ON FUZZY GENERALIZED CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

Mrs. M. Sentamilselvi,
Assistant Professor,
Department of Mathematics,
Vivekanandha College of Arts and Sciences for Women(Autonomous)
Tiruchengode, Namakkal(Dt).
Sentamilselvi85@gmail.com

G. Ramya
M.phil Research Scholar,
Department of Mathematics
Vivekanandha College of Arts and Sciences for Women(Autonomous),
Tiruchengode, Namakkal(Dt).
ramyaguru1595@gmail.com

Abstract

In this paper, we study the concept of fuzzy generalized closed sets. We will considered the question of when some classes of fuzzy generalized closed sets coincide. Also we introduce some fuzzy lower separation axioms, fuzzy exactly discontinuity, fuzzy sub-maximal space, fuzzy $T_{1/2}$-space and fuzzy T_1-space and study some of the properties.

Keywords:

Fuzzy generalized sets, fuzzy lower separation axioms, fuzzy exactly discontinuity, fuzzy sub-maximal space, fuzzy $T_{1/2}$-space and fuzzy T_1-space.

Introduction

Fuzzy set theory was introduced by zadeh [30] as a generalization of crisp or classical set theory. In fuzzy topological spaces [19] introduced and studied fuzzy generalized closed sets. Fuzzy sets have application in application in applied fields such as information. One of the application was the study of fuzzy topological spaces [6], introduced and studied by chang in 1968. This notion has been studied extensively in recent years by many fuzzy topology because fuzzy generalized closed sets are not only natural generalizations of fuzzy closed sets. More importantly, they also suggest several new properties of fuzzy topological spaces. Most of these new properties are separation axioms weaker than fuzzyT_1-space, some of which have been found to be useful in computer science and fuzzy digital topology. For example, the well-known digital line is fuzzy$T_{3/4}$-space but not fuzzyT_1-space. Other new properties are defined by variations of the property of fuzzy sub-maximality. Furthermore, the study of fuzzy generalized closed sets also provides new characterizations of some known fuzzy classes of spaces, for example, the class of extremallyfuzzy disconnected spaces.

For the sake of convenience, we begin with some basic concepts, althoughmost of these concepts can be found from the references of this paper.

A subset $S$of a fuzzy topological space$X$is called $f\alpha$-open [resp. fuzzy semi-open, fuzzy preopen, fuzzy semi-preopen] if $S \leq int(cl(int\, S))$[resp.$S \leq cl(int\, S), \leq int(cl\, S), S \leq cl(int(cl\, S))$]. Moreover, $S$is said to be$f\alpha$-closed [resp. fuzzy semi-closed, fuzzy preclosed,fuzzy semi-preclosed] if $X/_S$ is $f\alpha$-open [resp.fuzzy semi-open, fuzzy preopen, fuzzy semi-preopen] or equivalently, if $cl(int(cl\, S)) \leq S$ [resp.$int(cl\, S) \leq S, cl(int\, S) \leq S, int(cl(int\, S)) \leq S$]. The $f\alpha$-closure [resp. fuzzy semi-closure, fuzzy preclosure, fuzzy semi-preclosure] of $S \leq X$ is the smallest $f\alpha$-closed [resp. fuzzy semi-closed, fuzzy preclosed, fuzzy semi-preclosed]set containing$S$. It is well-known that $\alpha cl\, S = S \vee int(cl(int\, S))$. $S = S \vee int(cl\, S)$, $pcl\, S = S \vee cl(int\, S)$and $spcl\, S = S \vee int(cl(int\, S))$. The α-interiorof $S \leq X$ is the largest α-open set contained in S, and we have $\alpha int\, S = S \wedge int(cl(int\, S))$. It is worth mentioning that the collection $f\alpha(X)$of all $f\alpha$-open subsets of X is a fuzzy topology on X[21] which is finer thanthe original one, and that a subset fuzzy S of X is $f\alpha$-open if and only if S is fuzzy semi open and fuzzy preopen [8].

