μ-β-generalized α-continuous mappings in generalized topological spaces

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Abstract: In this paper, we have introducedμ-β-generalized α-continuous mapsand also introduced almost μ-β-generalized α-continuous mapsin generalized topological spaces by using μ-β-generalized α-closed sets (briefly μ-βGαCS). Also we have introduced some of their basic properties.

Keywords: Generalized topology, generalized topological spaces, μ-α-closed sets, μ-β-generalizedα-closed sets, μ-α-continuous, μ-β-generalized α-continuous,almost μ-α-continuous, almost μ-β-generalized α-continuous.

1. Introduction

In 1970, Levin [4] introduced the idea of continuous function. He also introduced the concepts of semi-open sets and semi-continuity [3] in a topological space. Mashhour [5] introduced and studied α-continuous function in topological spaces. The notation of μ-β-generalized α-closed sets (briefly μ-βGαCS) was defined and investigated by Kowsalya. M and Jayanthi. D [2]. In this paper, we have introduced μ-β-generalized α-continuous mapsand also introduced almost μ-β-generalized α-continuous mapsin generalized topological spaces. Also we have investigated some of their basic properties and produced many interesting theorems.

2. Preliminaries

Let us recall the following definitions which are used in sequel.

Definition 2.1: [1] Let X be a nonempty set. A collection μ of subsets of X is a generalized topology (or briefly GT) on X if it satisfies the following:

(1) Ø, X∈ μ and

(2) If $\{M_i : i\in I\} \subseteq \mu$, then $\cup_{i\in I} M_i \in \mu$.

If μ is a GT on X, then (X, μ) is called a generalized topological space(or briefly GTS) and the elements of μ are called μ-open sets and their complement are called μ-closed sets.

Definition 2.2: [1] Let (X, μ) be a GTS and let A⊆ X. Then the μ-closure of A, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A.

Definition 2.3: [1] Let (X, μ) be a GTS and let A⊆ X. Then the μ-interior of A, denoted by $i_\mu(A)$, is the union of all μ-open sets contained in A.

Definition 2.4: [1] Let (X, μ) be a GTS. A subset A of X is said to be

i. μ-semi-closed set if $i_\mu(c_\mu(A)) \subseteq A$
ii. μ-pre-closed set if $c_\mu(i_\mu(A)) \subseteq A$
iii. μ-α-closed set if $c_\mu(i_\mu(c_\mu(A))) \subseteq A$
iv. μ-β-closed set if $i_\mu(c_\mu(i_\mu(A))) \subseteq A$
v. μ-regular-closed set if $A = c_\mu(i_\mu(A))$

Definition2.5:[7] Let (X, μ_1) and (Y, μ_2) be GTSs. Then a mapping f: (X, μ_1) → (Y, μ_2) is called

i. μ-Continuous mappingif $f^{-1}(A)$ is μ-closed in (X, μ_1) for each μ-closed in (Y, μ_2).
ii. μ-Semi-continuous mapping if $f^{-1}(A)$ is μ-semi-closed in (X, μ_1) for every μ-closed in (Y, μ_2).
iii. μ-pre-continuous mapping if $f^{-1}(A)$ is μ-pre-closed in (X, μ_1) for every μ-closed in (Y, μ_2).
iv. μ-α-continuous mapping if $f^{-1}(A)$ is μ-α-closed in (X, μ_1) for every μ-closed in (Y, μ_2).
v. μ-β-continuous mappingif $f^{-1}(A)$ is μ-β-closed in (X, μ_1) for every μ-closed in (Y, μ_2).

Definition 2.6:[6]Let (X, μ_1) and (Y, μ_2) be GTSs. Then a mapping f: (X, μ_1) → (Y, μ_2) is called

i. almost μ-Continuous mapping if $f^{-1}(A)$ is μ-closed in (X, μ_1) for every μ-regular closed set A of (Y, μ_2).
ii. almost μ-semi continuous mappings if $f^{-1}(A)$ is μ-semi closed in (X, μ_1) for every μ-regular closed set A of (Y, μ_2).
iii. almost μ-pre-continuous mappings if $f^{-1}(A)$ is μ-pre closed in (X, μ_1) for every μ-regular closed set A of (Y, μ_2).
iv. almost μ-α-continuous mapping if $f^{-1}(A)$ is μ-α-closed in (X, μ_1) for every μ-regular closed set A of (Y, μ_2).
v. almost μ-β-continuous mapping if $f^{-1}(A)$ is μ-β-closed in (X, μ_1) for every μ-regular closed set A of (Y, μ_2).

