STUDY OF RESERVOIR PERFORMANCE USING PRESSURE TRANSIENT, MATERIAL BALANCE&DECLINE CURVE ANALYSIS

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ABSTRACT

Several methods of analyzing pressure build up data in wells have been presented by various authors. We dealt with the theory and method of Horner and Miller- Dyes-Hutchinson and presented calculations of the same and worked on data obtained during testing of various kinds of wells. These calculations include determination of formation permeability (K), skin factor(S), wellbore storage coefficient (Cs), additional pressure drop due to skin (ΔPs), flow efficiency (F.E) and initial reservoir pressure.

The semi-log analysis technique has been used to determine the permeability, apparent radius and skin from the pressure drawdown data. The drainage area, pore volume of the reservoir portion being drained by the test well and initial oil in place in the drainage area are calculated from the Cartesian plot of ΔP(initial reservoir pressure – flowing bottom hole pressure) versus time.

Deliverability tests are used for estimating the Absolute Open Flow Potential (AOFP) in gas wells. In this paper, the back pressure (flow after flow) test data are used for the estimation of the stabilized Absolute Open Flow Potential of gas wells.

Material balance equations have been used in the Oil industry for many years to evaluate the performance of oil and gas reservoirs. An attempt has been made to estimate the original hydrocarbon in place (N) and the size of the gas cap (m) for oil reservoirs under various drive mechanisms such as gas cap drive, depletion drive and water drive with the help of material balance straight line method.

Plotting P/Z versus cumulative gas production technique has been used to obtain original gas in place in gas reservoir.

Decline curve analysis is one of the most extensively used oil and gas reserve evaluation technique. The main aim of the method is to find an equation, which fits the observed rate-time performance graph using which future performance of the well is determined.

In this project production decline data of a gas well is used and type of decline is analyzed. The decline rate (D_i), production rate (q_i) and gas in place are estimated using the same.

INTRODUCTION

1.1: Background:

Transient well testing is one of practical methods to characterize reservoir properties or the ability of the formation to produce fluid. It has advantages compared to other techniques in determining reservoir properties. One of them is that well testing covers wider area to interpret, even more it can reach up to its boundaries if any. In addition, many types of test are also available depending on specific parameter to be analyzed.

One major purpose of well testing is to determine the ability of a formation to produce reservoir fluids. Further, it is important to determine a well's productivity. A properly executed and analyzed well test usually can provide information about formation permeability, extent of wellbore damage or stimulation, reservoir pressure and reservoir boundaries and heterogeneities.

The basic test method is to create a pressure drawdown in the wellbore; this causes formation fluids to enter the wellbore. If we measure the flow rate and the pressure in the wellbore during production or the pressure during a shut in period following production, we usually have sufficient information to characterize the tested well. Types of tests discussed are pressure build up tests, pressure drawn-down test and gas well tests.

Deliverability testing of gas wells for the estimation of stabilized Absolute Open flow (AOF) Potential is generally performed using backpressure (flow after flow) tests. The Absolute open Flow Potential represents a hypothetical gas flow rate that may be produced if the FBHP value is atmospheric pressure i.e. the maximum flow rate that the wellcan produce. The other techniques for deliverability testing are- Isochronal and modified Isochronal tests.

The general form of the material balance equation was presented by Schilthuis in 1941. In this equation, the cumulative withdrawal of the reservoir fluids is equated to the combined effects of fluid expansion, pore volume compaction and water influx.

In 1963, Havlena and Odeh presented techniques for interpreting the material balance equation as a straight line, which makes it easy to apply graphical techniques. Material balance method is used for estimating original oil in place, gas reserves (P/Z method), and recovery factors under different reservoir drive mechanisms.

A common method of predicting gas reserves is the graphical solution to the gas material balance equation. A special case of the material balance equation is linear in P/Z with cumulative gas production (G_p) which predicts the initial gas in place when P/Z is extrapolated to zero.

Decline curve method is used for estimating ultimate gas recoveries and predicting performance from the analysis of long term gas production data either individual well or for entire field. The first conventional analysis technique was presented by Arps. This conventional technique includes equation for exponential, harmonic and hyperbolic decline.