Definition: 1

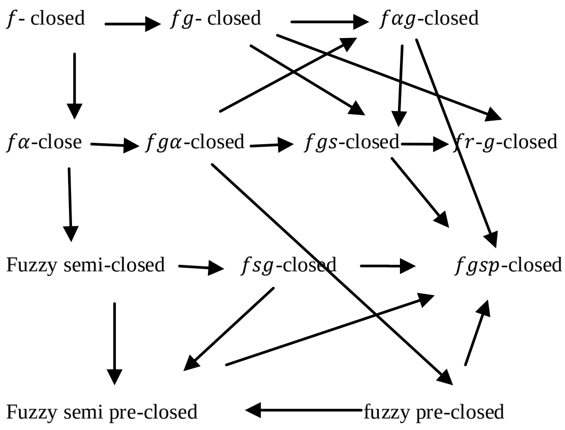
A fuzzy set A of (X,τ) is called

1. Fuzzy generalized closed (briefly, g-closed) [17] if $cl(\lambda)\leq\mu$, whenever $\lambda\leq\mu$ and μ is fuzzy open set in X.
2. Fuzzy generalized semi-closed (briefly, fgs-closed) [21] if $scl(\lambda)\leq\mu$, whenever $\lambda\leq\mu$ and μ is fuzzy open set in X.0
3. Fuzzy generalized α-closed (briefly, $fg\alpha$-closed) [22] if $\alpha cl(\lambda)\leq\mu$, whenever $\lambda\leq\mu$ and μ is fuzzy open set in X.
4. Fuzzy α-generalized closed (briefly, $f\alpha g$-closed) [22] if $\alpha cl(\lambda)\leq\mu$, whenever $\lambda\leq\mu$ and μ is fuzzy $f\alpha$-open set in X.
5. Fuzzy generalized semi-preclosed (briefly, $fgsp$-closed) [8] if $spcl(\lambda)\leq\mu$, whenever $\lambda\leq\mu$ and μ is fuzzy open set in X.
6. Fuzzy regular generalized closed set (briefly, frg-closed) [25] if $cl(\lambda)\leq\mu$, whenever $\lambda\leq\mu$ and μ is fuzzy regular open set in X.
7. Fuzzy semi-generalized closed set (briefly, fsg-closed) [3] if $scl(\lambda)\leq\mu$, whenever $\lambda\leq\mu$ and μ is fuzzy semi-open set in X.

In addition to Definition 1 above, a subset $A$of X is fg-open [7] (fsg -open[4]) if $X/_S$ is fg -closed [fsg -closed].

Other classes of fuzzy generalized open sets can be defined in a similar manner. Recall that a space X is said tobe fuzzy submaximal if every dense subset of X is fuzzy open. As variations of fuzzy submaximality, we obtain the notions of $f\alpha$-submaximality, fg-submaximality and fsg -submaximality. A space X is fsg -submaximal [resp. fg -submaximal, fsg -submaximal] if every dense subset is $f\alpha$-open (resp.fg -open, fsg -open).$f\alpha$ -submaximal spaces have been studied by Ganster in [14]. Obviously, every fuzzy submaximal space is fg-submaximal, and it has been pointed out in [5],Corollary 3.4., that if $(X;\alpha(X))$ is fg-submaximal, then $(X;\alpha(X))$ is also fsg-submaximal. Note that any indiscrete space with at least two points is fg-submaximal but not fuzzy submaximal, and an fsg-submaximal space which isnot fg-submaximal was given in [5].

In [8], Dontchev summarized the fundamental relationships between various types of fuzzy generalized closed sets in the following diagram. It was noted in [8] that, in general, none of the implications in that diagram is reversible.



Concerning possible converses of some implications in the above diagram Dontchev [8] posed two questions asking for the class of spaces in which every fuzzy semi-preclosed subset is fsg-closed, and for the class of spaces in which every fuzzy preclosed subset is $fg\alpha$-closed. These two questions have been considered and answered by Cao, Ganster and Reilly in [3]. Other possible converses of implications were investigated in [5]. This lead to some new characterizationsof fuzzy T_{fgs}-spaces, extremally disconnected spaces and fsg-submaximal spaces.

