Multi item Fuzzy Inventory Model with shortages through Just in Time; Karush Kuhn Tucker Conditions Approach

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Abstract: In this paper, a multi item objective function fuzzy inventory model through just in time with warehouse storage space constraint is developed in both crisp and fuzzy sense. The order quantity and shortage cost are taken as a decision variables. Here shortages and lead time are allowed. The total cost is minimized under the limitation on storage space using Karush Kuhn Tucker conditions. Finally, a numerical example and sensitivity analysis for crisp and fuzzy environment is given to illustrate this model. Graded mean representation method is used to defuzzify the results.

IndexTerms - Just – In –Time, Back order, Deteriorating items, Lead time, Safety Stock, Trapezoidal fuzzy numbers, Graded Mean Representation method, Karush Kuhn Tucker Condition.

1. INTRODUCTION

Formerly, organizations have focused their efforts on making effectual decisions within a facility. In this case, the various functions of an organization, including accumulate, storage space and provision are generally decoupled into their purposeful and geographic components through buffers of immense inventories. Avoiding these module dependencies conversely can have precious consequences. This becomes progressively more perceptible with market globalization.

Backorder costs are important for companies to track, as the relationship between holding costs of inventory and backorder costs will determine whether a company should over- or under-produce. If the carrying cost of inventory is less than backorder costs, the company should over-produce and keep an inventory. Backorder costs are generally computed and displayed on a per-unit basis.

Just in Time is set of tactical activities, which are formulated to attain maximum production with minimal perpetuation of inventory. Just in Time as philosophy is applicable to various types of organization but on implement side it is more applicable with manufacturing operations. For Just in Time system to be successful, there are two critical essentials, attitude of workers/suppliers/manufacturers/ management and practice.

The lead time is the time between processing of orders made and the delivery of such orders. This time period is sometimes seen as a reflection of the manufacturing cycle time. The lead time is the delay applicable for inventory control purposes. This lead time is usually computed in days.

In mathematical optimization, the Karush–Kuhn–Tucker (KKT) conditions, also known as the Kuhn–Tucker conditions, are first-order necessary conditions for a solution in a nonlinear programming to be optimal, provided that some regularity conditions are satisfied. Allowing inequality constraints, the KKT approach to nonlinear programming generalizes the method of Lagrange multipliers, which allows only equality constraints. The system of equations and inequalities corresponding to the KKT conditions is usually not solved directly, except in the few special cases where a closed-form solution can be derived analytically. The KKT conditions were originally named after Harold W. Kuhn, and Albert W. Tucker, who first published the conditions in 1951. Later scholars discovered that the necessary conditions for this problem had been stated by William Karush in his master's thesis in 1939.

Bellman and Zadeh [1] developed that a decision-making in a fuzzy environment.
Ben-Daya and Raouf [2] opined that inventory models involving lead time as a decision variable. Cheng [3] discussed that an economic order quantity with demand-dependent unit cost. Das [4] established that effect of lead time on inventory; a static analysis. Gupta and Mohan [5] discussed that problems in operations research (Methods & Solutions). Harris [6] established that operations and cost. Kar, Roy, and Maiti [7] asserted that multi item fuzzy inventory model for deteriorating items with finite time-horizon and time dependent demand. Kasthuri and Seshaiah [8]established that multi item EOQ model with demand dependent on unit price. Kasthuri and Seshaiah [9] opined that multi item inventory lot-size model with increasing varying holding cost: A Karush-Kuhn Tucker conditions approach. Mandal, Roy and Maiti [10] established that inventory model of deteriorated items with constraints: a geometric programming approach. Maragatham and Jayanthi [11] developed that integrated supply chain model in deteriorating inventory items and waste reduction contemplations through jit with fuzzy approach. Maragatham and Jayanthi [12] discussed that fuzzy inventory model for deterioration items through Just in Time with shortages

allowed. Ranganayaki and Seshaiah [13] asserted that multi objective fuzzy inventory model with demand dependent unit cost and lead time constraints - A Karush-Kuhn Tucker conditions approach. Taha [14] discussed that operations Research: An Introduction. Vasanthi and Seshaiah [15] established that multi item inventory model with demand dependent on unit cost and varying lead time under fuzzy unit production cost; a Karush-Kuhn Tucker conditions approach. Vasanthi and Seshaiah [16] asserted that multi item inventory model with shortages under limited storage space and set up cost constraints via Karush-Kuhn Tucker conditions approach. Zadeh [17] introduced that fuzzy sets and Zimmermann [18] introduced the description and optimization of fuzzy systems.