BASIC THEORY OF TRANSIENT WELL TESTING

2.1 Introduction:

The change of production rate of a well at the surface or subsurface creates pressure diffusion or transient in porous and permeable formations. The pressure diffuses away from the wellbore deep into the formation. From this change in pressure diffusion or transient, information about properties and characteristics of the reservoir are found out. This process is traditionally called pressure transient analysis.

The whole process of well test requires specific set-up. The standard well test set-up consists of surface rates, wellhead pressure (WHP), bottom hole pressure (BHP), acquisition and interpretation. Transient well testing applies the inverse solution of indirect measurement where input and output are known from the test, while the system is going to predict or estimate from interpretations. The system means well and reservoir characteristics, output represents pressure responses and input shows a change of rate.

As part of field data, well test data contributes in production analysis model (i.e. well test models, material balance models and decline curve analysis). This model allows engineers to simulate production forecast and run various scenarios with different production strategies.

Well test can investigate a much larger volume of the reservoir compared to cores and logs. Approximately depth of investigation of coring is only 10 cm and logging is 50cm, while well testing may reach 50-500 meters of investigation. Its larger area allows the estimations of reservoir permeability, skin factor, average pressure, fracture length, heterogeneities, drainage area, open flow potential and distance to the boundaries and many more.

2.2: Types of tests:

Certain types of tests are dedicated to specific stage of reservoir discovery, development and production. In exploration and appraisal wells, Drill Stem Tests (DSTs) and wireline formation tests are normally run. During primary, secondary and enhanced recovery stages, the conventional transient well tests (i.e. drawdown, buildup, and pulse and interference tests) are run. Step-rate, injectivity, fall-off, interference and pulse tests are executed during secondary and enhanced recovery stages. Some tests are implemented throughout the life of reservoir, such as multilayer and vertical permeability tests.

Each type of tests has various reservoir properties that canbeobtained. Some tests interpret same parameters, but the level of accuracy might be different. Table 2.1 lists the types of tests and various data which can be obtained from each test.

Table 2.1 – Reservoir Properties Obtainable from Various well Tests

Types of Tests	Data Obtained
DSTs	Reservoir Behavior Fluid samples Permeability Skin Fracture length Reservoir pressure Boundaries
Wire-line Formation Tests	Pressure profiles Fluid samples Some reservoir properties
Drawdown tests	Reservoir behavior Permeability Skin Fracture length Reservoir limit Boundaries
Buildup tests	Reservoir behavior Permeability Skin Fracture length Reservoir limit Boundaries
Step-rate tests	Formation parting pressure Permeability Skin
Falloff tests	Mobility in various banks Skin Reservoir pressure Fracture length Boundaries
Interference and pulse tests	Reservoir type behavior Porosity Vertical permeability

2.3: Flow Regimes categories:

With production, fluid flows in the reservoir with different ways at different times, generally based on the shape and size of the reservoir. Flow behavior classification is studied in terms rate of change of pressure with respect to time. Three main flow regimes will be described in this sub-chapter; they are steady state flow, pseudo steady state flow and transient state flow.

2.3.1 Steady State Flow:

In steady state flow, there is no pressure change anywhere with time i.e. the pressure change is zero.

Mathematically,

$\left(\frac{\partial P}{\partial t}\right) i = 0$ …………………………………… 2.1

It occurs during the late time region when the reservoir has gas cap or aquifer support. This type of flow regime can also be achieved when the reservoir is completely recharged by some pressure maintenance operations.

2.3.2: Pseudo-Steady State Flow:

This is the flow condition when the pressure at different locations in the reservoir declines linearly as a function of time i.e. at a constant decline rate.

Mathematically,

$\left(\frac{\partial P}{\partial t}\right) i = constant$ …………………2.2

This flow regime also occurs in late- time region, but it forms when there is no flow in the reservoir outer boundaries. No flow boundaries can be caused by the effect of presence of nearby sealing faults. This flow regime is also called as semi steady state, quasi-steady state or depletion state. During buildup or falloff tests, pseudo steady state is not occurred.