3. μ-β-generalized α-continuous mappings in topological spaces

In this section we have introduced μ-β-generalized α-continuous mappings in generalized topological spaces and discussed some of their basic properties.

Definition 3.1: A mapping f: (X, μ_1) → (Y, μ_2) is called a μ-β-generalized α-continuous (brieflyμ-βGα-continuous) if f^{-1} (A)is a μ-β-generalized α-closed set (briefly μ-βGαCS) in (X, μ_1) for each μ-closed set A in (Y, μ_2).

Example 3.2: Let X = Y = {a, b, c} with μ_1= {Ø, {a}, {b}, {a, b}, X} and μ_2 = {Ø, {a, b}, Y}. Let f: (X, μ_1) → (Y, μ_2) be a mapping defined by f(a) = a, f(b) = b, f(c) = c. Now, μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Let A = {c}, then A is a μ-closed set in (Y, μ_2). Then f^{-1}({c}) is a μ-βGαCS in (X, μ). Hence f is a μ-βGα-continuous mapping.

Theorem 3.3: Every μ-continuous mapping is a μ-βGα-continuous mapping but not conversely in general.

Proof: Let f: (X, μ_1) → (Y, μ_2) be a μ-continuous mapping. Let A be μ-closed set in (Y, μ_2). Since f is a μ-continuous mapping, f^{-1} (A) is a μ-closed set in (X, μ_1). Since every μ-closed set is a μ-βGαCS, f^{-1} (A) is a μ-βGαCS in (X, μ_1). Hence f is a μ-βGα-continuous mapping.

Example 3.4: Let X = Y = {a, b, c, d} with μ_1 = {Ø, {a}, {c}, {a, c}, X} and μ_2 = {Ø, {a, b, c}, Y}. Let f: (X, μ_1) → (Y, μ_2) be a mapping defined by f(a) = a, f(b) = b, f(c) = c, f(d) = d. Now, μ-βO(X) = {Ø, {a}, {c}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, c, d}, {a, b, d}, X}.Let A = {d}, then A is a μ-closed set in (Y, μ_2). Then f^{-1} ({d}) is a μ-βGαCS in (X, μ_1), but not μ-closed as $c_\mu(f^{-1}$ (A)) = cd}) = {b, d} ≠ f^{-1}(A). Hence f is a μ-βGα-continuous mapping, but not a μ-continuous mapping.

Theorem 3.5: Every μ-α-continuous mapping is a μ-βGα-continuous mapping in general.

Proof: Let f: (X, μ_1) → (Y, μ_2) be a μ-α-continuous mapping. Let A be any μ-closed set in (Y, μ_2). Since f is a μ-α-continuous mapping, f^{-1} (A) is a μ-α-closed set in (X, μ_1). Since every μ-α-closed set is a μ-βGαCS, f^{-1} (A) is μ-βGαCS in (X, μ_1). Hence f is a μ-βGα-continuous mapping.

Remark 3.6: A μ-pre-continuous mapping is not a μ-βGα-continuous mapping in general.

