SOME CONTRA gsp-CLOSED FUNCTIONS VIA SEMIOPEN SETS IN TOPOLOGY

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Abstract :

In 1986, D. Andrejevic had defined and studied the concepts of semipre-open sets and semipre-closed sets in topology. In 2002, Navalagi has defined and studied the classes of various continuous functions, open functions and closed functions in between topological spaces using semipre-open sets and semipre-closed sets. In this paper, some new classes of functions called p-semipreclosed functions and contra-p-semipreopen functions using semipreopen sets, preopen sets. Also, We study some of their basic properties.

Keywords and Phrases :

Preopen sets, semipre-open sets, preopen functions, pre-semiopen functions, p-semipreopen functions, preclosed sets, semipre-closed sets, preclosed functions, pre- semiclosed functions, p-semipreclosed functions.

INTRODUCTION

Andrijevic introduced the class of semipre-open and semipre-closed sets in topological spaces. Since then many authors including Andrijevic have studied these sets by defining their neighbourhoods, separation axioms and functions. The purpose of this paper is to study some more properties of semipre-open functions and pre-semipre-open functions due to Navalagi and introduce a new classes of functions called p-semipre-open functions and contra-p-semipreopen functions.

PRELIMINARIES

Throughout the present paper, the sets X, Y, Z always means topological spaces on which no separation axioms are assumed unless explicity stated and $f: X \to Y$ represents a single valued function. Let A be a subset of a space X.

The closure and the interior of $A \subset X$ are denoted by $Cl(A)$ and $Int(A)$, respectively.

The following definitions and results are useful in the sequel.

Definition 2.1

The subset A of X is said to be

i. A semi-open set, if $A \subset Cl\big(Int(A)\big)$.
ii. A pre-open set, if $A \subset Int\big(Cl(A)\big)$.
iii. A semi-pre-open set, if $A \subset Cl\Big(Int\big(Cl(A)\big)\Big)$.

The complement of a preopen sets is called preclosed sets of space X. And also,The complement of a semipre-open sets is called semipreclosed sets of a space X.

The family of all preopen sets of a space X is denoted by $PO(X)$.And also, The family of all semipre-open sets of a space X is denoted by $SPO(X)$.

The preclosed sets of a space X is denoted by $PF(X)$. And also,The semipreclosed sets of a space X is denoted by $SPF(X)$.

Definition 2.2 :

The pre-closure of a set A of space X is the intersection of all pre-closed sets that contain A and is denoted by $pCl(A)$.

The semi-pre-closure of a set A of space X is the intersection of all semi-pre-closed sets that contain A and is denoted by $spCl(A)$.

Definition 2.3 :

The union of all preopen sets which are contained in A is called the pre-interior of A and is denoted by $pInt(A)$.

Definition 2.4 :

A function $f: X \to Y$ is called semi-precontinuous if the inverse image of every open set in Y is semipre-open set in X.

Definition 2.5:

A function $f: X \to Y$ is called semipre-open in sense of Cammaroto and Noiri, if image of each semiopen set in X is a semipre-open in Y.

Definition 2.6 :

A function $f: X \to Y$ is called semipre-open, if image of each open set in X is a semipre-open in Y.

Definition 2.7 :

A function $f: X \to Y$ is called pre-closed. If for every closed set B of X,$f(B)$ pre-closed in Y.

Definition 2.8 :

A function $f: X \to Y$ is called contra-semiclosed. If for every closed set B of X, $f(B)$ is semiopen in Y.

Definition 2.9 :

A function $f: X \to Y$ is called pre-semiopen. If $f(A) \in SO(Y)$ for every $A \in SO(X)$.

Definition 2.10:

A function $f: X \to Y$ is called pre-semi-pre-closed. If the image of each semipre-closed set in X is a semipre-closed set in Y.

Definition 2.11 :

A function $f: X \to Y$ is called N-preclosed if the image of each pre-closed set of X is pre-closed in Y.

CONTRA gsp-CLOSED FUNCTIONS

Definition 3.1 :

A function $f: (X, \sigma) \to (Y, \tau)$ is said to be generalized semipre-closed if for each closed set F of X, $f(F)$ is gsp-closed in Y. Clearly, every semipre-closed function is gsp-closed function.

Definition 3.2 :

A function $f: (X, \sigma) \to (Y, \tau)$ is said to be pre-generalized semipre-closed if for each semipre-closed set U of $X, f(U)$ is gsp-closed in Y.

It is clear that the both semipre-closedness and pre-generalized semipre-closedness imply gsp-closedness.

Some more properties of gsp-closed and pgsp-closed functions. We prove in the following.

Theorem 3.3 :

A surjective function $f: (X, \sigma) \to (Y, \tau)$ is pre-generalized semipre-closed if and only if for each subset B of Y and each semipre-open F of X containing $f^{-1}(B)$, there exists gsp-open set H of Y such that $B \subset H$ and $f^{-1}(H) \subset F$.

Proof :

Necessary part :

Suppose f is pre-generalized semipre-closed. Let B be any subset of Y and F is semipre-open set of X containing $f^{-1}(B)$. Put $H = Y - f(X - F)$. Then H is gsp-open in Y, $B \subset H$ and $f^{-1}(H) \subset U$.