2.3.3: Transient State Flow:

This flow is also called as unsteady state flow. This is the fluid flow condition rate in which the change of pressure with respect to time at any location is not zero or constant i.e. the pressure changes irregularly at any location with respect to time. The pressure change is a function of time “t” and location “i”.

$\left(\frac{\partial P}{\partial t}\right) i = f(t.i)$…………………………2.3

Such flow occurs due to well geometry and the reservoir properties (i.e. permeability and heterogeneity). It is observed before boundary effects are reached or called infinite acting time period. Higher compressibility of the fluid leads to more pronounced unsteady state effect of the reservoir fluid. This part of flow usually becomes a focus on well test interpretation.

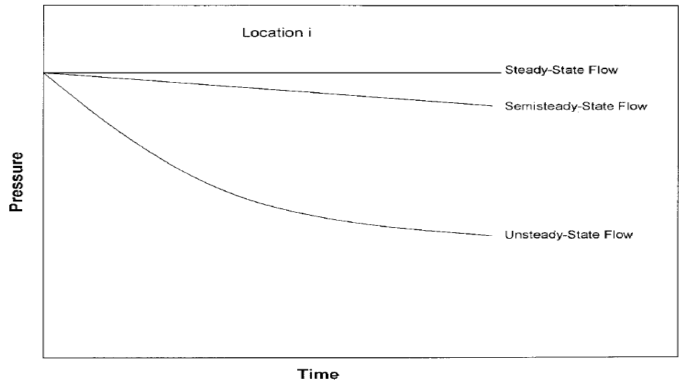

Figure 2.1: Flow Regimes

The typical pressure plot with respect to time for the entire fluid flow behavior in the reservoir is shown in Figure 2.1.

PRESSURE BUILDUP, DRAWDOWN AND GAS WELLTESTS

3.1: Introduction

Pressure build-up is a type of pressure transient test. This test is conducted by producing a well at constant rate for some time and then shutting the well in (usually at the surface) allowing the pressure to build up in the wellbore. Bottom-hole pressure of the well is recorded during this process as a function of time. From the data obtained it is possible to determine formation permeability, current drainage area pressure, reservoir heterogeneities or boundaries and to characterize damage or stimulation.

The methods used to analyze the data of pressure buildup test are-

Horner plot: This procedure is strictly correct for infinite acting reservoir but these plots can also be interpreted correctly for finite reservoirs.

Type curves: This is another important technique for buildup tests

Pressure drawdown test is conducted by producing well, starting ideally with uniform pressure in the reservoir. Rate and pressure are recorded as function of time.

The objective of a drawdown test usually includes estimates of permeability, skin factor and also reservoir volume in closed systems. These tests are particularly applicable to- new wells, wells that have been shut in sufficiently long to allow the pressure to stabilize and wells in which loss of revenue incurred in a buildup test would be difficult to accept. Exploratory wells are frequent candidates for lengthy drawdown tests, with a common objective of determining minimum or total volume being drained the well.

Deliverability testing of gas wells for the estimation of stabilized Absolute Open flow (AOF) Potential is generally performed using backpressure (flow after flow), isochronal and modified isochronal tests. The Absolute open Flow Potential represents a hypothetical gas flow rate that may be produced if the BHFP value were equivalent to the atmospheric pressure i.e. the maximum flow rate that the reservoir may production

3.2: Ideal Buildup Test:

By ideal test we mean a test in an infinite, homogeneous, isotropic reservoir containing a slightly compressible, single-phase fluid with constant fluid properties. Any wellbore damage or stimulation is considered to be concentrated in a skin of zero thickness at the wellbore (i.e. skin is considered to be zero). At instant shut in, flow into the wellbore ceases totally (i.e. the well column is filled with fluid)

But the actual buildup will not follow this description, but analysis of idealized method will help to prove for more realistic situations if we recognize the effect of deviation from the ideal behavior on the actual test behavior.

Assumption: A well is producing from an infinite acting reservoir(i.e. no boundary effects) are felt during the entire flow and later during shut in period. The formation and fluid have uniform properties.