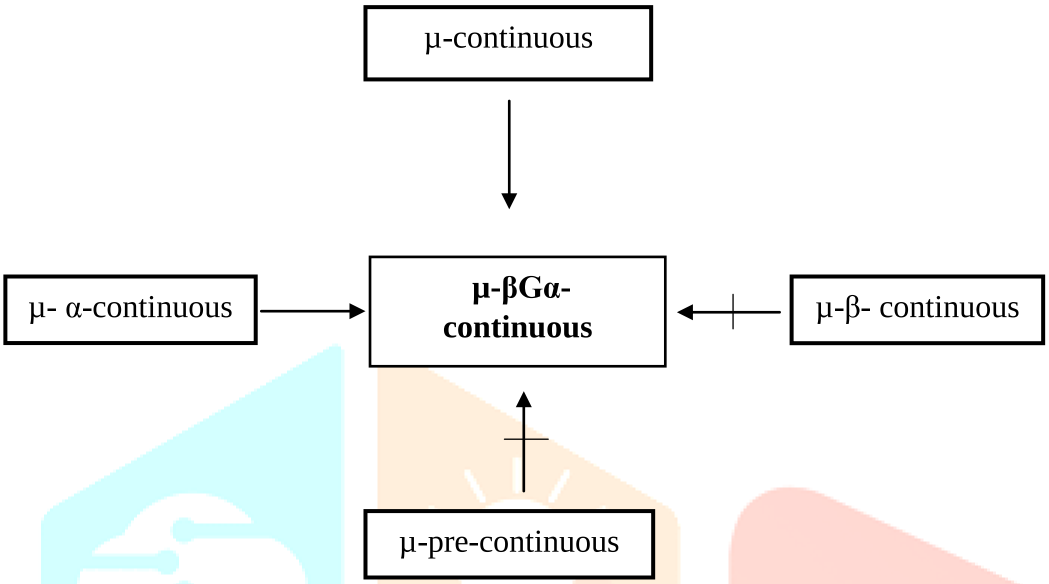
Example 3.7: Let X = Y = {a, b, c} with μ_1 = {Ø, {a, b}, X} and μ_2 = {Ø, {b, c}, Y}. Let f: (X, μ_1) →(Y, μ_2) be a mapping defined by f(a) = a, f(b) = b, f(c) = c. Now, μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Let A = {a}, then A is a μ-closed set in (Y, μ_2). Then f^{-1}({a}) is a μ-pre-closed set in (X, μ_1) as $c_\mu(i_\mu(f^{-1}$ (A))) = $c_\mu(i_\mu$({a})) = Ø ⊆ f^{-1} (A), but not μ-βGαCS as $\alpha c_\mu(f^{-1}$ (A)) = X ⊈ U = {a, b} in (X, μ_1). Hence f is a μ-pre-continuous mapping, but not a μ-βGα-continuous mapping.

Remark 3.8: A μ-β-continuous mapping is not a μ-βGα-continuous mapping in general.

Example 3.9: Let X = Y = {a, b, c} with μ_1 = {Ø, {a, b}, X} and μ_2 = {Ø, {b, c}, Y}. Let f: (X, μ_1) → (Y, μ_2) be a mapping defined by f(a) = a, f(b) = b, f(c) = c. Now,μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}.Let A = {a}, then A is a μ-closed set in (Y, μ_2). Then f^{-1}({a}) is a μ-β-closed in (X, μ_1) as $i_\mu(c_\mu(i_\mu(f^{-1}$

(A)))) $=i_\mu(c_\mu(i_\mu(\{a\}))) = \emptyset \subseteq f^{-1}(A)$, but not a μ-βGαCS as $\alpha c_\mu(f^{-1}(A)) = X \not\subseteq U = \{a, b\}$ in (X, μ_1). Hence f is a μ-β-continuous mapping, but not a μ-βGα-continuous mapping.

In the following diagram, we have provided relation between various types of μ-continuous mappings.



Theorem 3.10: A mapping f: $(X, \mu_1) \rightarrow (Y, \mu_2)$ is aμ-βGα-continuous mapping if and only if the inverse image of every μ-open set in (Y, μ_2) is a μ-βGαOS in (X, μ_1).

Proof: Necessity: Let U be a μ-open set in (Y, μ_2). Then Y-U is a μ-closed set in (Y, μ_2). Since f is aμ-βGα-continuous mapping, $f^{-1}(Y-U) = X - f^{-1}(U)$ is a μ-βGαCS in (X, μ_1).Hence $f^{-1}(U)$ is a μ-βGαOS in (X, μ_1).

Sufficiency: Assume that $f^{-1}(V)$ is a μ-βGαOS in (X, μ_1) for each μ-open set V in (Y, μ_2). Let V be any μ-closed set in (Y, μ_2). Then Y-V is μ-open in (Y, μ_2). By assumption, $f^{-1}(Y-V) = X - f^{-1}(V)$ is aμ-βGαOS in (X, μ_1) which implies that $f^{-1}(V)$ is a μ-β-generalized α-closed set in (X, μ_1). Hence f is aμ-βGα-continuous mapping.