The main purpose of this paper is to summarise recent work in this direction,and also to present some new results. Throughout this paper, no Separation axioms are assumed unless stated explicitly.

2.Dontchev's Questions

In a recent paper [8], Dontchev posed the following two fuzzy open questions concerning fuzzy generalized closed sets:

Question 2.1

[8] Characterize the spaces in which
1) Every fuzzy semi-pre-closed set is fsg-closed.
2) Every fuzzy pre-closed set is $fg\alpha$-closed.
In order to answer these questions we need some preparation. Let S be a fuzzy subset of a space X. A fuzzy resolution of S is a pair $\langle E_1,E_2\rangle$ of disjoint fuzzy dense subsets of S. The subset S is said to be fuzzy resolvable if it possesses aresolution, otherwise S is called fuzzy irresolvable. In addition, S is called strongly fuzzy irresolvable, if every fuzzy open subspace of S is fuzzy irresolvable.

Observe that if $\langle E_1, E_2\rangle$ is a resolution of Sthen $E_1$and E_2 are co-dense in X, i.e. have empty interior. We also note that every fuzzy submaximal space is hereditarily fuzzy irresolvable.

Lemma 2.2

[15, 13] Every fuzzy topological space X has a unique decomposition $X = F \vee G$, where F is fuzzy closed and fuzzy resolvable and G is fuzzy open and here ditarily fuzzy irresolvable.

In this paper, the representation$X = F \vee G$, where F and G are as in Lemma 2.2, will be called the Hewitt decomposition of X.

Lemma 2.3

[15] Let X be a fuzzy topological space. Then every fuzzy singleton of X is either nowhere fuzzy dense or fuzzy preopen.

For a fuzzy topological space X, we now define X_1 ={ $x \in X$: $\{x\}$ is nowhere fuzzy dense}, and X_2 ={ $x \in X$: $\{x\}$is fuzzy preopen}. Then $X = X_1 \vee X_2$ is a decomposition of X, which will be called the Jankovi´c-Reilly decomposition. Recall that a fuzzy topological space X sis said to be locally indiscrete if every fuzzy open subset is fuzzy closed. We are now ready to state the following theorem which was proved in [14] answering Question 2.1.

Theorem 2.4

[4] For a fuzzy topological space X with Hewitt decomposition$X = F \vee G$ the following are equivalent:

(1) Every fuzzy semi-pre closed subset of X is fsg-closed,

(2) $X_1 \wedge scl\ A \leq spcl\ A$for $A \leq X$,

(3) $X_1 \leq int(cl\ G)$,

(4) $\approx Y \oplus Z$, where Y is locally indiscrete and Z is strongly fuzzy irresolvable,

(5) Every fuzzy pre closed subset of X is $fg\alpha$-closed,

(6) X is sfg-submaximal with respect to $f\alpha(X)$.

3.Fuzzy Lower separation axioms

A fuzzy topological space X is called fuzzy $T_{1/2}$-space [17] if every fg-closed subset of X is fuzzy closed. Dunham [12] proved that a space X is fuzzy $T_{1/2}$-space if and only if fuzzy singletons of X are either fuzzy open or fuzzy closed. For further results concerning this class of spaces we referthe reader to [10] and [24].

Theorem 3.1

[7] Every fuzzy submaximal space is $T_{1/2}$. It is obvious that in any fuzzy topological space X, every fsg-closed subset of X is fgs-closed. In [12], the class of T_{fgs}-spaces was introduced where a space X is T_{fgs} called if every fgs-closed subset of X is fsg-closed. The following result exhibits the relationship between fuzzy T_{fgs}-spaces and fuzzy $T_{1/2}$-spaces.

