OPERATION APPROACHES ON $\beta - \gamma$ OPEN SETS IN TOPOLOGICAL SPACES

Mrs.M.Sentamilselvi,

Assistant professor

Department of Mathematics,

Vivekanandha College of Arts and

Sciences for women (Autonomous)

Tiruchengode, Namakkal (Dt) .

E.mail:sentamilselvi85@gmail.com

P.Gomathi

M.Phil Research Scholar,

Department of Mathematics,

Vivekanandha College of Arts and

Sciences for women (Autonomous)

Tiruchengode, Namakkal (Dt) .

E-mail:jpgomathi1995@gmail.com

ABSTRACT

In this paper the notion of $\beta - \gamma$ open sets in a topological space together with its corresponding interior and closure operations are introduced. Further some of their basic properties are studied.

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1. INTRODUCTION

O. Najastad [10] introduced β open sets in a topological space and studied some of their properties. The concept of semiopen sets, preopen sets and semi-preopen sets were introduced respectively by Levine [8],Mashhour [9] and Andrijevic [1].Andrijevic [2] introduced a new class of topology generated by preopen sets and the corresponding closure and interior operators.Kasahara defined the concept of an operation on topological spaces and introduced β −closed graphs of an operation.Ogata [11] called the operation β as γ operation and introduced the notion of T_γ which is the collection of all γ-open sets in a topological space (X, T).

In this paper in section 3 we introduce the notion of $T_{\beta-\gamma}$ which is the collection of all $\beta - \gamma$ open sets in a topological space (X, T).Further we introduce the concept of $T_{\beta-\gamma}$ interior and $T_{\beta-\gamma}$closure operator and study some of their properties.

2. PRELIMINARIES

In this section we recall some of the basic Definitions and Theorems

DEFINITION 2.1 Let(X, T) be a topological space and A be a subset of X. Then A is said to be

(i)[10] β-open set if $A \subseteq cl\left(int\left(cl(A)\right)\right)$

(ii)[7] semi-open set if $A \subseteq cl(int(A))$

(iii)[9] pre-open set if $A \subseteq int(cl(A))$

(iv)[9]semi-preopen set if $\left(cl\left(int(cl(A))\right)\right)$

DEFINITION 2.2

Let(X, T) be a topological space, an operation γ on the topology T is a mapping from T on the power set $P(X)$ of X such that $V \subseteq V^\gamma$ for each $V \in T$, where V^γ denotes the value of γ at V.

DEFINITION 2.3

Let (X, T)be a topological space and A be a subset of $X$and γ be an operation on T.Then A is said to be:

(i)[11] a γ-open set if for each $x \in A$ there exists an open set U such that $x \in U$ and $U^\gamma \subseteq A$.T_γ denotes the set of all γ-open sets in (X, T).

(ii)[14] γ-semi open if and only if

$$A \subseteq T_\gamma - cl\left(T_\gamma - int(A)\right)$$

(iii)[12] γ −preopen if and only if

$$A \subseteq \left(T_\gamma - int\left(T_\gamma - cl(A)\right)\right)$$

(iv)[12] γ-semi preopen if and only if

$$A \subseteq T_\gamma - cl(T_\gamma - int\left(T_\gamma - cl(A)\right)$$

DEFINITION 2.4

(i)[14] Let (X,T) be a topological space and γ be an operation on T.Then T_γ-interior of A is defined as the union of all γ-open sets contained in A and it is denoted

$T_\gamma - int(A)$. That is $T_\gamma - int(A) = \cup\{U: U$ is a γ −open set and $U \subseteq A\}$

(ii)[11] Let (X,T) be a topological space and γ be an operation on T.Then T_γ-closure of A is defined as the intersection of all γ-closed sets containing in $A$and it is denoted $T_\gamma - cl(A)$. That is $T_\gamma - cl(A) = \cap\{F: F$ is a γ −closed set and $A \subseteq F\}$

THEOREM 2.5

Let (X,T) be a topological space. Then

(i)[12] A subset A is γ −preclosed if and only if

$$T_\gamma - cl\left(T_\gamma - int(A)\right) \subseteq A$$

(ii)[12] A subset A is γ −semi preclosed if and only if

$$T_\gamma - int(T_\gamma - cl\left(T_\gamma - int(A)\right)) \subseteq A$$

3.$\beta - \gamma$ OPEN SET

DEFINITION 3.1

Let (X,T) be a topological space and γ be an operation on T.Then a subset$A$ of X is said to be a $\beta - \gamma$ open set if and only if $A \subseteq T_\gamma - cl(T_\gamma - int\left(T_\gamma - cl(A)\right)$.