Sufficient part :

Let U be any semipre-closed set in X. Put $B = Y - f(U)$, then we have $f^{-1}(B) \subset X - U$ and $X - U$ semi-preopen set of X. There exists pgsp-open set H of Y such that $B = Y - f(U) \subset H$ and $f^{-1}(H) \subset X - U$. Therefore, we obtain $f(U) = Y - H$. Hence, $f(U)$ is pre-generalized semipre-closed in Y. This shows f is pre-generalized semipre-closed.

Theorem 3.4 :

If $f: X \to Y$ is continuous, pre-generalized semipre-closed and A is generalized semipre-closed set in X, then $f(A)$ is generalized semipre-closed in Y.

Proof :

Let F be any open set of Y contained in $f(A)$. Then,
$A \supset f^{-1}(F)$ and $f^{-1}(F)$ is open in X. Since f is continuous function. Since A is gs-closed in

$X, f^{-1}(F)\ spCl(A)$. Hence, $F \subset f\big(spCl(A)\big) \subset f(A)$. Since f is pgsp-closed and $spCl(A)$ is semiclosed in $X, f(spCl(A))$ is gs-closed in Y and hence $F \subset spCl\Big(f\big(sCl(A)\big)\Big) \subset spCl\big(f(A)\big)$. This shows $f(A)$ is gsp-closed in Y.

Definition 3.5 :

A function $f(X,\sigma) \to (Y,\tau)$ is called contra generalized semipre-closed. If the image of each closed set U of $X, f(U)$ is generalized semipre-open in Y.

Clearly, every contra semipre-closed function is contra generalized semipre-closed.

Definition 3.6 :

A function $f(X,\sigma) \to (Y,\tau)$ is called contra pre-semipre-closed. If the image of each semipre-closed set U of $X, f(U)$ is semipre-open in Y.

Definition 3.7 :

A function $f(X,\sigma) \to (Y,\tau)$ is called contra pre-generalized semipre-closed. If the image of each semipre-closed set F of $X, f(F)$ is generalized semipre-open in Y.Clearly, every contra pre-semipre-closed function is contra pre-generalized semipre-closed.Now, we can see some decompositions of contra generalized semipre-open functions and contra pre-generalized semipre-open functions in the following.

Theorem 3.8 :

A function $f: X \to Y$ is semipre-closed and $g: Y \to Z$ is pre-generalized semipre-closed function, then the $gof: X \to Z$ is generalized semipre-closed.

Proof :

Let U be an closed set in X then $f(U)$ is semipre-closed in Y. Since, f is semipre-closed function. As $f(U)$ is semipre-closed set in Y and g is a pre-generalized semipre-closed function then $g(f(U))$ is generalized semipre-closed in Z. The composition function shows that $gof: X \to Z$ is generalized semipre-closed.

Theorem 3.9 :

A function $f: X \to Y$ is pre-semipre-closed and $g: Y \to Z$ is pre-generalized semipre-closed function, then the $gof: X \to Z$ is pre- generalized semipre-closed.

Proof :

Let U be an closed set in X then $f(U)$ is pre-semipre-closed in Y. Since f is pre-semipre-closed function. As $f(U)$ is pre-semipre-closed set in Y and g is a pre-generalized semipre-closed function then $g(f(U))$ is gsp-closed in Z. The composition function shows that $gof: X \to Z$ is pre-generalized semipre-closed.

Definition 3.10 :

A function $f: X \to Y$ is said to be pre-generalized semipre-open (in brief, pgsp-open) if the image of each semipre-open set of X is gsp-open in Y.

Definition 3.11 :

A function $f: X \to Y$ is said to be always generalized semipre-open (in brief, always gsp-open) if the image of each gsp-open set of X is gsp-open in Y.We discuss about some decompositions of contra gsp-closed and pgsp-closed functions in the following.

Theorem 3.12 :

Let $f: (X,\sigma) \to (Y,\tau)$ and $g: (Y,\gamma) \to (Z,\sigma)$ be two functions. Then, if f is contra-gsp-closed and g is always gsp-open function, the composition gof is contra-gsp-closed.

Proof :

Obvious.

Theorem 3.13 :

Let $f: (X,\sigma) \to (Y,\tau)$ and $g: (Y,\gamma) \to (Z,\sigma)$ be two functions. If for a semipre-closed function f and pgsp-closed function g, then their composition gof is gsp-closed.

Proof :

Let F be any closed set in X. Since f is semipre-closed, then $f(F)$ is semipre-closed set in Y. As g is pgsp-closed function, $g\big(f(U)\big) = gof(U)$ is gsp-closed set in Z. This shows that gof is gsp-closed.

Theorem 3.14 :

Let $f: (X,\sigma) \to (Y,\tau)$ and $g: (Y,\gamma) \to (Z,\sigma)$ be two functions. If f is pre-semipre-closed and g is contra pgsp-closed, then their composition gof is contra pgsp-closed.

Theorem 3.15 :

Let $f: (X,\sigma) \to (Y,\tau)$ and $g: (Y,\gamma) \to (Z,\sigma)$ be two functions. If f is closed and g is contra-gsp-closed,then the composition gof is contra gsp-closed.

Theorem 3.16 :

Let $f:(X,\sigma) \to (Y,\tau)$ and $g:(Y,\gamma) \to (Z,\sigma)$ be two functions. If f is contra-gsp-closed and g is always gsp-open, then the composition gof is contra gsp-closed.

Proof :

The proof of the above three theorems are easy. Hence, The above three theorems proof are omitted.

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