This paper considers a simple and practical situation and derives the minimum optimal solution with deteriorating items with multi item fuzzy inventory model with allowed shortages through just in time. Here lead time crashing cost is considered.

We focus here on one particular approach (i.e) Karush Kuhn Tucker conditions approach. Based on this approach we derive an optimal cost. Some of the costs are taken as a trapezoidal fuzzy numbers and for defuzzification we use graded mean representation method.

II. METHODOLOGY

2.1 Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_A = 0, -\infty < x \le a_1,$

(iii) $\mu_A =$ L(x) is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A, a_2 \le x \le a_3$

(v) $\mu_A =$ R(x) is strictly decreasing on $[a_3, a_4]$

(vi) $\mu_A = 0, a_4 \le x < \infty$

where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$; When $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$

2.2 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \frac{x-a}{b-a}, & when\ a \le x \le b; \\ 1, & when\ b \le x \le c; \\ R(x) = \frac{d-x}{d-c}, & when\ c \le x \le d; \\ 0, & otherwise \end{cases}$$

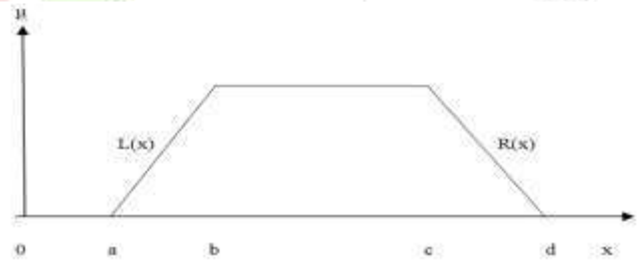

Fig.1: Trapezoidal Fuzzy Number

2.3 The Function Principle

The function principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ *and* $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$
3. $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\tilde{A} \oslash \tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$

5. $\alpha \otimes \tilde{A} = \begin{cases} (\alpha a1, \alpha a2, \alpha a3, \alpha a4), \alpha \geq 0 \\ (\alpha a4, \alpha a3, \alpha a2, \alpha a1), \alpha < 0 \end{cases}$

2.4 Karush – Kuhn Tucker Conditions

Given general problem,

$$\min \ f(x)$$
$$x \in R^n$$
$$subject\ to \ \ h_i(x) \leq 0, i=1,2,...,m$$
$$l_j(x) = 0, \ j=1,2,...,r$$

The Karush – Kuhn Tucker conditions are:

- $0 \in \partial f(x) + \sum_{i=1}^{m} u_i \partial h_i(x) + \sum_{j=1}^{r} v_j \partial l_j(x)$ (stationarity)
- $u_i.h_i(x)=0, \ for\,all\,i$ (complementary slackness)
- $.h_i(x) \leq 0, \ l_j(x)=0, \ for\,all\,i, j$ (primal feasibility)
- $.u_i(x) \geq 0, \ for\,all\,i$ (dual feasibility)

Then,

$$f(x^*) = g(u^*, v^*)$$
$$= \min f(x) + \sum_{i=1}^{m} u_i^* h_i(x) + \sum_{j=1}^{r} v_j^* l_j(x)$$
$$x \in R^n$$
$$\leq f(x^*) + \sum_{i=1}^{m} u_i^* h_i(x^*) + \sum_{j=1}^{r} v_j^* l_j(x^*)$$
$$\leq f(x^*)$$

2.5 Graded Mean Integration Representation Method

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number then the graded mean integration representation of $\tilde{A}$ is,

$$p(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

2.6 Notations and Assumptions

The mathematical model in this paper is developed on the basis of the following notations and assumptions.