EXAMPLE 3.2

Let $X = \{a,b,c,d\}$,

$$T = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}.$$

We define an operation $\gamma: T \to P(X)$ as follows**:** for every $A \in T$,

$$A^\gamma = \begin{cases} int\big(cl(A)\big) & if\ A \neq \{a\} \\ cl(A) & if\ A = \{a\} \end{cases}$$

Then $T_\gamma = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,c\}, \{a,b,d\}\}$ and

$$T_{\beta-\gamma} = \{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

THEOREM 3.3

Let(X,T) be a topological space and γ be an operation on T.Then every γ −open set in $(X,T)$is a $\beta - \gamma$ open set. However, the converse need not be true.

PROOF:

Proof is straight forward from the definition 3.1

In example 3.2 {a,b},{a,d},{a,b,c},{a,c,d} are $\beta - \gamma$ open sets but not γ-open sets.

THEOREM 3.4

Let (X,T) be a topological space and γ be an operation on T and $\{A_\beta : \beta \in J\}$ be a family of $\beta - \gamma$ open sets in (X,T).Then $U_{\beta \epsilon J} A_\beta$ is also a $\beta - \gamma$ open set.

PROOF:

Given $\{A_\beta : \beta \in J\}$ be the family of $\beta - \gamma$ open sets in (X,T).Then for each $A_\beta, A_\beta \subseteq T_\gamma - cl(T_\gamma - int\left(T_\gamma - cl(A_\beta)\right))$.This implies that $\cup A_\beta \subseteq \cup \left[T_\gamma - cl\left(T_\gamma - int\left(T_\gamma - cl(A_\beta)\right)\right)\right]$. and hence $\cup A_\beta \subseteq [T_\gamma - cl(T_\gamma - int\left(T_\gamma - cl(\cup A_\beta)\right))]$.Therefore we have $U_{\beta \epsilon J} A_\beta$ is also a $\beta - \gamma$ open set.

REMARK: 3.5

(i)Let (X,T) be a topological space and γ be an operation on T.If A,B are any two $\beta - \gamma$ open sets in (X,T),then the following example shows that $A \cap B$ need not be a $\beta - \gamma$ open set.

Let $X = \{a,b,c\}$,

$T = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$, define an operation γ on T such that

$$A^\gamma = \begin{cases} cl(A) & if\ b \notin A \\ A & if\ b \in A \end{cases}$$

Then$T_{\beta-\gamma} = \{\emptyset, X, \{b\}, \{a,b\}, \{a,c\}\}$. $A = \{a,b\}$and $B = \{a,c\}$ are $\beta - \gamma$ open sets but $A \cap B = \{a\}$ is not a$\beta - \gamma$ open set.

(ii) the following example shows that the concepts of β −open set are independent.

Let $X = \{a,b,c\}, T = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$, the β −open sets are $\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$.we define an operation γ on T such that $\gamma(B) = cl(B)$.Then

$T_\gamma = \{\emptyset, X, \{b\}, \{a,c\}\}$ and

$T_{\beta-\gamma} = \{\emptyset, X, \{a\}, \{a,c\}\}$.Here {a},{a,b} are β −open sets but not $\beta - \gamma$ open sets.

Similarly in example 3.2 $\{a,d\}, \{a,c,d\}$ are $\beta - \gamma$ open sets but not β −open sets.

THEOREM 3.6

If (X,T) is a γ −regular space,then the concept of $\beta-\gamma$ open set and β −open set coincide.

PROOF:

Proof follows from the proposition 2.4[9] and the theorem 3.6[9].

DEFINITION 3.7

Let (X,T) be a topological space and γ be an operation on $T$and A be a subset of X. A is said to be $\beta-\gamma$ closed if and only if $X-A$ is $\beta-\gamma$ open,which is equivalently A is $\beta-\gamma$ closed if and only if $A\supseteq T_\gamma-int(T_\gamma-cl(T_\gamma-int(A)))$.

THEOREM: 3.8

Let(X,T) be a topological space and γ be an operation on T.

(i) Every $\beta-\gamma$ open set is γ −semi-open.

(ii) Every $\beta-\gamma$ open set is γ −preopen.