Theorem 3.11: If f: $(X, \mu_1) \rightarrow (Y, \mu_2)$ is a μ-βGα-continuous mapping and g: $(Y, \mu_2) \rightarrow (Z, \mu_3)$ is a μ-continuous mapping then $g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3)$ is a μ-βGα-continuous mapping.

Proof: Let V be any μ-closed set in (Z, μ_3). Since g is a μ-continuous mapping, $g^{-1}(V)$ is a μ-closed set in (Y, μ_2). Since f is a μ-βGα-continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a μ-βGαCSin (X, μ_1). Therefore $g \circ f$ is a μ-βGα-continuous mapping.

Theorem 3.12: If f: $(X, \mu_1) \rightarrow (Y, \mu_2)$ is a μ-continuous mapping and g: $(Y, \mu_2) \rightarrow (Z, \mu_3)$ is a μ-continuous mapping then $g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3)$ is a μ-βGα-continuous mapping.

Proof: Let V be any μ-closed set in (Z, μ_3). Since g is a μ-continuous mapping, g^{-1} (V) is a μ-closed set in (Y, μ_2). Since f is a μ-continuous mapping, $(g \circ f)^{-1}$ (V) = f^{-1} (g^{-1}(V)) is a μ-closed set in (X, μ_1). Since every μ-closed set is aμ-βGαCS, $(g \circ f)^{-1}$ (V) is aμ-βGαCS. Therefore g ∘ f is a μ-βGα-continuous mapping.

Theorem 3.13: If f: (X, μ_1) → (Y, μ_2) is a μ-α-continuous mapping and g: (Y, μ_2) → (Z, μ_3) is a μ-continuous mapping then g ∘ f: (X, μ_1) → (Z, μ_3) is a μ-βGα-continuous mapping.

Proof: Let V be any μ-closed set in (Z, μ_3). Since g is a μ-continuous mapping g^{-1} (V) is a μ-closed in (Y, μ_2). Since f is a μ-α-continuous mapping, $(g \circ f)^{-1}$ (V) = f^{-1} (g^{-1} (V)) is a μ-α-closed in (X, μ_1). Since every μ-α-closed set is aμ-βGαCS, $(g \circ f)^{-1}$ (V) is a μ-βGαCSin (X, μ_1). Therefore g ∘ f is a μ-βGα-continuous mapping.

4. ALMOST μ-β-GENERALIZED α-CONTINUOUS MAPPINGS

In this section we have introduced almost μ-β-generalized α-continuous mappings in generalized topological spaces and studied some of their basic properties.

Definition 4.1: A mapping f: (X, μ_1) → (Y, μ_2) is called an almost μ-β-generalized α-continuous mapping (brieflyalmost μ-βGα-continuous) if f^{-1} (A)is a μ-β-generalized α-closed set (briefly μ-βGαCS) in (X, μ_1) for eachμ-regular closed set A in (Y, μ_2).

Example 4.2: Let X = Y = {a, b, c} with μ_1= {Ø, {a}, {b}, {a, b}, X} and μ_2 = {Ø, {c}, {a, b}, Y}. Let f: (X, μ_1) → (Y, μ_2) be a mapping defined by f(a) = a, f(b) = b, f(c) = c. Now,μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}.Let A = {c}, then A is a μ-regular closed set in (Y, μ_2). Then f^{-1}({c}) is μ-βGαCSin (X, μ_1). Hence f is an almost μ-βGα-continuous mapping.

Theorem 4.3: Every almost μ-continuous mapping is an almost μ-βGα-continuous mapping but not conversely in general.

Proof: Let f: (X, μ_1) → (Y, μ_2) be an almost μ-continuous mapping. Let A be a μ-regular closed set in (Y, μ_2). Since f is an almost μ-continuous mapping, f^{-1} (A) is a μ-closed set in (X, μ_1). Since every μ-closed set is a μ-βGαCS, f^{-1}(A) is a μ-βGαCS in (X, μ_1). Hence f is an almost μ-βGα-continuous mapping.