2.6.1 Notations

For i^{th} item (i = 1, 2, 3, …,n)

n - number of items
q_i - production quantity batch (decision variable)
R_i - annual demand rate (function of unit cost)
P_i - unit purchase or production cost
w_i - storage space per item
W - floor (or) shelf space available
c_i - unit holding cost per item
s_i - set up or ordering cost per order
r_i - shortage cost

K_i - shortage level (decision variable)
L_i - leading rate time
$k\sigma\sqrt{L_i}$ - safety stock
TC - total annual cost
α - screening cost per unit
β - reworking cost per unit
θ - percentage of defective items
$\tilde{R}_i$ - fuzzy annual demand rate
$\tilde{q}_i$ - fuzzy production quantity (fuzzy decision variable)
$\tilde{P}_i$ - fuzzy production cost
$\tilde{c}_i$ - fuzzy holding cost
$\tilde{s}_i$ - fuzzy ordering cost
$\tilde{r}_i$ - fuzzy shortage cost
$\tilde{K}_i$ - fuzzy shortage level (fuzzy decision variable)
$T\tilde{C}$ - fuzzy total annual cost

2.6.2 Assumptions

The following basic assumptions about the model are made.

- Demand rate is related to unit price as $R_i = A_i P_i^{-\gamma_i}$, where $A_i > 0$ and γ_i $(0 < \gamma_i < 1)$ are constants and real numbers related to provide the best fit of the estimated price function.
- Time horizon is finite.
- Shortages are allowed.
- Lead time crashing cost is related to the lead time by a function of the form $R(L_i) = a L_i^{-b}$, i = 1, 2, 3, …, n, a > 0, 0 < b ≤ 0.5, where a and b are real constants.
- Safety stock is considered.
- Transportation cost is not considered.
- Screening cost and reworking cost is constant.
- In screening process, if defective items are found, then the duplicate costs is paid by the purchaser.
- Holding cost (c_i), production cost (P_i), Ordering cost (s_i), and shortage cost (r_i) are taken as a trapezoidal fuzzy numbers.
- Our aim is to minimize the annual relevant total cost.

III. MODEL FORMULATION

3.1 Proposed Inventory Model in Crisp Sense

From the above notations and assumptions, we obtain the total annual cost for the proposed inventory model in crisp environment.

Then, the total annual cost is given by,

TC (q_i, K_i) = production cost + ordering cost + holding cost + shortage cost + lead time crashing cost + screening cost + reworking cost

Total annual cost is,

$$TC(q_i, K_i) = \sum_{i=1}^{n} \{ P_i R_i + \frac{s_i R_i}{q_i} + \left[\frac{(q_i - K_i)^2}{2q_i} + k\sigma\sqrt{L_i} \right] c_i + \frac{r_i K_i^2}{2q_i} + \alpha q_i + \beta\theta q_i + \frac{R_i}{q_i} R(L_i) \} \quad (1)$$

Substituting R_i and $R(L_i)$ in equation (1), yields,

$$TC(q_i, K_i) = \sum_{i=1}^{n}\{A_i P_i^{1-\gamma_i} + \frac{s_i A_i P_i^{-\gamma_i}}{q_i} + \left[\frac{(q_i - K_i)^2}{2q_i} + k\sigma\sqrt{L_i}\right] c_i + \frac{r_i K_i^2}{2q_i} + \alpha q_i + \beta\theta q_i + \frac{A_i P_i^{-\gamma_i}}{q_i} a L_i^{-b}\} \quad (2)$$

There is a limitation on the available warehouse floor space where the items are to be stored and these available resources in inventory problems that cannot be ignored to derive the optimal total cost