Horner's pseudo producing time approximation is applicable. If the well has produced for a time (tp) and rate (q) before shut in and the elapsed since shut in is Δt then-

$$P_i\text{-}P_{ws}=-70.6\frac{q\mu B}{Kh}\left[\ln\left(\frac{1688\Phi\mu Ctrw^2}{K(tp+\Delta t)}\right)-2S\right]-70.6\frac{(-q)\mu B}{Kh}\left[\ln\left(\frac{1688\Phi\mu Ctrw^2}{K\Delta t}\right)-2S\right]$$

$$P_i\text{-}P_{ws} = \frac{-70.6q\mu B}{Kh}\left[ln\frac{\Delta t}{(\Delta t+tp)}\right]$$

$$P_i\text{-}P_{ws} = \frac{70.6q\mu B}{Kh}\left[ln\frac{(\Delta t+tp)}{\Delta t}\right]$$

$$P_{ws} = P_i - 162.6\frac{q\mu B}{Kh}\left[log\frac{(\Delta t+tp)}{\Delta t}\right] \quad \text{......................................3.1}$$

Where, P_{ws}= shut in bottom hole pressure

Equation 3.1 suggests that P_{ws} recorded during a pressure buildup test should plot as a straight line function of $\log\frac{(\Delta t+tp)}{\Delta t}$.

Further, the slope m of this straight line should be-

$$m = -162.6\frac{q\mu B}{Kh}$$

It is convenient to use the absolute value of m in test analysis,

$$m = 162.6\,\frac{q\mu B}{Kh} \quad \text{..3.2}$$

Thus formation permeability, K can be determined from a buildup test by measuring the slope m. in addition, if we extrapolate this straight line to infinite shut-in time [i.e. $\frac{(\Delta t+tp)}{\Delta t} = 1$] the pressure at this time will be the original formation pressure. P_i (P*).

Conventional practice in the industry is to plot P_{ws} Vs $\frac{(\Delta t+tp)}{\Delta t}$, figure 3.1, on semi logarithmic paper with values of $\frac{(\Delta t+tp)}{\Delta t}$ decreasing from left to right. The slope m on such a plot is found by simply subtracting the pressures at any two points on the straight line that are one cycle(i.e. a factor of 10) apart on the semi log paper.

The skin factor S can also be determined from the data available in the idealized pressure buildup test. At the constant a well is shut in, the flowing BHP (P_{wf}) is-

$$Pwf = Pi + 70.6\,\frac{q\mu \boldsymbol{B}}{Kh}\left[\left(ln\,\frac{1688\Phi\mu ctrw^{2}}{Ktp}\right) - 2S\right]$$

$$Pwf = Pi + m\left[log\left[ln\,\frac{1688\Phi\mu ctrw^{2}}{Ktp}\right] - .869S\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.3$$

At shut in times Δt in the buildup test-

$$Pws = Pi - m\,\log\left[\frac{(\Delta t + tp)}{\Delta t}\right]$$

Combining these two equations and solving for the skin factor S, we have-

$$S = 1.151((Pws-Pwf)/m) + 1.151\log((1688\Phi\mu ctrw^{2})/K\Delta t) + 1.151\log((tp+\Delta t)/tp) \ldots\ldots\ldots...3.4$$