Example 4.4: Let X = Y = {a, b, c, d} with μ_1 = {Ø, {a}, {c}, {a, c}, X} and μ_2 = {Ø, {a, b}, {a, b, c}, Y}. Let f: (X, μ_1) → (Y, μ_2) be a mapping defined by f(a) = a, f(b) = b, f(c) = c. Now,μ-βO(X) = {Ø, {a}, {a, b}, {a, c}, X}.Let A = {b}, then A is a μ-regular closed set in (Y, μ_2). Then f^{-1} (A) is aμ-βGαCS, but not μ-closed as $c_\mu(f^{-1}(A)) = c_\mu(\{b\}) = \{b, c\} \neq f^{-1}(A)$ in (X, μ_1). Hence f is an almost μ-βGα-continuous mapping, but not an almost μ-continuous mapping.

Theorem 4.5: Every almost μ-α-continuous mapping is an almost μ-βGα-continuous mapping in general.

Proof: Let f: $(X, \mu_1) \rightarrow (Y, \mu_2)$ be an almost μ-α-continuous mapping. Let A be any μ-regular closed set in (Y, μ_2). Since f is an almost μ-α-continuous mapping, $f^{-1}(A)$ is a μ-α-closed set in (X, μ_1). Since every μ-α-closed set is aμ-βGαCS, $f^{-1}(A)$ is a μ-β-generalized α-closed set in (X, μ_1). Hence f is an almost μ-βGα-continuous mapping.

Remark 4.6: An almost μ-pre-continuous mapping is not an almost μ-βGα-continuous in general.

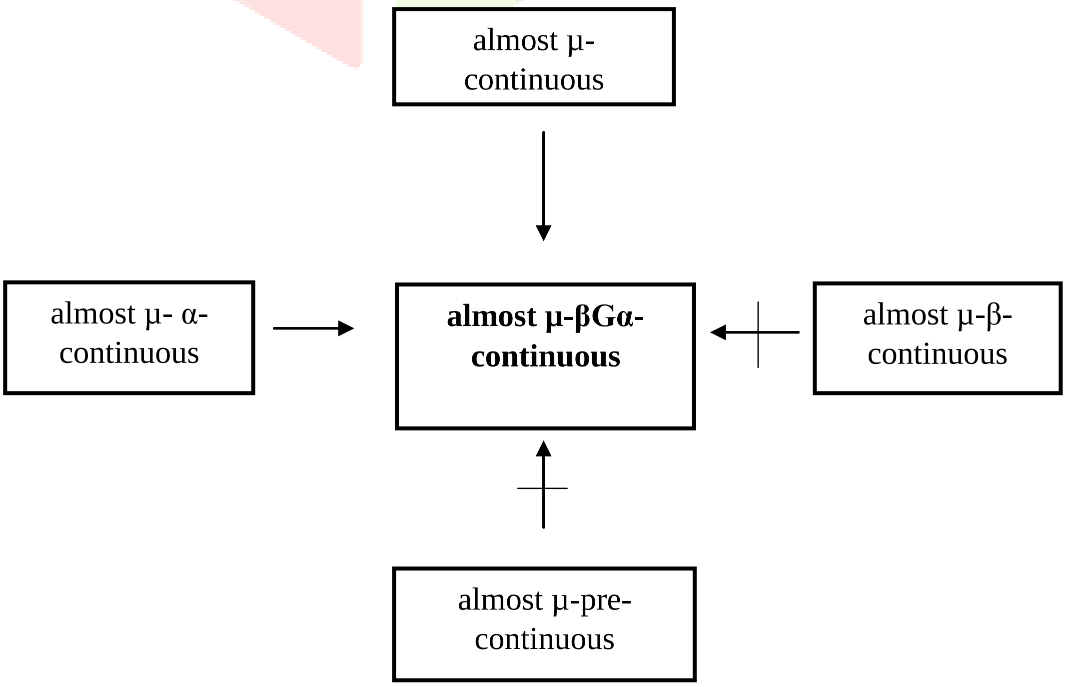
Example 4.7: Let X = Y = {a, b, c} with μ_1 = {Ø, {a, b}, X} and μ_2 = {Ø, {a}, {b, c}, Y}. Let f: $(X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by f(a) = a, f(b) = b, f(c) = c. Now,μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}.Let A = {a}, then A is a μ-regular closed set in (Y, μ_2). Then $f^{-1}(A)$ is a μ-pre closed set as $c_\mu(i_\mu(f^{-1}(A)))= c_\mu(i_\mu(\{a\})) = Ø \subseteq f^{-1}(A)$, but not μ-βGαCS as $\alpha c_\mu(f^{-1}(A)) = X \nsubseteq U = \{a, b\}$ in (X, μ_1). Hence f is an almost μ-pre-continuous, but not an almost μ-βGα-continuous mapping.