(i.e) $\sum_{i=1}^{n} w_i q_i \le W$

$$TC(q_i, K_i) = \sum_{i=1}^{n}\{A_i P_i^{1-\gamma_i} + \frac{s_i A_i P_i^{-\gamma_i}}{q_i} + \left[\frac{(q_i - K_i)^2}{2q_i} + k\sigma\sqrt{L_i}\right] c_i + \frac{r_i K_i^2}{2q_i} + \alpha q_i + \beta\theta q_i + \frac{A_i P_i^{-\gamma_i}}{q_i} a L_i^{-b} - \lambda(w_i q_i - W)\}$$

$$TC(q_i, K_i) = \sum_{i=1}^{n}\{A_i P_i^{1-\gamma_i} + \frac{s_i A_i P_i^{-\gamma_i}}{q_i} + \frac{q_i c_i}{2} - K_i c_i + \frac{K_i^2 c_i}{2q_i} + k\sigma\sqrt{L_i} + \frac{r_i K_i^2}{2q_i} + \alpha q_i + \beta\theta q_i + \frac{A_i P_i^{-\gamma_i}}{q_i} a L_i^{-b} - \lambda(w_i q_i - W)] \quad (3)$$

Partially differentiate equation (3) with respect to ‘q_i’, we get

$$\frac{\partial TC}{\partial q_i} = -\frac{s_i A_i P_i^{-\gamma_i}}{q_i^2} + \frac{c_i}{2} - \frac{K_i^2 c_i}{2q_i^2} - \frac{r_i K_i^2}{2q_i^2} + \alpha + \beta\theta - \frac{A_i P_i^{-\gamma_i}}{q_i^2} a L_i^{-b} - \lambda w_i \quad (4)$$

Partially differentiate equation (3) with respect to ‘K_i’, we get,

$$\frac{\partial TC}{\partial K_i} = -c_i + \frac{K_i}{q_i}(c_i + r_i) \quad (5)$$

and the constraint is,

$$\lambda(w_i q_i - W) = 0$$

$$w_i q_i - W \ne 0 \Rightarrow \lambda = 0 \quad (6)$$

Again, differentiate equation (4) w.r.to ‘q_i’, we get,

$$\frac{\partial^2 TC}{\partial q_i^2} = \frac{2 A_i P_i^{-\gamma_i}}{q_i^3}(s_i + a L_i^{-b}) + \frac{K_i^2}{q_i^3}(c_i + r_i) \quad (7)$$

Again, differentiate equation (5) w.r.to ‘K_i’, we get,

$$\frac{\partial^2 TC}{\partial K_i^2} = \frac{c_i + r_i}{q_i} > 0 \quad (8)$$

Now, set the equation (5) to zero and we compute for ‘K_i’, then,

$$K_i = \frac{q_i c_i}{c_i + r_i} \quad (9)$$

And also set the equation (4) to zero, we get,

$$\lambda = \frac{1}{w_i}\left[\frac{-A_i P_i^{-\gamma_i}}{q_i^2}\left(s_i + a L_i^{-b}\right) + \frac{c_i}{2} - \frac{K_i^2}{q_i^2}\frac{(c_i + r_i)}{2} + \alpha + \beta\theta\right] \quad (10)$$

Substituting equation (6) in (10) , we get,

$$q_i = \left[\frac{A_i P_i^{-\gamma_i} \left(s_i + aL_i^{-b}\right)}{\frac{c_i r_i}{2(c_i + r_i)} + \alpha + \beta\theta} \right]^{1/2} \qquad (11)$$

And the minimum total cost is

$$Min\ TC = \sum_{i=1}^{n} \{A_i P_i^{1-\gamma_i} + \frac{s_i A_i P_i^{-\gamma_i}}{q_i} + \left[\frac{(q_i - K_i)^2}{2q_i} + k\sigma\sqrt{L_i}\right] c_i + \frac{r_i K_i^2}{2q_i} + \alpha q_i + \beta\theta q_i + \frac{A_i P_i^{-\gamma_i}}{q_i} aL_i^{-b}\} \qquad (12)$$

3.2 Proposed Inventory Model in Fuzzy Sense

Here, we consider the proposed inventory model in fuzzy environment. Here, Holding cost (c_i), production cost (P_i), ordering cost (s_i), and shortage cost (r_i) are taken as a trapezoidal fuzzy numbers.

