2x_5) + x_5 \\ \hline z = & 120 + 2(20 - x_4 + 2x_5) - 3x_5 \end{array} \qquad \longrightarrow \qquad \begin{array}{rlll} x_1 = & 40 & & - x_5 \\ x_2 = & 20 & - x_4 & + 2x_5 \\ x_3 = & 20 & + x_4 & - x_5 \\ \hline z = & 160 & - 2x_4 & + x_5 \end{array}$$

X_5 incoming variable, ratio test

$$x_1 : \frac{40}{1} = 40$$

X_2 : positive coefficient $\to$ no constant

$$x_3 : \frac{20}{1} = 20$$

Minimum achieved for $x_3 \to$ outgoing variable

$x_3 = 20 + x_4 - x_5 \qquad \longrightarrow \qquad x_5 = 20 + x_4 - x_3$

$$
\begin{aligned}
x_1 &= 40 & & - & (20 + x_4 - x_3) \\
x_2 &= 20 - & x_4 + & 2(20 + x_4 - x_3) \\
x_5 &= & & & (20 + x_4 - x_3) \\
\hline
z &= 160 - & 2x_4 + & (20 + x_4 - x_3)
\end{aligned}
\qquad \longrightarrow \qquad
\begin{aligned}
x_1 &= 20 + & x_3 - x_4 \\
x_2 &= 60 - & 2x_3 + x_4 \\
x_5 &= 20 - & x_3 + x_4 \\
\hline
z &= 180 - & x_3 - x_4
\end{aligned}
$$

No more improved possible →optimal solution.

$x_1=20, x_2=60, x_3=0, x_4=0, x_5=20$ of the value $z=180$

Why setting x_3, x_4 to any non zero values results in a smaller value of z.

Each dictionary is equivalent to the original system (the two have the same set of solutions.

3.5 Simplex algorithm:

Preparation: find a starting feasible solution/dictionary

        1. Convert to the canonical form (constraints are equalities) by adding slack variables $x_{n+1}, \ldots, x_{n+m}$

        2. Construct a starting dictionary - express slack variables and objective function z

        3. If the resulting dictionary is feasible, then we are done with preparation

          If not, try to find a feasible dictionary using the Phase I. method.

3.6 Simplex step (maximization LP): try to improve the solution

1. (Optimality test): If no variable appears with a positive coefficient in the equation for z→ STOP, current solution is optimal

        • set non-basic variables to zero

        • read off the values of the basic variables and the objective function z→ Hint: the values are the   constant terms in respective equations

        • report this (optimal) solution

2. Else pick a variable xi having positive coefficient in the equation for z xi ≡ incoming variable

3. Ratio test: in the dictionary, find an equation for a variable x_j in which

         • xi appears with a negative coefficient −a

         • the ratio b/ a is smallest possible .(where b is the constant term in the equation for xj)

4. If no such such x_j exists → stop, no optimal solution, report that LP is unbounded

5. Else x_j ≡ outgoing variable→ construct a new dictionary by pivoting:

        • express xi from the equation for x_j ,

        • add this as a new equation,

        • remove the equation for x_j,

        • substitute xi to all other equations (including the one for z)

6. Repeat from 1.

3.7.Two phase Simplex method

Canonical form = equations, non-negative variables

n = number of variables

m = number of equations

basic solution = at least $(n − m)$ variables are zero

basic solutions = dictionaries

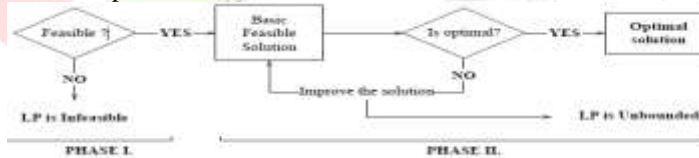basic feasible solutions = corner/extreme points = feasible dictionaries

Figure 3.7.Two phase simplex method

Example: 3.71:

$$
\begin{aligned}
\max\ & 3x_1 + 2x_2 \\
& x_1 + x_2 \le 80 \\
& 2x_1 + x_2 \le 100 \\
& x_1 \quad\quad \le 40 \\[4pt]
& x_1, x_2 \ge 0
\end{aligned}
$$