Theorem 3.2

[8, 10] For a fuzzy topological space X, the following statements are equivalent:

(1) X is a fuzzy T_{fgs}-space,

(2) Every nowhere fuzzy dense subset of X is a union of fuzzy closed subsets of X (i.e. X is T_1^* [17]),

(3) Every fuzzy generalized semi-preclosed subset of X is fuzzy semi-preclosed (i.e. X is semi-pre-$T_{1/2}$[8]),

(4) Every fuzzy singleton of X is either fuzzy preopen or fuzzy closed.

Corollary 3.3

Every fuzzy $T_{1/2}$-space is T_{fgs}. A space X is called fuzzy semi-T_1-space [11] if each fuzzy singleton is fuzzy semi-closed, it is called fuzzy semi-$T_{1/2}$-space [3] if every fuzzy singleton is either fuzzy semi-closed or fuzzy semi-open. Let $s(X)$ be the fuzzy semi-regularization of a space X. The closure of a subset A of X with respect to $s(X)$ will be denoted by $\delta cl\ A$. A subset A of X is called fuzzy δ-generalized closed if $\delta cl\ A \leq U$when $A \leq U$and U is fuzzy open in X.

Moreover, X is called a fuzzy $T_{3/4}$-space [10] if every fuzzy δ-generalized closed subset of X is closed in $s(X)$. The well-known digital line, also called the Khalimsky line, is a fuzzy $T_{3/4}$-space which fails to be fuzzy T_1-space.

Theorem 3.4

For any fuzzy topological space X,

(1) [10] $T_{3/4} = T_{fgs} + semi\ T_1$.

(2) [12] $T_{1/2} = T_{fgs} + semi\ T_{1/2}$.

The results above clarify some connections between the T_{fgs}-spaceproperty andother fuzzy lower separation axioms. Note, however, that not every T_{fgs}-space is fuzzy T_0-space. For example, a three point space $X = \{a, b, c\}$in which the only proper fuzzy open subset is $\{a, b\}$, is a fuzzy T_{fgs}-space, but not a fuzzy T_0-space. To obtain more characterizations of fuzzy T_{fgs}-spaces, we need the following lemma.

Lemma 3.5

A fuzzy subset A of a space X is $fg\alpha$-closed if and only if $X_1 \wedge \alpha cl\ A \leq A$.

Proof:

Suppose that A is $fg\alpha$-closed, and let $x \in X_1 \wedge \alpha cl\ A$, then $X/\{x\}$ is an $f\alpha$-open set containing A and so $\alpha cl\ A \leq X/\{x\}$, which is impossible.

Conversely, suppose that $X_1 \wedge \alpha cl\ A \leq A$. Let U be an $f\alpha$-open set containing A, and let $x \in \alpha cl\ A$. If $x \in X_1$, then $x \in A \leq U$. Now let $x \in X_2$ and assume that $x \notin U$. Then X/U is an $f\alpha$-closed set containing x, and thus, $\alpha cl(\{x\}) = \{x\} \vee cl(int(cl\{x\})) \leq X/U$. Since $\{x\}$ is fuzzy preopen, we have $int(cl(\{x\})) \wedge A \neq \emptyset$. Pick a point $y \in int(cl(\{x\})) \wedge A$.

Then $y \in A \wedge (X/U) \leq U \wedge (X/U)$, which is a contradiction.

Theorem 3.6

A fuzzy topological space X is T_{fgs}-space if and only if every $f\alpha g$-closed subset of $X$is $fg\alpha$-closed.

Proof:

Suppose that X is T_{fgs}. Let A be $f\alpha g$-closed and let $x \in X_1 \wedge \alpha cl\ A$. Then $\{x\}$is fuzzy closed by Theorem 3.2. Assume that $x \notin A$, i.e. $A \leq X/\{x\}$.Since $A$is $f\alpha g$-closed and $X/\{x\}$ is open we have $x \in \alpha cl \leq X/\{x\}$, which is a contradiction. Therefore, $X_1 \wedge \alpha cl\ A \leq A$. By Lemma 3.5, A is$fg\alpha$-closed.