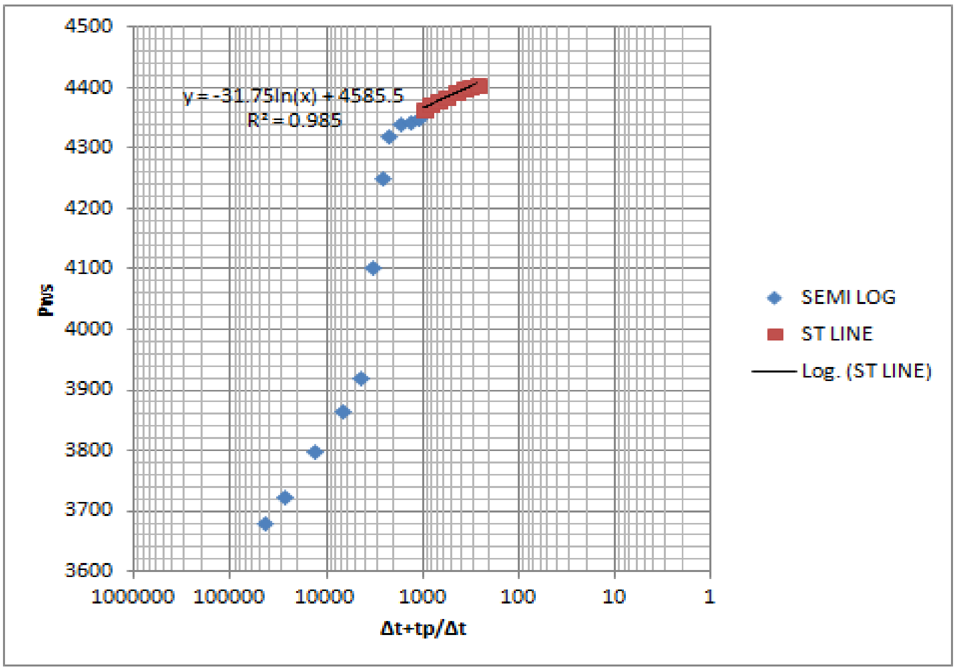

Figure: 3.1 Semi-log plot P_{ws}(shut in Pressure) Vs (tp+Δt)/Δt(Horner time ratio)

It is conventional practice in the petroleum industry to choose a fixed shut in time Δt of 1hour and the corresponding shut in pressure, P_{1hr}, to use in this equation. The pressure, P_{1hr} must be on the straight line or its extrapolation. We usually can assume further that $\log\left[\frac{(\Delta t+tp)}{\Delta t}\right]$ is negligible. With these simplifications-

$$S = 1.151\left[\frac{(P1hr - Pwf)}{m} - log\left(\frac{K}{\Phi\mu Ctrw^{2}}\right) + 3.23\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.5$$

The slope m is considered to be a positive number in this equation.

In summary, from the ideal buildup test, we can determine formation permeability, original reservoir pressure (Pi) and skin factor(S) which is a measure of damage or stimulation.

3.3: Actual Buildup Tests:

Encouraged by the simplicity and ease of application of the ideal buildup test theory, we mat test an actual well and obtain a most discouraging result. Instead of a single line for all times, we obtain a curve with a complicated shape. To explain what went wrong, the radius of investigation concept is useful. Based on this concept, we logically can divide a buildup curve into three regions, Figure 3.2- an early time region (ETR) during which a pressure transient is moving through the formation nearest the wellbore; a middle time region (MTR) during which the pressure transient has moved away from the wellbore and into the bulk formation and a late time region (LTR) in which the radius of investigation has reached the well's drainage boundaries.

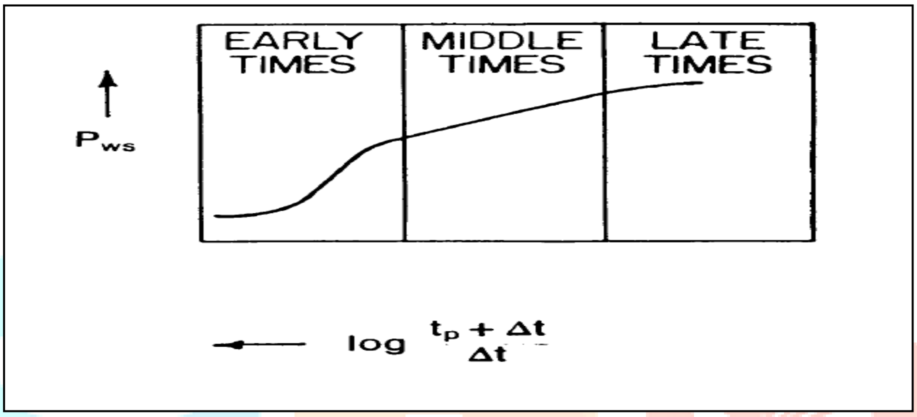

Figure 3.2: Actual buildup test graph

3.3.1 Early Time Region:

We know most of the wells have altered permeability near the wellbore. Until the pressure transient caused by shutting in the well for the build-up test moves through this region of altered permeability, there is no reason to expect a straight line that is related to formation permeability (we should note that ideal build up curve i.e. one with a single straight line over virtually all time is possible for a damaged well only when the damage is concentrated in a very thin skin at the sand face).