Feasible dictionary: $x_3 = x_4 = 0$

$$
\begin{aligned}
x_1 &= 20 + x_3 - x_4 \\
x_2 &= 60 - 2x_3 + x_4 \\
x_5 &= 20 - x_3 + x_4 \\
\hline
z &= 180 - x_3 - x_4
\end{aligned}
$$

$$\max 3x_1 + 2x_2$$
$$x_1 + x_2 + x_3 = 80$$
$$2x_1 + x_2 + x_4 = 100$$
$$x_1 \qquad + x_5 = 40$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Infeasible dictionary: $x_3 = x_5 = 0$

$$x_1 = 40 \qquad - x_5$$
$$x_2 = 40 - x_3 + x_5$$
$$x_4 = -20 + x_3 + x_5$$
$$z = 200 - 2x_3 - x_5$$

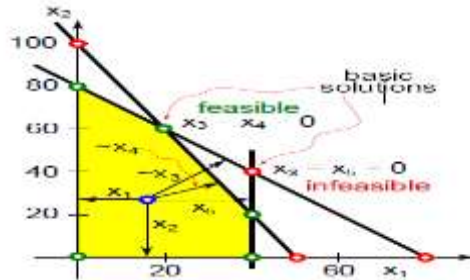(it is infeasible since $x_4 = -20$)

Figure 3.7.Graphical representation of two phase simplex method example

IV.Transportations problem:

A special class of linear programming problem is Transportation Problem, where the objective is to minimize the cost of distributing a product from a number of sources (e.g. factories) to a number of destinations (e.g. warehouses) while satisfying both the supply limits and the demand requirement. Because of the special structure of the Transportation Problem the Simplex Method of solving is unsuitable for the Transportation Problem. The model assumes that the distributing cost on a given rout is directly proportional to the number of units distributed on that route. Generally, the transportation model can be extended to areas other than the direct transportation of a commodity, including among others, inventory control, employment scheduling, and personnel assignment. The transportation problem special feature is illustrated here with the help of following Example 1.1.

Example 4.1:

Suppose a manufacturing company owns three factories (sources) and distribute his products to five different retail agencies (destinations). The following table shows the capacities of the three factories, the quantity of products required by the various retail agencies and the cost of shipping one unit of the product from each of the three factories to each of the five retail agencies.

Factories	Retail Agency					Capacity
	1	2	3	4	5	
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300

Usually the above table is referred as Transportation Table, which provides the basic information regarding the transportation problem. The quantities inside the table are known as transportation cost per unit of product. The capacity of the factories 1, 2, 3 is 50, 100 and 150 respectively. The requirement of the retail agency 1, 2, 3, 4, 5 is 100,60,50,50, and 40 respectively.

In this case, the transportation cost of one unit

from factory 1 to retail agency 1 is 1,

from factory 1 to retail agency 2 is 9,

from factory 1 to retail agency 3 is 13, and so on

A transportation problem can be formulated as linear programming problem using variables with two subscripts.

Let X_{11}=Amount to be transported from factory 1 to retail agency 1

X_{12}= Amount to be transported from factory 1 to retail agency 2

……..

……..

……..

……..

X_{35}= Amount to be transported from factory 3 to retail agency 5.

Let the transportation cost per unit be represented by C_{11}, C_{12}, …..C_{35} that is C_{11}=1, C_{12}=9, and so on.

Let the capacities of the three factories be represented by a_1=50, a_2=100, a_3=150.

Let the requirement of the retail agencies are b_1=100, b_2=60, b_3=50, b_4=50, and b_5=40.

Thus, the problem can be formulated as

Minimize

$$C_{11}x_{11} + C_{12}x_{12} + \ldots\ldots\ldots + C_{35}X_{35}$$

Subject to the constraints
$$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}=a_1$$
$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25}=a_2$$
$$X_{31}+X_{32}+X_{33}+X_{34}+X_{35}=a_3$$
$$X_{11}+X_{21}+X_{31}=b_1$$
$$X_{12}+X_{22}+X_{32}=b_2$$
$$X_{13}+X_{23}+X_{33}=b_3$$
$$X_{14}+X_{24}+X_{34}=b_4$$
$$X_{15}+X_{25}+X_{35}=b_5$$
$$X_{11}, X_{12}, X_{13}\ldots\ldots\ldots\ldots X_{35} \geq 0$$

Thus, the problem has 8 constraints and 15 variables. So, it is not possible to solve such a problem using simplex method. This is the reason for the need of special computational procedure to solve transportation problem. There are varieties of procedures, which are described in the next section.

4.1 Transportation Algorithm

The steps of the transportation algorithm are exact parallels of the simplex algorithm, they are:

Step 1: Determine a starting basic feasible solution, using any one of the following three methods
1. North West Corner Method
2. Least Cost Method
3. Vogel Approximation Method

Step 2: Determine the optimal solution using the following method
   MODI (Modified Distribution Method) or UV Method.