Conversely, assume that every $f\alpha g$-closed subset of $X$is $fg\alpha$-closed. Let $x \in X_1$ and suppose that $\{x\}$is not fuzzy closed. Then $X/\{x\}$is fuzzy dense and $f\alpha g$-closed, thus $fg\alpha$-closed. It follows from Lemma 3.5 that
$X_1 \wedge \alpha cl(X/\{x\})X_1 \wedge X = X_1 \leq X/\{x\}$. So we obtain $x \in X/\{x\}$, a contradiction. By Theorem 3.2, $X$is fuzzyT_{fgs}-space.

Remark 3.7

A fuzzy topological space whose $f\alpha g$-closed subsets are $fg\alpha$ -closed may be called a fuzzy $T_{f\alpha g}$-space. Theorem 3.6 shows, however, that the fuzzy class of $T_{f\alpha g}$-spaces is precisely the fuzzy class of T_{fgs}-spaces.

Proposition 3.8

A space $X$is fuzzy T_{fgs} and extremally disconnected if and only if every fgs-closed subset of X is fuzzy preclosed.

Proof:

Suppose that X is fuzzy T_{fgs} and extremally disconnected. Let A be a fgs-closed subset. Then A is fsg-closed. By Theorem 2.3 (3) of [4], A is fuzzy preclosed. Now assume that every fgs-closed subset is fuzzy preclosed. Let $x \in X$ and suppose that $\{x\}$ is not fuzzy closed. Then $X/\{x\}$ is not fuzzy open, hence it is fgs-closed. By assumption, $X/\{x\}$ is fuzzy preclosed, and so $\{x\}$ is fuzzy preopen. ByTheorem 3.2, X is a fuzzy T_{fgs}-space. Moreover, since every fsg-closed subset is fuzzy preclosed, again by Theorem 2.3(3) of [5], X is extremally disconnected.By Theorem 3.6, in a fuzzy T_{fgs}-space, every fg-closed subset is $fg\alpha$-closed. This suggests a natural question: Characterize those spaces whose $fg\alpha$-closed subsets are fg-closed. Clearly, if X is nodec, i.e. every nowhere fuzzy dense subset of X is fuzzy closed [29], then every $fg\alpha$-closed subset of $X$is fg-closed since in that case $f\alpha(X)$ coincides with the given topology on X. Also observe that X is nodec if and only if every nowhere fuzzy dense subset is discrete as a subspace.

Theorem 3.9

For a fuzzy topological space X the following are equivalent:
(1) Every $fg\alpha$-closed set is fg-closed,
(2) Every nowhere fuzzy dense subset is locally indiscrete as a fuzzy subspace,
(3) Every nowhere fuzzy dense subset is fg-closed,
(4) Every $f\alpha$-closed set is fg-closed.

Proof:

(1) → (2) : Let $N \leq X$ be nowhere fuzzy dense, let Ube fuzzy open and let $N_1 = U \wedge N$.We have to show that N_1 is fuzzy closed in N. Since N_1 is nowhere fuzzy dense it is $f\alpha$-closed, hence $fg\alpha$-closed and so fg-closed. Since $N_1 \leq U$, we have$clN_1 \leq U$and hence $clN_1 \wedge N = N_1$, i.e. N_1 is fuzzy closed in N.

(2) → (3): Let $N \leq U$where $N$is nowhere fuzzy dense and U is open. Let $x \in clN$. Then $cl\{x\} \leq clN$. Since clN is nowhere fuzzy dense, by

(2) we havethat $cl\{x\}$ is also fuzzy open in clN, i.e. there exists an fuzzy open set W such that $cl\{x\} = W \wedge clN$. Suppose that $x \notin U$. Then $cl\{x\} \leq X/U$ and so $W \wedge N \leq W \wedge clN \wedge U = \emptyset$ a contradiction. Therefore $clN \leq U$.

(3) → (4) : Let $F \leq X$be $f\alpha$-closed. Then $F = A \vee N$ where A is fuzzy closed and $N$is nowhere fuzzy dense. If $F \leq U$where U is fuzzy open then, by our assumption, $clN \leq U$and so $clF \leq U$, i.e. F is fg-closed.