There is another complication at earliest times in a pressure buildup test. Continued movement of fluids into a wellbore (after flow, a form of wellbore storage) following the usual surface shut-in compresses the fluids (gas, oil and water) in the wellbore.

The answer to the reasons how this affect the character of a buildup curve at earliest times, perhaps the clearest answer lies in the observation that the idealized theory leading to the equation-

$P_{ws} = P_i - m\, log \left[\frac{tp + \Delta t}{\Delta t}\right]$……………………………………………….3.6

Explicitly assumed that at Δt=0, flow rate abruptly changed from q to zero. In practice, of decline towards zero but, at the instant of surface shut-in, the downhole rate is in fact still q, figure 3.3. Thus, one of the assumptions we made in deriving the buildup equation is violated in the actual test. One more question arise is that does that after flow ever diminish to such an extent that data obtained in a pressure buildup test can be analyzed as an ideal test. The answer for this question is yes. But the important problem of finding the point at which after-flow ceases distorting buildup data remains. This is the point at which the early time region usually ends, because after-flow frequently lasts longer than the time required for a transient to move through the altered zone near a well.

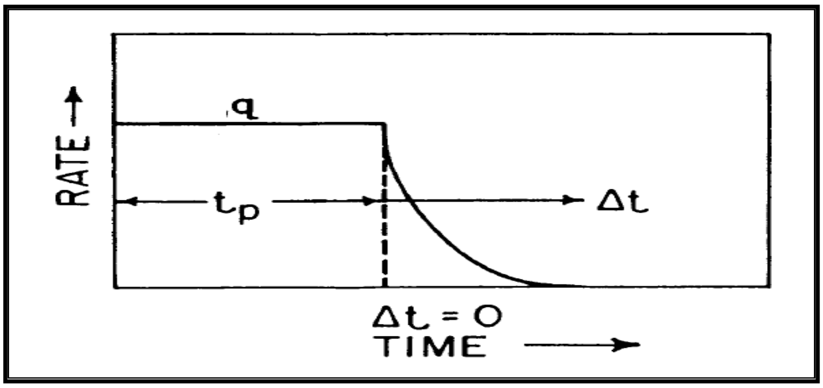

Figure 3.3: Rate history for actual buildup test.

3.3.2: Middle Time Region:

When the radius of investigation has moved beyond the influence of the altered zone near the tested well, and when after-flow has ceased distorting the pressure buildup test data, we usually observe the ideal straight line whose slope is related the formation permeability. This straight line ordinarily will continue until the radius of investigation reaches one or more reservoir boundaries, massive heterogeneities or a fluid/ fluid contact.

Systematic analysis of a pressure buildup test using the Horner method of plotting p_{ws}Vs log $\frac{tp+\Delta t}{\Delta t}$ requires that we recognize this middle time line and that, in particular, we do not confuse it with false straight lines in the early and late time regions. As we have seen determination of reservoir permeability and skin factor depends on recognition of the middle time line, estimation of the average drainage
is pressure for a well in a developed field also requires that this line to be defined.

3.3.3: Late Time Region:

Given enough time, the radius of investigation eventually will reach the drainage boundaries of a well. In this late time region pressure behavior is influenced by boundary configuration, interference from nearby wells, significant reservoir heterogeneities and fluid/ fluid contacts.

3.4: Deviations from assumptions in Ideal Test theory:

In suggesting that test logically can be divided into early, middle and late time regions, we have recognized that several assumptions made in developing the theory of ideal buildup test behavior are not valid for actual tests. In this section, we examine further the implications of three over idealized assumptions- the infinite reservoir assumptions, the single phase liquid assumptions and the homogeneous reservoir assumption.

