ON SUPRA g*bω - CONTINUOUS MAPS IN SUPRA TOPOLOGICAL SPACES

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ABSTRACT :

In this paper, we introduce the concepts of supra g*bω - continuous maps and study their basic properties in supra topological spaces.

Keywords : supra topological spaces, supra g*b ω - continuous maps, supra g*bω - irresolute maps.

1 INTRODUCTION

Levine [1] introduced the concept of generalized closed sets (briefly g - closed) in topological spaces. In 1983, A.S.Mashhour et al [2] introduced the notion of supra topological spaces and studied S-S continuous functions and S* - continuous functions. In 2010, O.R.Sayed and Takashi Noiri [4] introduced supra b - open sets and supra b - continuity on topological spaces. In 2018, P.Priyadharsini et al. [3] introduced supra g*b ω - closed sets in supra topological spaces.

In this paper, we introduce the concepts of supra g*bω - continuous maps in supra topological spaces.

2 PRELIMINARIES

Definition 2.1 [2, 4] A subfamily μ of X is said to be a ***supra topology*** on X if

(i) $X, \varphi \in \mu$
(ii) if $A_i \in \mu$ for all i, then $\cup A_i \in \mu$

The pair (X, μ) is called ***supra topological space***. The elements of μ are called *supra open* sets in (X, μ) and complement of a supra open set is called a *supra closed* set.

Definition 2.2 [2] The *supra closure* of a set A is defined as $cl^{\mu}(A) = \cap\{B : B$ is supra closed and $A \subseteq B\}$ and the *supra interior* of a set A is defined as $int^{\mu}(A) = \cup\{ B : B$ is supra open and $A \supseteq B\}$.

Throughout this paper we shall denote by (X, μ) a supra topological space. For any subset A $\subseteq$ X, $int^{\mu}(A)$ and $cl^{\mu}(A)$ denote the supra interior of A and the supra closure of A with respect to μ.

We shall require the following known definitions:

Definition 2.3 [2] Let (X, μ) be a supra topological spaces. A subset A of X is called

- ***supra semi - open*** if $A \subseteq cl^{\mu}(int^{\mu}(A))$ and ***supra semi - closed*** if $int^{\mu}(cl^{\mu}(A)) \subseteq A$
- ***supra pre open*** if $A \subseteq int^{\mu}(cl^{\mu}(A))$ and ***supra pre closed*** if $cl^{\mu}(int^{\mu}(A)) \subseteq A$
- ***supra α - open*** if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$ and ***supra α - closed*** if $cl^{\mu}(int^{\mu}(cl^{\mu}(A))) \subseteq A$
- ***supra regular open*** if $A = int^{\mu}(cl^{\mu}(A))$ and ***supra regular closed*** if $A = cl^{\mu}(int^{\mu}(A))$
- ***supra b - open*** if $A \subseteq cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$ and ***supra b - closed*** if $cl^{\mu}(int^{\mu}(A)) \cap int^{\mu}(cl^{\mu}(A)) \subseteq A$.

Let (X, μ) or simply X denote a supra topological space. For any subset A $\subseteq$ X, the intersection of all supra b - closed sets containing A is called the *supra b - closure* of A, denoted by $bcl^{\mu}(A)$. The union of all b-open sets contained in A is called the *supra b - interior* of A, denoted by $bint^{\mu}(A)$.

Definition 2.4: [2] Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subseteq \mu$.

Definition 2.5 [2] A set A of a supra topological space (X, μ) is called ***supra generalized semi closed*** (briefly gs^{μ} - *closed*) if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ).

Definition 2.6 [3] A set A of a supra topological space (X, μ) is called ***supra generalized star b omega closed*** (briefly, g*bω^μ - closed) if bcl^μ(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra gs - open in (X, μ).

3 g*bω^μ- CONTINUOUS MAPS

Definition 3.1 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A map f : (X, τ) → (Y, σ) is supra **generalized star b omega - continuous** (briefly, g*bω^μ - continuous) if f^{-1}(U) is g*bω^μ- closed in (X, μ) for each closed set U in (Y, σ).

Theorem 3.2 Every continuous map is g*bω^μ - continuous.

Proof: Let f : (X, τ) → (Y, σ) be continuous. Let U be a closed set in (Y, σ). Then f^{-1}(U) is closed in (X, τ). Since μ is associated with supra topology τ then $\tau \subseteq \mu$. Therefore f^{-1}(U) is closed in (X, μ) and it is g*bω^μ- closed, f is g*bω^μ- continuous.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.3 Let X = Y = {a, b, c} with the topologies τ = {φ, X, {a}, {a, b}} and μ = {φ, Y, {a}, {a, b}, {a, c}}. Let f : (X, τ) → (Y, σ) be a map defined by f(a) = c, f(b) = a, f(c) = b. Then f is g*bω^μ - continuous but not continuous, since {c} is g*bω^μ - closed but f^{-1}({c}) = {a} is not closed.

Theorem 3.4 Every supra continuous map is g*bω^μ - continuous.

Proof: Obvious.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.5 In example 3.3, f is g*bω^μ - continuous but not supra continuous, since {c} is g*bω^μ - closed but f^{-1}({c}) = {a} is not supra closed.