(4) → (1) : Let A be f$g\alpha$-closed with $A \leq U$where $U$is fuzzy open. By assumption $\alpha clA = A \vee cl(int(clA)) \leq U$. It is easily checked that $N = A/cl(int(clA))$ is nowhere fuzzy dense, hence $f\alpha$-closed and so fg-closed by assumption. Since $N \leq U$ we have $clA \wedge (X/cl(int(clA))) \leq U$. It follows readily that $clA/\alpha clA \leq U$and so $clA \leq U$, i.e. A is fg-closed.

In our next example we will show that there exist spaces whose nowhere fuzzy dense subsets are fg-closed but which are not nodec.

Example 3.10

Let X be the real line and let $X_1 = \{x \in X: x > 0\}$ and $X_2 = \{x \in X: x < 0\}$. We now define a fuzzy topology on Xin the following way. Let $\{0\}$ be fuzzy open. If $x \in X_1$, a basic (minimal) open neighbourhood of x is $X_1 \vee \{0\}$. If $x \in X_2$, a basic (minimal) open neighbourhood of $x$is $X_2 \vee \{0\}$. Clearly, if $x \in X_1$ then $\{0\}$ is nowhere fuzzy dense but not fuzzy closed, so X fails to be nodec. Now let $N \leq U$ where $N$is nowhere fuzzy dense and $U$is fuzzy open. Then $0 \notin N$. Let $N_1 = N \wedge X_1$and $N_2 = N \wedge X_2$. If $x \in N_1$ then $x \in U$ and so $X_1 \leq U$. Hence $clN_1 \leq X_1 \leq U$. In the same manner, $clN_2 \leq U$and so $clN \leq U$, i.e. N is fg-closed.

4. More Characterizations

We now return to the diagram in Section 1 to consider other possible conversesof some of the implications in that diagram. The following result aboutthe class of extremallyfuzzy disconnected spaces was proved in [5].

Theorem 4.1

[5] For a fuzzy topologicalspace$X$, the following statements are equivalent:

(1) (X, τ) is extremally fuzzy disconnected,
(2)$(A \vee B) = sclA \vee sclB$ for all $A, B \leq X$,
(3) The union of two fuzzy semi-closed subsets of X is fuzzy semi-closed,
(4) The union of two fuzzysg-closed subsets of X is fuzzy sg-closed,
(5) Every fuzzy semi-preclosed subset of X is Fuzzy preclosed,
(6) Every fuzzysg -closed subset of X is fuzzy preclosed,
(7) Every fuzzy semi-closed subset of X is fuzzy preclosed,
(8) Every fuzzy semi-closed subset of X is fuzz yα-closed,
(9) Every fuzzy semi-closed subset of X is Fuzzy $g\alpha$-closed.

Next we present one more characterization of extremal fuzzy disconnectedness using fuzzy generalized closed subsets.

Theorem 4.2

A space X is extremally fuzzy disconnected if and only if everyfgs-closed subset of $X$is $f\alpha g$-closed.

Proof:

Suppose that X is extremally fuzzy disconnected. Let A be fgs-closed andlet U be an fuzzy open set containing A. Then $sclA \leq U$, i.e. $int(clA) \leq U$. Since $int(clA)$ is fuzzy closed, we have

$$\alpha clA = A \vee cl(int(clA)) \leq A \vee int(clA) \leq U.$$

Hence A is$f\alpha g$-closed.

To prove the converse, let every fgs-closed subset of Xbe $f\alpha g$-closed. Let$A \leq X$be fuzzy regular open. Then $A$is fgs-closed and so $f\alpha g$-closed. It follows that$clA = cl(int(clA)) = \alpha clA \leq A$. Therefore A is fuzzy closed and $X$is extremally fuzzydisconnected.

We now consider the property of fsg-submaximality. First we give some elementary characterizations of fsg-submaximal spaces. Since the proof of the following result is straightforward, we will omit it.