Let $\tilde{P}_i$ - fuzzy production cost per unit item

$\tilde{c}_i$ - fuzzy carrying or holding cost per unit quantity per unit time

$\tilde{s}_i$ - fuzzy set up or ordering cost per order

$\tilde{r}_i$ - fuzzy shortage cost or stock out cost per unit quantity per unit time

Now we fuzzify the total annual cost given in equation (1), the fuzzy total annual cost is given by,

$$T\tilde{C}(\tilde{q}_i, \tilde{K}_i) = \sum_{i=1}^{n} \{\tilde{P}_i \tilde{R}_i + \frac{\tilde{s}_i \tilde{R}_i}{\tilde{q}_i} + \left[\frac{(\tilde{q}_i - \tilde{K}_i)^2}{2\tilde{q}_i} + k\sigma\sqrt{L_i}\right] \tilde{c}_i + \frac{\tilde{r}_i \tilde{K}_i^2}{2\tilde{q}_i} + \alpha\tilde{q}_i + \beta\theta\tilde{q}_i + \frac{\tilde{R}_i}{\tilde{q}_i} R(L_i)\} \qquad (13)$$

Substituting $\tilde{R}_i$ and $R(L_i)$ in equation (13), yields,

$$T\tilde{C}(\tilde{q}_i, \tilde{K}_i) = \sum_{i=1}^{n} \{A_i \tilde{P}_i^{1-\gamma_i} + \frac{\tilde{s}_i A_i \tilde{P}_i^{-\gamma_i}}{\tilde{q}_i} + \left[\frac{(\tilde{q}_i - \tilde{K}_i)^2}{2\tilde{q}_i} + k\sigma\sqrt{L_i}\right] \tilde{c}_i + \frac{\tilde{r}_i \tilde{K}_i^2}{2\tilde{q}_i} + \alpha\tilde{q}_i + \beta\theta\tilde{q}_i + \frac{A_i \tilde{P}_i^{-\gamma_i}}{\tilde{q}_i} aL_i^{-b}\} \qquad (14)$$

There is a limitation on the available warehouse floor space where the items are to be stored and these available resources in inventory problems that cannot be ignored to derive the optimal total cost

(i.e) $\sum_{i=1}^{n} w_i q_i \leq W$

To pertain Graded Mean Representation method to defuzzify the total annual cost, optimal order quantity, demand and shortage level.

Suppose $\tilde{P} = (P_1, P_2, P_3, P_4)$, $\tilde{c} = (c_1, c_2, c_3, c_4)$, $\tilde{s} = (s_1, s_2, s_3, s_4)$ and $\tilde{r} = (r_1, r_2, r_3, r_4)$ are fuzzy trapezoidal numbers in LR form, where 0 < r < c < P < s and $r_1, r_2, r_3, r_4, c_1, c_2, c_3, c_4, P_1, P_2, P_3, P_4$ and s_1, s_2, s_3, s_4 are known positive numbers.

Then, the total annual cost is given by,

$T\tilde{C}(q_i, K_i)$ = fuzzy production cost + fuzzy ordering cost + fuzzy holding cost + fuzzy shortage cost + lead time crashing cost + screening cost + reworking cost

Total annual cost is,

$$T\widetilde{C}(\widetilde{q}_i,\widetilde{K}_i) = \sum_{i=1}^{n}\{A_i\,\widetilde{P}_i^{1-\gamma_i} + \frac{\widetilde{s}_i A_i \widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i} + \left[\frac{(\widetilde{q}_i-\widetilde{K}_i)^2}{2\widetilde{q}_i} + k\sigma\sqrt{L_i}\right]\widetilde{c}_i + \frac{\widetilde{r}_i\widetilde{K}_i^{\,2}}{2\widetilde{q}_i} + \alpha\widetilde{q}_i + \beta\theta\widetilde{q}_i + \frac{A_i\widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i}\,aL_i^{-b} - \lambda(w_i\,\widetilde{q}_i - W)\}$$