(iii) Every $\beta-\gamma$ open set is γ −semi preopen

PROOF

(i) Let$A$ be a $\beta-\gamma$ open set in (X,T).Then it follows that $A\subseteq T_\gamma-cl(T_\gamma-int(T_\gamma-cl(A)))$.and hence $A\subseteq T_\gamma-int\left(T_\gamma-cl(A)\right)$.Therefore A is γ −semi-open

(ii)Let A be a$\beta-\gamma$ open set in (X,T).since $T_\gamma-cl(A)\subseteq A$,implies that $T_\gamma-int\left(T_\gamma-cl(A)\right)\subseteq T_\gamma-int(A)$ and hence $T_\gamma-cl(T_\gamma-int(T_\gamma-cl(A)))\subseteq T_\gamma-cl(T_\gamma-int(A))$.this implies that $A\subseteq T_\gamma-cl\left(T_\gamma-int(A)\right)$.therefore A is γ −preopen.

(iii)Proof is obvious using the (i),(ii) results, Definition 3.11[10] and Remark 3.2[10]

REMARK: 3.9[10]

Let $X=\{a,b,c\}$,

$T=\{\emptyset,X,\{a\},\{c\},\{a,b\},\{a,c\}\}$, define an operation γ on T such that

$$A^\gamma=\begin{cases} A & if\ A=\{a\} \\ A\cup\{c\} & if\ A\neq\{a\}\end{cases}$$

Then $T_\gamma=\{\emptyset,X,\{a\},\{c\},\{a,c\}\}$,

$T_\gamma-SO(X)=\{\emptyset,X,\{a\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $T_{\beta-\gamma}=\{\emptyset,X,\{a\},\{c\},\{a,c\}\}$. Here $\{a,b\}$ and $\{b,c\}$ are γ −semi-open sets but they are not $\beta-\gamma$ open sets.

REMARK: 3.10

Let $X=\{a,b,c\},T=\{\square,X,\{a\},\{b\},\{a,b\},\{a,c\}\}$ define an operation γ on T such that

$$A^\gamma=\begin{cases} A & if\ b\in A \\ cl(A) & if\ b\notin A\end{cases}$$

Then $T_\gamma=\{\emptyset,X,\{b\},\{a,b\},\{a,c\}\}$,

$T_\gamma-PO(X)=\{\emptyset,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$,

$T_\gamma-SPO(X)=\{\emptyset,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$, and $T_{\beta-\gamma}=\{\emptyset,X,\{a\},\{c\},\{a,b\},\{b,c\},\{a,c\}\}$. Here$\{b\}$ are γ −preopen sets, γ −semi-preopen sets but they are not $\beta-\gamma$ open sets.

THEOREM: 3.11

Let A be a subset of a topological space (X,T).If B is a γ −semi-open set of X such that $B\subseteq A\subseteq T_\gamma-cl\left(T_\gamma-int(B)\right)$, then$A$ is a $\beta-\gamma$ open set of X.

PROOF:

Given $B\subseteq A$ and B is a γ −semi-open set,implies that $T_\gamma-int(B)\subseteq T_\gamma-int(A)$ and

$B\subseteq T_\gamma-cl\left(T_\gamma-int(B)\right)$.This implies that $B\subseteq T_\gamma-cl\left(T_\gamma-int(A)\right)$ and hence$\left(T_\gamma-cl(B)\right)\subseteq T_\gamma-cl\left(T_\gamma-int(A)\right)$.Therefore $T_\gamma-int\left(T_\gamma-cl(B)\right)\subseteq T_\gamma-int\left(T_\gamma-cl(T_\gamma-int(A))\right)$.Hence by assumption A is a $\beta-\gamma$ open set of X.

THEROREM: 3.12

A subset A is $\beta-\gamma$ open if and only if it is γ −semi-open and γ −preopen.

PROOF:

By theorem 3.8(i) and (ii) it follows that if A is $\beta-\gamma$ open then A is γ −semi-open and γ −preopen.conversely if A is γ −semi-open and γ −preopen,then $A\subseteq T_\gamma-cl\left(T_\gamma-int(A)\right)$ and $A\subseteq T_\gamma-int\left(T_\gamma-cl(A)\right)$ .This implies that

$A\subseteq T_\gamma-cl(T_\gamma-int(T_\gamma-cl(A)))$ .Therefore A is $\beta-\gamma$ open.

REMARK: 3.13

The following statements are equivalent for subsets of a topological space (X,T):

(i) Every $\gamma-$preopen is $\gamma-$semi-open.A subset A of X is is $\beta-\gamma-$open if and only if it is is $\gamma-$preopen.

PROOF:

$(i)\Rightarrow(ii)$ If A is $\beta-\gamma-$ open then by the theorem 3.8(ii) A is $\gamma-$preopen.

Conversely if A is $\gamma-$preopen.,then by (i) and theorem 3.12,A is $\beta-\gamma-$ open.