Basic Feasible Solution of a Transportation Problem

The computation of an initial feasible solution is illustrated in this section with the help of the Example 4.1 discussed in the previous section. The problem in the example 1.1 has 8 constraints and 15 variables we can eliminate one of the constraints since $a_1 + a_2 + a_3 = b_1 + b_2 + b_3 + b_4 + b_5$. Thus now the problem contains 7 constraints and 15 variables. Note that any initial (basic) feasible solution has at most 7 non-zero X_{ij}. Generally, any basic feasible solution with m sources (such as factories) and n destination (such as retail agency) has at most m + n -1 non-zero X_{ij}. The special structure of the transportation problem allows securing a non artificial basic feasible solution using one the following three methods.

- North West Corner Method
- Least Cost Method
- Vogel Approximation Method

The difference among these three methods is the quality of the initial basic feasible solution they produce, in the sense that a better that a better initial solution yields a smaller objective value. Generally the Vogel Approximation Method produces the best initial basic feasible solution, and the North West Corner Method produces the worst, but the North West Corner Method involves least computations.

4.2 North West Corner Method : The method starts at the North West (upper left) corner cell of the tableau (variable x_{11}).

Step -1: Allocate as much as possible to the selected cell, and adjust the associated amounts of capacity (supply) and requirement (demand) by subtracting the allocated amount.

Step -2: Cross out the row (column) with zero supply or demand to indicate that no further assignments can be made in that row (column). If both the row and column becomes zero simultaneously, cross out one of them only, and leave a zero supply or demand in the uncrossed out row (column).

Step -3: If exactly one row (column) is left uncrossed out, then stop. Otherwise, move to the cell to the right if a column has just been crossed or the one below if a row has been crossed out. Go to step -1.

Example 4.2:

Consider the problem discussed in Example 4.1 to illustrate the North West Corner Method of determining basic feasible solution:

Factories	Retail Agency					Capacity
	1	2	3	4	5	
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300

The allocation is shown in the following tableau:

The arrows show the order in which the allocated (bolded) amounts are generated. The starting basic solution is given as

$x_{11} = 50$,

$x_{21} = 50$, $x_{22} = 50$

$x_{32} = 10$, $x_{33} = 50$, $x_{34} = 50$, $x_{35} = 40$

The corresponding transportation cost is $50 * 1 + 50 * 24 + 50 * 12 + 10 * 33 + 50 * 1 + 50 * 23 + 40 * 26 = 4420$

It is clear that as soon as a value of X_{ij} is determined, a row (column) is eliminated from further consideration. The last value of X_{ij} eliminates both a row and column. Hence a feasible solution computed by North West Corner Method can have at most $m + n - 1$ positive X_{ij} if the transportation problem has m sources and n destinations.

4.3 Least Cost Method

The least cost method is also known as matrix minimum method in the sense we look for the row and the column corresponding to which C_{ij} is minimum. This method finds a better initial basic feasible solution by concentrating on the cheapest routes. Instead of starting the allocation with the northwest cell as in the North West Corner Method, we start by allocating as much as possible to the cell with the smallest unit cost. If there are two or more minimum costs then we should select the row and the column corresponding to the lower numbered. If they appear in the same row we should select the lower numbered column. We then cross out the satisfied row or column, and adjust the amounts of capacity and requirement accordingly. If both a row and a column is satisfied simultaneously, only one is crossed out. Next, we look for the uncrossed-out cell with the smallest unit cost and repeat the process until we are left at the end with exactly one uncrossed-out row or column.

Example 4.4:

The least cost method of determining initial basic feasible solution is illustrated with the help of problem presented in the section 4.1.

The Least Cost method is applied in the following manner:

We observe that $C_{11}=1$ is the minimum unit cost in the table. Hence $X_{11}=50$ and the first row is crossed out since the row has no more capacity. Then the minimum unit cost in the uncrossed-out row and column is $C_{25}=1$, hence $X_{25}=40$ and the fifth column is crossed out. Next $C_{33}=1$ is the minimum unit cost, hence $X_{33}=50$ and the third column is crossed out. Next $C_{22}=12$ is the minimum unit cost, hence $X_{22}=60$ and the second column is crossed out. Next we look for the uncrossed-out row and column now $C_{31}=14$ is the minimum unit cost, hence $X_{31}=50$ and crossed out the first column since it was satisfied. Finally $C_{34}=23$ is the minimum unit cost, hence $X_{34}=50$ and the fourth column is crossed out. So that the basic feasible solution developed by the Least Cost Method has transportation cost is

$1 * 50 + 12 * 60 + 1 * 40 + 14 * 50 + 1 * 50 + 23 * 50 = 2710$

Note that the minimum transportation cost obtained by the least cost method is much lower than the corresponding cost of the solution developed by using the north-west corner method.