Theorem 3.6 The following are equivalent for a map f : (X, τ) → (Y, σ) and μ be an associated supra topology with τ.

(a) f is g*bω^μ- continuous
(b) f^{-1}(A) is g*bω^μ - open for each open set A in (Y, σ).

Theorem 3.7 If f : (X, τ) → (Y, σ) is g*bω^μ- continuous then f(g*bω^μcl(A)) ⊂ cl(f(A)) for every subset A of (X, τ).

Proof: Since A ⊂ f^{-1}f(A), we have A ⊂ f^{-1}(cl(f(A))). Now cl(f(A)) is a closed set in (Y, σ) and hence f^{-1}(cl(f(A))) is a g*b ω^μ - closed set containing A. Consequently, g*b ω^μ cl(A) ⊂ f^{-1}(cl(f(A))). Therefore f(g*bω^μcl(A)) ⊂ ff^{-1}(cl(f(A))) ⊂ cl(f(A)).

Theorem 3.8 The following are equivalent for a map f : (X, τ) → (Y, σ)

(a). For every subset A of (X, τ), f(g*bω^μcl(A)) ⊂ cl(f(A)).

(b). For every subset B of (Y, σ), g*b ω^μ cl(f^{-1}(B)) ⊂ f^{-1}(cl(B)).

Proof: Suppose that (a) holds and let B be any subset of Y. Replacing A by f^{-1}(B) we get from (a) f(g*bω^μcl(f^{-1}(B))) ⊂ cl(ff^{-1}(B)) ⊂ cl(B). Hence g*bω^μcl(f^{-1}(B)) ⊂ f^{-1}(cl(B)).

Conversely, suppose that (b) holds and let B = f(A) where A is a subset of X. Then g*bω^μcl(A) ⊂ g*bω^μcl(f^{-1}(B)) ⊂ f^{-1}(cl(B)). Therefore f(g*bω^μcl(A)) ⊂ cl(B) = cl (f(A)).

Definition 3.9 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A map f : (X, τ) → (Y, σ) is called **g*bω^μ- irresolute** if f^{-1}(U) is g*bω^μ- closed in (X, μ) for every g*bω^μ- closed set U in (Y, σ).

Theorem 3.10 A map f : (X, τ) → (Y, σ) is g*bω^μ- irresolute if and only if f^{-1}(V) is g*b ω^μ - open in (X, μ) for every g*bω^μ - open set V in (Y, σ).

Theorem 3.11 Every g*bω^μ- irresolute map is g*bω^μ- continuous.

Proof: Let U be any closed set in (Y, σ). Then U is a g*bω^μ- closed set in (Y, σ). Since f is g*bω^μ- irresolute, f^{-1}(U) is g*bω^μ - closed in (X, τ). Since μ is associated with supra topology τ then $\tau \subseteq \mu$. Therefore f^{-1}(U) is g*bω^μ- closed in (X, μ). Therefore f is g*bω^μ- continuous.

Definition 3.12: A supra topological space (X, μ) is called $_{g*b}\omega^\mu T_c$ ***- space*** if every g*bω^μ- closed subset of (X, μ) is closed in (X, μ).

Theorem 3.13 Let f : (X, τ) → (Y, σ) be a g*bω^μ- continuous map where (X, μ) is a $_{g*b}\omega^\mu T_c$ - space. Then f is continuous.

Proof: Let U be any closed set in (Y, σ). Since f : (X, τ) → (Y, σ) is g*bω^{μ}- continuous, f^{-1}(U) is g*bω^{μ}- closed in (X, τ). Since μ is associated with supra topology τ then $\tau \subseteq \mu$. Therefore f^{-1}(U) is g*bω^{μ}- closed in (X, μ). Since (X, μ) is a $_{g^*b}\omega^{\mu} T_c$ - space, f^{-1}(U) is closed in (X, μ). Hence f is continuous.

Theorem 3.14 If (X, μ) is a $_{g^*b}\omega^{\mu} T_c$ - space then g*bω^{μ}cl(B) = cl(B) for each subset B of (X, μ).

Proof: Since (X, μ) is a $_{g^*b}\omega^{\mu} T_c$ - space, every g*bω^{μ}- closed set is closed. Since every closed set is g*bω^{μ}- closed set in (X, μ), G*bω^{μ}C(X, μ) = C(X, μ). Hence g*bω^{μ}cl(B) = cl(B) for each subset B of (X, μ).

Theorem 3.15 If (X, μ) is a $_{g^*b}\omega^{\mu} T_c$ - space then for each x ∈ X either {x} is supra gs - closed or supra open.

Proof: Let x ∈ X and suppose {x} is not supra gs - closed in (X, τ). Then X \ {x} is not supra gs - open. Hence X is the only supra gs - open set containing X \ {x}. This implies that X \ {x} is a g*bω^{μ}- closed set of (X, τ). Since (X, μ) is a $_{g^*b}\omega^{\mu} T_c$ - space, X \ {x} is a closed set in (X, μ) or equivalently {x} is open in (X, μ).

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