(ii)$\Rightarrow(i)$proof follows from the theorem 3.12.

Similarly we can prove the following remark.

REMARK: 3.14

The following statements are equivalent for subsets of a topological space (X,T):

(i) Every $\beta-\gamma-$ open set is $\gamma-$preopen.

A subset A of X is $\beta-\gamma-$ open if and only if it is $\beta-\gamma-$ open.

THEOREM 3.15

Let A be a subset of a topological space (X,T).Then A is $\gamma-clopen$ if and only if it is $\beta-\gamma-open$ and $\gamma-$preclosed.

PROOF:

If A is $\gamma-$clopen, then by theorem 3.3 and theorem 2.12 [12] A is $\beta-\gamma-open$ and $\gamma-$preclosed.

Conversely if A is $\beta-\gamma-open$ and $\gamma-$preclosed then $A\subseteq T_\gamma-cl(T_\gamma-int(T_\gamma-cl(A)))$ and $(T_\gamma-cl(T_\gamma-int(A))\subseteq A$ implies that $A\subseteq T_\gamma-int(A)$.This implies that A is $\gamma-open$.since $A\subseteq T_\gamma-int(A), T_\gamma-cl\,(A)\subseteq(T_\gamma-cl(T_\gamma-int(A))\subseteq A$.Hence $T_\gamma-cl(A)\subseteq A$.Therefore A is $\gamma-clopen$.

DEFINITION: 3.16

(i)Let (X,T) be a topological space and γ be an operation on T and A be a subset of X.Then $T_{\beta-\gamma}-$interior of A is the union of all $\beta-\gamma$-open sets contained in A and it is denoted by $T_{\beta-\gamma}-$int(A).That is $T_{\beta-\gamma}-$int(A)=$\cup\{U: U\ is\ a\ \beta-\gamma-open\ set\ and\ U\subseteq A\}$

(ii)Let (X,T) be a topological space,S be a subset of X and x be a point of X.Then x is called an $\beta-\gamma-$interior point of S if there exists $V\in T_{\beta-\gamma}$ such that $x\in V$.

The set of all $\beta-\gamma-$interior points of S is called $\beta-\gamma-$interior of S and is also denoted by $\beta-\gamma-int(S)$.

REMARK: 3.17

Let (X,T) be a topological space and γ be an operation on T.Let A,B be subsets of X.Then the following holds good:

(i)$T_{\beta-\gamma}-int(A)$is the largest $\beta-\gamma$-open subset of X contained in A.

(ii) A is $\beta-\gamma$-open if and only if $T_{\beta-\gamma}-int(A)=A$

(iii) $T_{\beta-\gamma}-int(T_{\beta-\gamma}-int(A))$=$T_{\beta-\gamma}-int(A)$

(iv)If $A\subseteq B$ then $T_{\beta-\gamma}-int(A)\subseteq T_{\beta-\gamma}-int(B)$

$(v)T_{\beta-\gamma}-int(A)\cup T_{\beta-\gamma}-int(B)\subseteq T_{\beta-\gamma}-$int(A$\cup B$)

PROOF:

(i)Follows from the definition 3.16
(ii)Follows from the definition 3.16 and theorem 3.4
(iii)Follows from (ii)
(iv)Follows from the definition 3.16
(v)Follows from the theorem 3.4 and (i)

DEFINITION: 3.18

Let (X,T) be a topological space and γ be an operation on T.Let A be A subset of X.Then $T_{\beta-\gamma}-$closure of A is the intersection of $\beta-\gamma$ closed sets containing A and it is denoted by $T_{\beta-\gamma}-cl$(A).That is

$$T_{\beta-\gamma}-cl(A)=\cap\{F: F\ is\ a\ \beta-\gamma-closed\ set\ and\ A\subseteq F\}$$

REMARK: 3.19

(i)If A is a subset of (X,T).Then $T_{\beta-\gamma}-cl(A)$ is a $\beta-\gamma$-closed set containing A.

(ii)A is $\beta-\gamma$-closed if and only if $T_{\beta-\gamma}-cl(A)=A$.