4.4 Vogel Approximation Method (VAM):

VAM is an improved version of the least cost method that generally produces better solutions. The steps involved in this method are:

Step 1: For each row (column) with strictly positive capacity (requirement), determine a penalty by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

Step 2: Identify the row or column with the largest penalty among all the rows and columns. If the penalties corresponding to two or more rows or columns are equal we select the topmost row and the extreme left column.

Step 3: We select X_{ij} as a basic variable if C_{ij} is the minimum cost in the row or column with largest penalty. We choose the numerical value of X_{ij} as high as possible subject to the row and the column constraints. Depending upon whether ai or b_j is the smaller of the two ith row or jth column is crossed out.

Step 4: The Step 2 is now performed on the uncrossed-out rows and columns until all the basic variables have been satisfied.

Example 4.5:

Consider the following transportation problem

Origin	Destination				a_i
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
b_j	60	40	30	110	240

Note: a_i=capacity (supply),b_j=requirement (demand)

Now, compute the penalty for various rows and columns which is shown in the following table

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22	17	4	120	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

Look for the highest penalty in the row or column, the highest penalty occurs in the second column and the minimum unit cost i.e. c_{ij} in this column is $c_{12}=22$. Hence assign 40 to this cell i.e. $x_{12}=40$ and cross out the second column (since second column was satisfied. This is shown in the following table

Row Penalty	4	15	8	3	

The next highest penalty in the uncrossed-out rows and columns is 13 which occur in the first row and the minimum unit cost in this row is $c_{14}=4$, hence $x_{14}=80$ and cross out the first row. The modified table is as follows:

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 / 40	17	4 / 80	0	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

The next highest penalty in the uncrossed-out rows and columns is 8 which occurs in the third column and the minimum cost in this column is $c_{23}=9$, hence $x_{23}=30$ and cross out the third column with adjusted capacity, requirement and penalty values. The modified table is as follows

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 / 40	17	4 / 80	0	13
2	24	37	9 / 30	7	40	17
3	32	37	20	15	50	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the second row and the smallest cost in this row is $c_{24}=15$, hence $x_{24}=30$ and cross out the fourth column with the adjusted capacity, requirement and penalty values. The modified table is as follows:

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 / 40	17	4 / 80	0	13
2	24	37	9 / 30	7 / 30	10	17
3	32	37	20	15	50	17
b_j	60	40	30	110	240	
					80	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the second row and the smallest cost in this row is $c_{21}=24$, hence $x_{21}=10$ and cross out the second row with the adjusted capacity, requirement and penalty values. The modified table is as follows:

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 / 40	17	4 / 80	0	13
2	24 / 10	37	9	7 / 30	0	17
3	32	37	20	15	50	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the third row and the smallest cost in this row is $c_{31}=32$, hence $x_{31}=50$ and cross out the third row or first column. The modified table is as follows:

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 / 40	17	4 / 80	0	13
2	24 / 10	37	9	7 / 30	0	17
3	32 / 50	37	20	15	0	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The transportation cost corresponding to this choice of basic variables is

$$22 * 40 + 4 * 80 + 9 * 30 + 7 * 30 + 24 * 10 + 32 * 50 = 3520$$

V.CONCLUSION

Linear programming problem is like a vast ocean where many methods ,advantages ,uses requirement etc can seen .linear programming can be done in any sector where there is less waste and more profit .by this the production of anything is possible through the new method of L.P. Linear programming is a goods techniques ,especially in the business sector.

Transportation Problem is a special kind of linear programming problem. Because of the transportation problem special structure the simplex method is not suitable. But which may be utilized to make efficient computational techniques for its solution. Generally transportation problem has a number of origins and destination. A certain amount of consignment is available in each origin. Similarly, each destination has a certain demand/requirements. The transportation problem represents amount of consignment to be transported from different origins to destinations so that the transportation cost is minimized without violating the supply and demand constraints.

There are two phases in the transportation problem. First is the determination of basic feasible solution and second is the determination of optimum solution. There are three methods available to determine the basic feasible solution, they are1.North West Corner Method 2. Least Cost Method or Matrix Minimum Method 3.Vogel's Approximation Method (VAM) Transportation problem can be generalized into a Trans shipment Problem where transportation of consignment is possible from origin to origin or destination as well as destination to origin or destination. The Trans shipment problem may be result in an economy way of shipping in some situations.

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