Theorem 4.3

For a fuzzy topological space X, the following are equivalent:

(1) X is fsg -submaximal,
(2) Every fuzzy subset of X is an intersection of a fuzzy closed subset and an fsg-open subset of X,
(3) Every fuzzy subset of X is a union of an open subset and an fsg -closed subset of X,
(4) Every fuzzy co-dense subset A of X is fsg – closed,

(5) clA/A isfsg -closed for every fuzzy subset A of X.

A more advanced result about fsg -submaximality was obtained in [5].

Theorem 4.4

[5] For a fuzzy topological space X with Hewitt decomposition$X = F \cup G$, the following are equivalent:

(1) $X_1 \leq clG$,

(2) Every fuzzy preclosed subset of X is fsg – closed,

(3) X isfsg -submaximal,

(4) X isfsg -submaximal with respect to $f\alpha(X)$.

We shall now improve the equivalence of (2) and (3) in Theorem 4.4 Thereby providing a new characterization of fsg -submaximal spaces.

Theorem 4.5

A fuzzy topological space X is fsg -submaximal if and only if every fuzzy preclosedsubset of X is fgs -closed.

Proof:

The necessity is trivial by Theorem 4.4 (2). For the sufficiency, supposethat every fuzzy preclosed subset is fgs -closed. Let $X = F \vee G$ be the Hewitt decomposition of X, and let $\langle E_1, E_2 \rangle$ be a resolution of $intF$.

We first claim that every open set $V \leq intF$is regular open. In fact, $V \wedge E_1$is fuzzy co-dense and contained in V. Since fuzzy co-dense sets are fuzzy preclosed, byassumption, they are fgs-closed. Thus $int\big(cl(V \wedge E_1)\big) \leq V$. On the otherhand, E_1 is fuzzy dense in $intF$, hence we have $int\big(cl(V \wedge E_1)\big) = int(clV)$. Itfollows that $V = int(clV)$.

Now let $x \in intF$and let $V = intF \wedge (X/cl(\{x\}))$. Suppose that $\{x\}$ is nowhere fuzzy dense. Then $X/cl\{x\}$ is fuzzy dense and $int(clV) = int\big(cl(intF)\big) = intF$.By our claim, $intF = V$. Hence $intF \leq X/\{x\}$, a contradiction. Therefore$\{x\}$ has to be fuzzy preopen. We have thus proved that $intF \leq X_2$, i.e. $X_1 \leq clG$. By Theorem 4.4, X is fsg-submaximal.

Remark 4.6

One may define a space $X$to be fgs -submaximal if each fuzzy subset of $X$is fgs-open. Similar to the proof of Theorem 4.4, one checks easilythat a space $X$is fgs -submaximal if and only if each fuzzy preclosed subset of $X$is fgs -closed. In the light of Theorem 4.5, the notion of fgs -submaximality coincides with that of fgs -submaximality.

Conclusion:

In this paper, we discussed properties of generalized closed sets in fuzzy topological spaces and also discussed between fuzzy lower separation axioms, fuzzy exactly discontinuity, fuzzy sub-maximal space, fuzzy $T_{1/2}$-space and fuzzy T_1-space etc. In this paper through in future we introduced some applications.

References

[1] S. Arya and T. Nour, Characterizations of fuzzys-normal spaces, Indian J.Pure Appl. Math., 21 (1990), 717-71

[2]O. Bedre Ozbakir, On generalized fuzzy strongly semi-closed sets i fuzzy topological spaces, Itf.J.Math sci.30. No 11(2002)651-657.

[3] P. Bhattacharya and B. K. Lahiri, Fuzzy Semi-generalized closed sets in fuzzy topology, Indian J. Math., 29 (1987), 375-382.

[4] J. Cao, M. Ganster and I. L. Reilly, Onfsg-closed sets and $fg\alpha$-closedsets, Mem. Fac. Sci. Kochi Univ. Ser A, Math., to appear.

[5] J. Cao, M. Ganster and I. L. Reilly, Submaximality, extremal Disconnectedness and fuzzy generalized closed sets, Houston J. Math., to appear.

