AFFINE CONTROL SYSTEMS ON NON-COMPACT LIE GROUP

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Abstract:

In this paper we deal with affine control systems on a non-compact Lie group cx+e group. First we study topological properties of the state space Ef(1) and the automorphism orbit of Ef(1). Affine control system, non-compact Lie group state space Ef(1). Affine control systems on the generalized Heisenberg Lie groups are studied. Affine algebra, automorphism.

Introduction:

The purpose of this paper affine control systems on some specific lie group is called cx+e group by relating to associated bilinear parts.

Related to the affine control system on lie groups, in Ef(1). The authors Ayala and San Martin have the subalgebra of the Lie algebra Ef(G) generated by the vector fields of a linear control system the drift vector field X is an infinitesimal automorphism i.e.,$(X_K)_{K\in M}$ is a one-parameter subgroup of Aut(G); have lifted the system itself to a right-invariant control system on Lie group Ef(1) for compact connected and non-compact semi-simple Lie group.

The affine control systems on a non-compact Lie group cx+e group have been investigated and given characterization.

1. Affine Control Systems On Lie Groups

If G is a connected Lie group with Lie algebra L(G), the affine group Ef(G) of G is the semi-direct product of Aut(G) with G itself i.e.,Ef(G) = Aut(G)× G. The group operation of Ef(G) .

The identity element of Aut (G) and e denotes the neutral element of G, then the group identity of Ef(G) is $(1, a)$ and $(\Phi^{-1}, \Phi^{-1}(h^{-1}))$ In the invers of $(\Phi, h) \in$ Ef(G). Hence, h $\rightarrow (1, h)$ and $\Phi \rightarrow (\Phi, a)$ embed G into Ef(G) and Aut(G) into Af(G) respectively. Therefore, G and Aut(G) are subgroups of Ef(G). The natural transitive action

$$\mathrm{Ef}(G) \times G \rightarrow G$$

$$(\Phi, h_1).h_2 \rightarrow h_1\Phi(h_2)$$

Where $(\Phi, h_1) \in$ Ef(G) and $h_2 \in$ G.

"Affine in the control" is used to describe class system.

$\frac{dx}{dt}$ = n(x) +h(x)v is considered affine control.

Theorem:1

Let $\sum = (Ef(1), D)$ be an affine control system. Then, the state space Ef(1) is a locally compact Hausdorff space.

Proof:

Ef(1) is a Hausdorff space is a lie group. The compactness for a given x ∈ Ef(1) and neighborhood Z of x, the existence of some neighborhood Z of x such that. The topology on Ef(1) half plane is homomorphic to the standard topology of M^2.

Therefore, ∀ x ∈ Ef(1), the neighborhood Z of x is homeomorphic to an open ball.For each neighborhood Z of x, there is neighborhood W of x such x ∈ W. Since W is also homeomorphic to an open ball the closure of U is a closed ball.

Theorem:2

The automorphism orbit of the state space Ef(1) is dense.

Proof:

The set

J = exp (cf(1) – [cf(1), cf(1)])

Aut(Ef(1))-orbit of Ef(1). The exponential mappingfrom the tangent plane to the surface of diffeomorphism. Then two elements h_1,h_2 ∈ J the line segment $h_1 h_2$ which is parallel to [Ef(1), Ef(1)],

$$\Phi : J \rightarrow J$$

Defined by

$$h_1 \rightarrow k_1 h_1 + k_2 = J, k_{1,} k_2 \in M$$

Also it is possible to connect those segments with the perpendicular segments .Aut(Ef(1)) orbits open the center[Ef(1), Ef(1)] for any element x ∈ [Ef(1), Ef(1)] and every neighborhood Q (x, γ) of x have some element of Ef(1) different then x.

Ef(1) – [Ef(1), Ef(1)] = Ef(1).

Theorem:3

The affine control system Σ_c on the state space Ef(1) is not have any equilibrium point and the associated bilinear system

Σ_c = (Ef(1), D_e) is control on the Aut (Ef(1)) orbit.

Proof:

For the control not having equilibrium point is necessary. Now consider the associated bilinear system

Σ_e= (Ef(1), D_e is control on the Aut(Ef(1)) orbit.