Remark 4.8: An almost μ-β-continuous mapping is not an almost μ-βGα-continuous mapping in general.

Example 4.9: Let X = Y = {a, b, c} with μ_1 = {Ø, {a, b}, X} and μ_2 = {Ø, {a}, {b, c}, Y}. Let f: $(X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by f(a) = a, f(b) = b, f(c) = c.

Now, μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Let A = {a}, then A is a μ-regular closed set in (Y, μ_2). Then $f^{-1}(A)$ is a μ-β-closed set as $i_\mu(c_\mu(i_\mu(f^{-1}(A)))) = i_\mu(c_\mu(i_\mu(\{a\}))) = Ø \subseteq f^{-1}(A)$, but not a μ-βGαCS as $\alpha c_\mu(f^{-1}(A)) = X \nsubseteq U = \{a, b\}$ in (X, μ_1). Hence f is an almost μ-β-continuous mapping, but not an almost μ-βGα-continuous mapping.

In the following diagram, we have provided the relation between various types of almost μ-continuous mappings.



Theorem 4.10: A mapping f: (X, μ_1) → (Y, μ_2) is an almost μ-βGα-continuous mapping if and only if the inverse image of every μ-regular open set in (Y, μ_2) is μ-βGαOS in (X, μ_1).

Proof:Necessity: Let U be a μ-regular open set in (Y, μ_2). Then Y-U is μ-regular closed in (Y, μ_2). Since f is an almostμ-βGα-continuous mapping,f^{-1} (Y-U) = X - f^{-1}(U) is a μ-βGαCS in (X, μ_1). Hence f^{-1} (U) is a μ-βGαOS in (X, μ_1).

Sufficiency: Let V be any μ-regular closed set in (Y, μ_2). Then Y-V is a μ-regular open in (Y, μ_2). By hypothesis, f^{-1}(Y-V) = X - f^{-1}(V) is aμ-βGαOS in (X, μ_1) which implies that f^{-1}(V) is a μ-βGαCS in (X, μ_1). Hence f is an almost μ-βGα-continuous mapping.

Theorem 4.11: If f: (X, μ_1) → (Y, μ_2) is a μ-continuous mapping and g: (Y, μ_2) → (Z, μ_3) is an almost μ-continuous mapping then g ∘ f: (X, μ_1) → (Z, μ_3) is an almost μ-βGα-continuous mapping.

Proof: Let V be any μ-regular closed set in (Z, μ_3). Since g is an almost μ-continuous mapping, g^{-1}(V) is a μ-closed set in (Y, μ_2). Since f is μ-continuous, $(g \circ f)^{-1}$ (V) = f^{-1}(g^{-1}(V)) is a μ-closed set in (X, μ_1). Since every μ-closed set is aμ-βGαCS, $(g \circ f)^{-1}$ (V) is a μ-βGαCS in (X, μ_1). Therefore g ∘ f is an almost μ-βGα-continuous mapping.

Theorem 4.12: If f: (X, μ_1) → (Y, μ_2) is a μ-α-continuous mapping and g: (Y, μ_2) → (Z, μ_3) is an almost μ-continuous mapping then g ∘ f: (X, μ_1) → (Z, μ_3) is an almost μ-βGα-continuous mapping.

Proof: Let V be any μ-regular closed set in (Z, μ_3). Since g is almost μ-continuous, g^{-1}(V) is μ-closed in (Y, μ_2). Since f is μ-α-continuous, $(g \circ f)^{-1}$ (V) = f^{-1}(g^{-1} (V)) is μ-α-closed in (X, μ_1). Since every μ-α-closed set is aμ-βGαCS, $(g \circ f)^{-1}$ (V) is aμ-βGαCS. Therefore g ∘ f is an almost μ-βGα-continuous mapping.

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