$$T\widetilde{C}(\widetilde{q}_i,\widetilde{K}_i) = \sum_{i=1}^{n}\{A_i\,\widetilde{P}_i^{1-\gamma_i} + \frac{\widetilde{s}_i A_i \widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i} + \frac{\widetilde{q}_i\widetilde{c}_i}{2} - \widetilde{K}_i\widetilde{c}_i + \frac{\widetilde{K}_i^{\,2}\widetilde{c}_i}{2\widetilde{q}_i} + k\sigma\sqrt{L_i} + \frac{\widetilde{r}_i\widetilde{K}_i^{\,2}}{2\widetilde{q}_i} + \alpha\widetilde{q}_i + \beta\theta\widetilde{q}_i + \frac{A_i\widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i}\,aL_i^{-b} - \lambda(w_i\,\widetilde{q}_i - W)\} \quad (15)$$

Partially differentiate equation (15) with respect to 'q_i', we get

$$\frac{\partial T\widetilde{C}}{\partial\widetilde{q}_i} = -\frac{\widetilde{s}_i A_i\widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i^{\,2}} + \frac{\widetilde{c}_i}{2} - \frac{\widetilde{K}_i^{\,2}\widetilde{c}_i}{2\widetilde{q}_i^{\,2}} - \frac{\widetilde{r}_i\widetilde{K}_i^{\,2}}{2\widetilde{q}_i^{\,2}} + \alpha + \beta\theta - \frac{A_i\widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i^{\,2}}\,aL_i^{-b} - \lambda w_i \quad (16)$$

Partially differentiate equation (15) with respect to 'K_i', we get,

$$\frac{\partial T\widetilde{C}}{\partial\widetilde{K}_i} = -\widetilde{c}_i + \frac{\widetilde{K}_i}{\widetilde{q}_i}(\widetilde{c}_i + \widetilde{r}_i) \quad (17)$$

and the constraint is,

$$\lambda(w_i\,\widetilde{q}_i - W) = 0$$

$$w_i\,\widetilde{q}_i - W \neq 0 \Rightarrow \lambda = 0 \quad (18)$$

Again, differentiate equation (16) w.r.to 'q_i', we get,

$$\frac{\partial^2 T\widetilde{C}}{\partial\widetilde{q}_i^{\,2}} = \frac{2A_i\widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i^{\,3}}(\widetilde{s}_i + aL_i^{-b}) + \frac{\widetilde{K}_i^{\,2}}{\widetilde{q}_i^{\,3}}(\widetilde{c}_i + \widetilde{r}_i) \quad (19)$$

Again, differentiate equation (17) w.r.to 'K_i', we get,

$$\frac{\partial^2 T\widetilde{C}}{\partial\widetilde{K}_i^{\,2}} = \frac{\widetilde{c}_i + \widetilde{r}_i}{\widetilde{q}_i} > 0 \quad (20)$$

Now, set the equation (17) to zero and we compute for 'K_i', then,

$$\widetilde{K}_i = \frac{\widetilde{q}_i\,\widetilde{c}_i}{\widetilde{c}_i + \widetilde{r}_i} \quad (21)$$

And also set the equation (16) to zero, we get,

$$\lambda = \frac{1}{w_i}\left[\frac{-A_i\,\widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i^{\,2}}(\widetilde{s}_i + a\,L_i^{-b}) + \frac{\widetilde{c}_i}{2} - \frac{\widetilde{K}_i^{\,2}}{\widetilde{q}_i^{\,2}}\frac{(\widetilde{c}_i + \widetilde{r}_i)}{2} + \alpha + \beta\theta\right] \quad (22)$$

Substituting equation (18) in (22) , we get,

$$\widetilde{q}_i = \left[\frac{A_i\widetilde{P}_i^{-\gamma_i}(\widetilde{s}_i + aL_i^{-b})}{\frac{\widetilde{c}_i\,\widetilde{r}_i}{2(\widetilde{c}_i + \widetilde{r}_i)} + \alpha + \beta\theta}\right]^{1/2} \quad (23)$$