3.4.1: Infinite Reservoir Assumption:

In developing the equation suggesting the Horner plot, we assumed that the reservoir was infinite acting during both the production period preceding the buildup test and the buildup test itself. Frequently the reservoir is at pseudo steady state before shut-in; if so, its logarithmic approximation should not be used to describe the pressure drawdown caused by the producing well.

$(Pi\text{-}Pwf)_{\text{prod well}} \neq 70.6\frac{q\mu B}{Kh}\left\{ln\left[\frac{1688\Phi\mu Ctrw^2}{K(tp+\Delta t)}\right]\right\}$……………………………….3.6

Instead, if the well is centered in a cylindrical reservoir-

$(Pi\text{-}Pwf)_{\text{prod well}} = 141.2\frac{q\mu \boldsymbol{B}}{Kh}\left[\frac{0.000527\,K(tp+\Delta t)}{\Phi\mu Ctrw^{2}} + ln\left(\frac{re}{rw}\right) - \frac{3}{4}\right]$…………………...3.7

Thus, we must conclude that in principle, the Horner plot is incorrect when the reservoir is not infinite acting during the flow period preceding the buildup test. Boundaries become important as ri→re.

The problem is compounded when ri→re after shut in. then too, the Horner plot is incorrect in principle.

This difficulty is resolved in different ways by different analysis. One method for analysis is the research of Cobb and Smith. Other analysis methods for finite acting reservoir are discussed by Miller, Dyes and Hutchinson (MDH) and Slider. Many analyses use the data plotting method suggested by MDH because it is simpler than the Horner method.

3.4.2: Single Phase Liquid Assumption:

The assumption that a petroleum reservoir contains only a single phase liquid must be modified. Even reservoirs in which only oil flows contain immobile water saturation; many also contain immobile gas saturation. Also, in many cases, compressibility of the formation cannot be ignored. These factors are taken into account if we use total compressibility (C_t) in solutions to flow equations-

$C_t = C_oS_o+C_wS_w+C_gS_g+C_f$ ……………………………………………….3.8

Even in simple phase flow, when $S_g \neq 0$, evaluation of oil compressibility, C_o and water compressibility, C_w, are somewhat complicated.

$C_o = \frac{-1}{Bo}\frac{dBo}{dP} + \frac{Bg}{Bo}\frac{dRs}{Dp}$ ……………………………………………………3.9

$C_w = \frac{-1}{Bw}\frac{dBo}{dP} + \frac{Bg}{Bw}\frac{dRsw}{dP}$……………………………………………...3.10

3.4.3: Homogeneous Reservoir Assumption:

No reservoir is homogeneous, yet solutions to the flow equations are valid only for homogeneous reservoirs. The solutions prove to be adequate for most real reservoirs, particularly early in time while conditions nearest the tested well dominate test behavior. Rate of pressure change is dominated by average rock and fluid properties. When massive heterogeneities are encountered (particularly in a localized portion of the reservoir), the simple solutions to flow equations lose accuracy. Examples include changes of depositional environment, with resultant changes in permeability or thickness and some fluid/fluid contacts. The longer a test is run, the higher the probability that a significant heterogeneity will be encompassed within the radius of investigation and thus influence the test.

3.5: Effects and duration of After-flow:

So far, we have noted several problems that after-flow causes the buildup test analyst. Summarizing, these problems include-

i. Delay in the beginning of the MTR, making its recognition more difficult.

ii. Total lack of development of the MTR in some cases, with relatively long periods of after-flow and relatively early onset of boundary effects.

iii. Development of several false straight lines, anyone of which could be mistaken for the MTR line.

We not further that recognition of the middle time line is essential for successful buildup analysis based on the Horner plotting method. P_{ws}Vs$\frac{tp+\Delta t}{\Delta t}$, the linemust be identified to estimate reservoir permeability,to calculate skin factor and estimate static drainage area pressure. The need for methods to determine when after-flow ceased distorting a buildup test is clear by the next few lines.The characteristic influence of after-flow on a pressure buildup plot is a lazy S shape at early times, Figure 3.4.