[6] C.L.Chang, Fuzzy topological spaces, (1968) 182-190.

[7] J. Dontchev, Onfuzzy submaximal spaces, Tamkang Math. J., (1995),243-250.

[8] J. Dontchev, On generating fuzzy semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A., Math. 16 (1995), 35-48.

[9] J. Dontchev, On some fuzzy separation axioms associated with the –α-Fuzzy topology, Mem. Fac. Sci. Kochi Univ. Ser A, Math., 18 (1997), 31–35.

[10] J. Dontchev and M. Ganster, On fuzzyδ-generalized closed sets And fuzzy $T_{3/4}$-spaces, Mem. Fac. Sci. Kochi Univ. Ser A, Math., 17 (1996), 15–31.

[11] J. Dontchev and H. Maki, On fuzzy sg-closed sets and fuzzy semi-λ-closed sets, Questions Answers Gen. Fuzzy topology, 15 (1997), 259-266.

[12] W. Dunham, $T_{1/2}$-spaces, Kyung pook Math. J.,17 (1977), 161-169.

[13] M. Ganster, Fuzzy preopen sets and fuzzy resolvable spaces, Kyungpook Math. J., 27 (1987), 135-143.

[14] M. Ganster, I. Reilly and M. Vamanamurthy, Remarks on fuzzy locally closed sets , Math.

Panon., 3 (1992), 107-113.
[15] E. Hewitt, A problem of set-theoretic fuzzy topology, Duke J. Math., 10 (1943), 309-333.
[16] D. Jankovi´c and I. L. Reilly, On fuzzy semi-separation properties, Indian J.Pure Appl. Math., 16 (1985), 957-964.
[17] N. Levine, Fuzzy Generalized closed sets in fuzzy topological spaces, Rend. Circ.Mat. Palermo, 19 (1970), 89-96.
[18] S. Maheshwari and R. Prasad, Some new fuzzy separation axioms,Ann. Soc. Sci. Bruxelles, 89 (1975), 395-402.
[19]H.Maki, T.Fukutake, M.Kojima, H.Harada, fuzzy Generalized closed set in fuzzy topological spaces, I.Meetings on topological spaces, theoryand its applications, (1998), 23-36.
[20] H. Maki, R. Devi and K. Balachandran, Fuzzy generalized α-closed sets in fuzzy topology, Bull. Fukuoka Univ. Ed. Part III, 42 (1993), 13-21.
[21] H. Maki, K. Balachandran and R. Devi, Remarks on fuzzy semi-generalizend closed sets and fuzzy generalized semi-closed sets, Kyungpook Math. J., 36(1996), 155-163.
[22] H. Maki, K. Balachandran and R. Devi, Associated topologies of fuzzy generalized α-closed sets and fuzzy α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 15 (1994), 51-63.
[23] S.R. Malghan and S.S. benchali open maos, fuzzy closed maps and local fuzzy compactness in fuzzy topological spaces. Int. J.Math.Anal.99. No.2 (1984) 74-79.
[24] O. Njastad, On some fuzzy classes of nearly open sets, Pacific J. Math., 15 (1965), 961–970.
[25] N. Palaniappan and K. Rao, Fuzzy regular generalized closed sets, Kyungpook Math. J., 33 (1993), 211-219.
[26] I. L. Reilly and M. K. Vamanamurthy, On fuzzy α-continuity in fuzzy topological spaces, Acta Math. Hungar., 45 (1985), 27-32.
[27] M.E El-shafei and A. zakari, fuzzy semi generalized continuous mappings in fuzzy topological spaces, J.Egpt.Mathsoc 15(1) (2007), 57-67.
[28] R.K. Saraf and M. Caldas preserving fuzzy sg-closed sets, Universidad catolica del note Antofagasta chile 25. No. 2 (2001) 127-138.
[29] E. van Douwen, Applications of maximal fuzzy topologies, fuzzy topology Appl., 51(1993), 125-240.
[30] L.A.Zadeh, Fuzzy set, Inform. Control 8 (1965), 338-353.