And the minimum total cost is

$$Min\ T\widetilde{C} = \sum_{i=1}^{n}\{A_i\ \widetilde{P}_i^{1-\gamma_i} + \frac{\widetilde{s}_i A_i \widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i} + \left[\frac{\left(\widetilde{q}_i - \widetilde{K}_i\right)^2}{2\widetilde{q}_i} + k\sigma\sqrt{L_i}\right]\widetilde{c}_i + \frac{\widetilde{r}_i \widetilde{K}_i^{\ 2}}{2\widetilde{q}_i} + \alpha\widetilde{q}_i + \beta\theta\widetilde{q}_i + \frac{A_i \widetilde{P}_i^{-\gamma_i}}{\widetilde{q}_i}\ aL_i^{-b}\} \qquad (24)$$

IV. NUMERICAL EXAMPLE

4.1 Numerical Example in Crisp Sense

The production rate is 20 unit / year. Annual inventory holding cost is Rs. 16 per unit, annual inventory ordering cost is Rs.50 per unit, shortage cost is Rs.12 per unit, lead time crashing cost is Rs.0.1 per unit, standard deviation is 4 unit/year, k = 8. If there is 10 % defective items then the duplicate cost for the defective items is Rs. 1/unit and the screening cost is Rs. 2/unit. The demand, order quantity, shortage level and minimum total annual cost are to be determined.

Here the number of items is 1, A = 100, a = 1, b = 0.1, γ = 0.5

Sol:

n = 1
A = 100
P = 20 unit / year
c = Rs. 16 / unit
s = Rs. 50 / unit
r = Rs. 12 / unit
L = Rs. 0.1 / unit
σ = 4
k = 8
a = 1
b = 0.1
γ = 0.5
θ = 10 %
α = Rs. 2 / unit
β = Rs. 1 / unit

Order Quantity $$q = \left[\frac{AP^{-\gamma}\ \left(s + aL^{-b}\right)}{\frac{c\,r}{2(c+r)} + \alpha + \beta\theta}\right]^{1/2}$$

$$q = 14.40$$

Demand

$$R = AP^{-\gamma}$$

$$R = 22.36$$

Shortage Level

$$K = \frac{q\,c}{c+r}$$

$$K = 8.23$$

Total Annual Cost

$$Min\ TC = AP^{1-\gamma} + \frac{sAP^{-\gamma}}{q} + \left[\frac{(q-K)^2}{2q} + k\sigma\sqrt{L_i}\right]c + \frac{rK^2}{2q} + \alpha q + \beta\theta q + \frac{AP^{-\gamma}}{q}\ aL^{-b}$$

Min TC = Rs. 768.30

4.2 Numerical Example in Fuzzy Sense

To validate the proposed model consider the data

Let n = 1

A = 100

$\tilde{P}$ = (16, 19, 21, 24)

$\tilde{c}$ = (Rs.12, Rs.15, Rs.17, Rs.20) unit/year

$\tilde{s}$ = (Rs.46, Rs.49, Rs.51, Rs.54) unit/year

$\tilde{r}$ = (Rs.8, Rs.11, Rs.13, Rs.16) unit/year

L = Rs. 0.1 / unit

σ = 4

k = 8

a = 1

b = 0.1

γ = 0.5

θ = 10 %

α = Rs. 2 / unit

β = Rs. 1 / unit

Sol:

Fuzzy Order Quantity

$$\tilde{q} = \left[\frac{A\tilde{P}^{-\gamma}\left(\tilde{s} + aL^{-b}\right)}{\frac{\tilde{c}\,\tilde{r}}{2(\tilde{c}+\tilde{r})} + \alpha + \beta\theta} \right]^{1/2}$$

$$\tilde{q} = (10.82, 12.31, 15.40, 17.95)$$

Graded Mean Representation Method:

$$P(\tilde{q}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

= 14.03

Fuzzy Demand

$$\tilde{R} = A\tilde{P}^{-\gamma}$$

$\tilde{R}$ = (25, 22.94, 21.82, 20.41)

Graded Mean Representation Method

$$P(\tilde{R}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

= 22.49

Fuzzy Shortage Level

$$\tilde{K} = \frac{\tilde{q}\,\tilde{c}}{\tilde{c} + \tilde{r}}$$

$\tilde{K}$ = (4.68, 7.02, 9.17, 14.03)