Φ_δ:∂L(G) × L(G)→ ∂L(G) × L(G)

Φ_δ = Id × $\frac{1}{\delta}$ ∀D + X ∈ cf(1) = $\partial L(G) \times L(G)$,we have

Φ_δ(D+X) = D $\frac{1}{\delta}$ X.

Since complete under the small permutations sufficiently large δ, $\Phi_\delta(\Sigma_c)$ is control on S(1_e,1) – [Ef(1), Ef(1)]. Therefore, since normally control finite system are open on S(1_e,1). The system$\Phi_\delta(\Sigma_c)$ is also control on B(1_e,1) – [Ef(1), Ef(1)]. Since the state space is connected, the affine system Σ_c is control on Ef(1).

Lemma :1

For the generalized Heisenberg lie group H =: H(W, X, ∝), the map φ_δ = $\sqrt{\delta}$Id × δId, i.e., Φ_δ(w,g) = ($\sqrt{\delta}$w,δg) is an automorphism.

Proof:

The mapping Φ_δ is 1-1 and onto its image.

$\Phi_\delta((w_1,g_1) * (w_{2,}g_2)) = \Phi_\delta(w_1 + w_2,\, g_1 + g_2 + \frac{1}{2}\alpha(w_1,w_2))$

= ($\sqrt{\delta}$Idw_1+ $\sqrt{\delta}$Idw_2,δIdg_1 + δIdg_2+$\frac{\delta Id}{2}$ $\alpha(w_1, w_2)$)

by bilinearity of α

($\sqrt{\delta}Id w_1$+$\sqrt{\delta}$Idw_2,$\delta Id g_1$+δIdg_2+$\frac{1}{2}\alpha(\sqrt{\delta}w_1,\sqrt{\delta}w_2)$)

= ($\sqrt{\delta}$Idw_1,δIdg_1) * ($\sqrt{\delta}$Idw_2,δIdg_2)

= $\Phi_\delta(w_1,g_1) * \Phi_\delta(w_2, g_2)$.

This proves that Φ_δ is an automorphism.

Lemma:2

Let H be a generalized Heisenberg Lie group. Then there exist a dense Aut(H)-orbit.

Proof:

The set φ = : exp (L(H)-[L(H), L(H)]) = H-[H, H]

Is an Aut(H)-orbit of H. The exponential map is a global diffeomorphism for simply connected nilpotent Lie groups. Two elements X,Y ∈ φ the line segment mod XY parallel to [H,H], can be connected via a line segment by taking once X as a initial point so that the function that connection $f_s : \varphi \to \varphi$ defined by X → k_1X + k_2 = Y, where k_1,k_2 ∈ IM, is an automorphism. Actually it is possible to connect these segments with the perpendicular segments to each oyher via the same way. That Aut(H)-orbit of H is φ is open. In fact, if dimZ = 1 the center [H,H] forms a line for any Heisenberg group [X,Y] = G, X, Y,G ∈ L(H). For the density, any x ∈ [H,H] every ball B(x, γ)

$$B(x, \gamma) \cap H - [H, H] \neq \emptyset.$$

Thus, H – [H, H] = H.

Theorem:4

Let G be a non-compact connected Lie group and L(G) be its Lie algebra. Then, compact subsets of G are not G_Σ-invariant, if the control system on G is an invariant system.

Proof:

For ∀x ∈ G, ∀X ∈ L(G) and ∀k ∈ IM, the differentiable curve ρx(;x) : (c,e) ⊂ IM → G is defined ρx(k,x) = X_K(x). Assume that F ⊂ G is a compact and G_Σ- invariant subset. Each vector field X ∈ L(G) is complete. Consider any open covering

E = {V_i\ i ∈ / Z^+}. Therefore, $\forall_i$ γx(k, V_i) is an open covering of K, since X_k(x), $\forall x \in K$. K is compact, therefore it can be covered by a finite subfamily of A_δ = {δx(k,V_i)| i ∈ /Z^+}. Then, inverse images of the elements of A_δ covers IM, which is a contradiction.

References

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