PROOF:

(i)Follows from the definition 3.18.(ii) follows from the definition 3.18 and definition 3.7

THEOREM: 3.20

Let A and B be subsets of (X,T).Then the following statements hold:

(i)$T_{\beta-\gamma}-cl\left(T_{\beta-\gamma}-cl(A)\right)=T_{\beta-\gamma}-cl(A)$

(i) If $A\subseteq B$,then $T_{\beta-\gamma}-cl(A)\subseteq T_{\beta-\gamma}-cl(B)$

$T_{\beta-\gamma}-cl(A)\cup T_{\beta-\gamma}-cl(B)\subseteq T_{\beta-\gamma}-cl(A\cup B)$

$T_{\beta-\gamma}-cl(A\cap B)\subseteq T_{\beta-\gamma}-cl(A)\cap T_{\beta-\gamma}-cl(B)$

PROOF:

(i)proof follows from the definition 3.18

(ii) given $A\subseteq B$,implies that $A\subseteq T_{\beta-\gamma}-cl(B)$ and by (i) $T_{\beta-\gamma}-cl(A)\subseteq T_{\beta-\gamma}-cl(B)$.

(iii) $A\subseteq A\cup B$,and $B\subseteq A\cup B$,implies that $T_{\beta-\gamma}-cl(A)\subseteq T_{\beta-\gamma}-cl(A\cup B)$ and $T_{\beta-\gamma}-cl(B)\subseteq T_{\beta-\gamma}-cl(A\cup B)$.This implies that $T_{\beta-\gamma}-cl(A)\cup T_{\beta-\gamma}-cl(B)\subseteq T_{\beta-\gamma}-cl(A\cup B)$

$(iv)A\subseteq T_{\beta-\gamma}-cl(A), B\subseteq T_{\beta-\gamma}-cl(B) and\ (A\cap B)\subseteq T_{\beta-\gamma}-cl(A)\cap T_{\beta-\gamma}-cl(B))$.This implies that $T_{\beta-\gamma}-cl(A\cap B)\subseteq T_{\beta-\gamma}-cl(T_{\beta-\gamma}-cl(A))\cap T_{\beta-\gamma}-cl(T_{\beta-\gamma}-cl(B))$.Hence $T_{\beta-\gamma}-cl(A\cap B)\subseteq T_{\beta-\gamma}-cl(A)\cap T_{\beta-\gamma}-cl(B)$.

THEOREM: 3.21

Let (X,T) be a topological space and γ be an operation on T.Then for a point x$\in X, x\in T_{\beta-\gamma}-cl(A)$ if and only if $V\cap A\neq\emptyset$ for any $V\in T_{\beta-\gamma}$ such that x$\in V$

PROOF:

Let F_0 be the set of all y$\in XV \cap A \neq \emptyset$ for every $V \in T_{\beta-\gamma}$ such that y$\in V$.to prove this theorem it is enough to prove that $F_0 = T_{\beta-\gamma} - cl(A)$.Let $X \in T_{\beta-\gamma} - cl(A)$.Let us assume that $X \notin F_0$ then there exists a $\beta - \gamma$-open set U of Xsuch that $U \cap A \neq \emptyset$.This implies that $A \subseteq X - U$ and hence $(T_{\beta-\gamma} - cl(A) \subseteq X - U$ $.Therefore, x \notin (T_{\beta-\gamma} - cl(A)$ which is a contradiction and hence $(T_{\beta-\gamma} - cl(A) \subseteq F_0.conversely, let\ F\ be\ a\ set\ such\ that\ A \subseteq$ F and $(X - F) \in T_{\beta-\gamma}$.Let $x \notin F$ then we have $x \in (X - F)$ and $(X - F) \cap \emptyset$.this implies $x \notin F_{0.}$Therefore $F_{0.} \subseteq$ F.Hence $F_{0.} \subseteq (T_{\beta-\gamma} - cl(A)$

Hence the proof

THEOREM: 3.22

Let (X, T) is a topological space and $A \subseteq X$. Then the following statements hold:

(i) $T_{\beta-\gamma} - int(X - A) = X - T_{\beta-\gamma} - cl(A)$

(ii) $T_{\beta-\gamma} - cl(X - A) = X - T_{\beta-\gamma} - int(A)$

PROOF:

Proof of (i) and (ii) is obvious.

DEFINITION: 3.23

A subset B_x of a topological space (X, T) is said to be the $\beta - \gamma$ neighbourhood of a point $x \in X$ if there exists an $\beta - \gamma$ open set U such that $x \in U \subseteq B_x$.

THEOREM: 3.24

A subset of a topological space (X, T) is $\beta - \gamma$ if and only if it is $\beta - \gamma$ neighbourhood of each of its points.

PROOF:

The proof follows from the definition 3.16 and definition 3.23

REMARK: 3.25

Let (X, T) be a topological space and γ be an operation on T and A be a subset of X.then from the theorem 3.3 and the definition 3.18 we have

$$A \subseteq T_{\beta-\gamma} - cl(A) \subseteq T_\gamma - cl(A).$$

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