Graded Mean Representation Method

$$P(\tilde{K}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

= 8.52

Fuzzy Total Annual Cost

$$Min\ \tilde{TC} = A\tilde{P}^{1-\gamma} + \frac{\tilde{s}A\tilde{P}^{-\gamma}}{\tilde{q}} + \left[\frac{(\tilde{q}-\tilde{K})^2}{2\tilde{q}} + k\sigma\sqrt{L_i}\right]\tilde{c} + \frac{\tilde{r}\tilde{K}^2}{2\tilde{q}} + \alpha\tilde{q} + \beta\theta\tilde{q} + \frac{A\tilde{P}^{-\gamma}}{\tilde{q}}\ aL^{-b}$$

$Min\tilde{TC}$ = (Rs. 668.66, Rs.743.95, Rs.792.60, Rs.865.04)

Graded Mean Representation Method

$$P(\tilde{TC}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

= Rs. 767.80

V. Sensitivity Analysis

Table:1 Optimal solution table for varying γ and a = 1, b = 0.1

S.No	γ	a	b	$\tilde{q}$	$\tilde{R}$	$\tilde{K}$	$Min\tilde{TC}$
1	0.46	1	0.1	14.91	25.34	9.05	834.65
2	0.48	1	0.1	14.46	23.87	8.77	800.43
3	0.5	1	0.1	14.03	22.49	8.52	767.80
4	0.52	1	0.1	13.62	21.19	8.27	737.36
5	0.54	1	0.1	13.22	19.96	8.06	708.31
6	0.56	1	0.1	12.83	18.81	7.79	680.92
7	0.58	1	0.1	12.44	17.72	7.55	654.92

Table: 2 Optimal solution table for varying a and γ = 0.5, b = 0.1

S.No	γ	a	b	$\tilde{q}$	$\tilde{R}$	$\tilde{K}$	$Min\tilde{TC}$
1	0.50	1	0.1	14.03	22.49	8.52	767.80
2	0.50	2	0.1	14.21	22.49	8.63	770.02
3	0.50	3	0.1	14.37	22.49	8.72	771.87
4	0.50	4	0.1	14.54	22.49	8.83	773.77
5	0.50	5	0.1	14.71	22.49	8.93	775.74

Table: 3 Optimal solution table for varying b and γ = 0.5, a = 1

S.No	γ	a	b	$\tilde{q}$	$\tilde{R}$	$\tilde{K}$	$Min\tilde{TC}$
1	0.5	1	0.1	14.03	22.49	9.52	767.80
2	0.5	1	0.2	14.08	22.49	8.55	768.55
3	0.5	1	0.3	14.14	22.49	8.58	769.08
4	0.5	1	0.4	14.21	22.49	8.63	770
5	0.5	1	0.5	14.29	22.49	8.67	771

Table: 4 Optimal solution table for varying γ, a and b

S.No	γ	a	b	$\tilde{q}$	$\tilde{R}$	$\tilde{K}$	$Min\tilde{TC}$
1	0.46	1	0.1	14.91	25.34	9.05	834.65
2	0.48	2	0.2	14.73	23.87	8.94	798.23
3	0.50	3	0.3	14.66	22.49	8.90	775.23
4	0.52	4	0.4	14.74	21.19	8.95	750.18
5	0.54	5	0.5	14.96	19.96	8.08	728.50

VI. CONCLUSION

Using Karush – Kuhn Tucker conditions we developed a total relevant annual inventory cost with backorder in the crisp sense as well as in the fuzzy sense. Annual production cost, annual inventory holding cost, annual inventory ordering cost and annual shortage cost are taken as a fuzzy numbers. Finally, the proposed model has been verified by the numerical example along with the sensitivity analysis. Here we are given the sensitivity analysis for getting the optimal solution for varying γ , a and b. From the above table, it implies that the dealers will varying γ only we get the minimum total annual relevant cost. In the future study, we acquire the decision variables for all provisions in this proposed model.

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