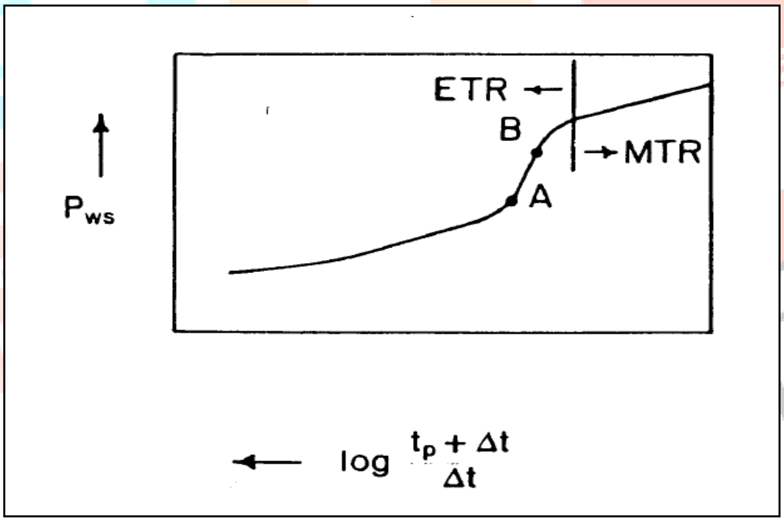

Figure 3.4: Characteristic influence of after-flow on Horner graph.

In some tests, parts of the S-shape may be missing in the time range during which data have been recorded. Example: data before Time A may be missing or data for times greater than Time B may be absent. Thus, the shape of the buildup test alone is not sufficient to indicate the pressure or absence of after-flow- it is merely a clue that sometimes indicated presence of after-flow

3.6: Analysis of Pressure Buildup test:

Conventional well test interpretation is done based on Horner or MDH pressure buildup analysis. Analysis is done based on basic wellbore reservoir model such as cylindrical homogeneous reservoir of infinite radial extent with impermeable upper and lower boundaries and uniform initial pressure throughout. The outer and inner boundary conditions are:

Outer boundary condition- provides a constant average pressure.

Inner boundary condition- it is the one which allows the well to flow at a constant rate q for a period of time tp and then to be shut in for a period of Δt. The inner boundary condition for this model is allow for a zone of altered K(skin) to exist near wellbore.

When a well is shut in, the pressure in the drainage area will recover to a stabilized value. The interpretation of the stabilized pressure depends on the number of wells producing from the reservoir.

If a single well reservoir, the reservoir pressure will recover to an average reservoir pressure.

For DST in a new reservoir, the average reservoir pressure should be equal to P_i if there is no depletion and if there is depletion, the average reservoir pressure will be lower than the initial reservoir pressure.

In multiple well reservoirs, the stabilized pressure at the shut in, the well will simply reflect the pressure drop due to other producing wells.

Most of the buildup tests are carried out during the initial completion or work-over to determine the skin factor if there is any damage suspected during the initial production test.

3.6.1: Horner Plot:

Using the principle of superposition for a well producing at a constant rate q during a producing time tp, Horner derived the following equation-

$P_{ws} = P_i - 162.6 \frac{q\mu B}{Kh} log\left(\frac{tp+\Delta t}{\Delta t}\right)$ ………………………………………….3.11

Where, $tp = \frac{24\ Np}{q}$ ………………………………………………………..3.12

P_{ws} = shut in well pressure (Psia)

Δt = shut in time (hours)

N_p = cumulative oil produced (STB) at a constant rate.

Equation 3.11 indicates if there is a plot between P_{ws} or ($\Delta P = P_i - P_{ws}$) Vs $\frac{tp+\Delta t}{\Delta t}$ on semi log paper should yield a straight line of slope m during infinite acting radial flow portion of the curve.

$m = \frac{162.6 q\mu B}{Kh}$…………………………………………………………..3.13

$K = \frac{162.6 q\mu \boldsymbol{B}}{mh}$…………………………………………………….……..3.14

This Figure 3.6 Plot is called as Horner Plot.

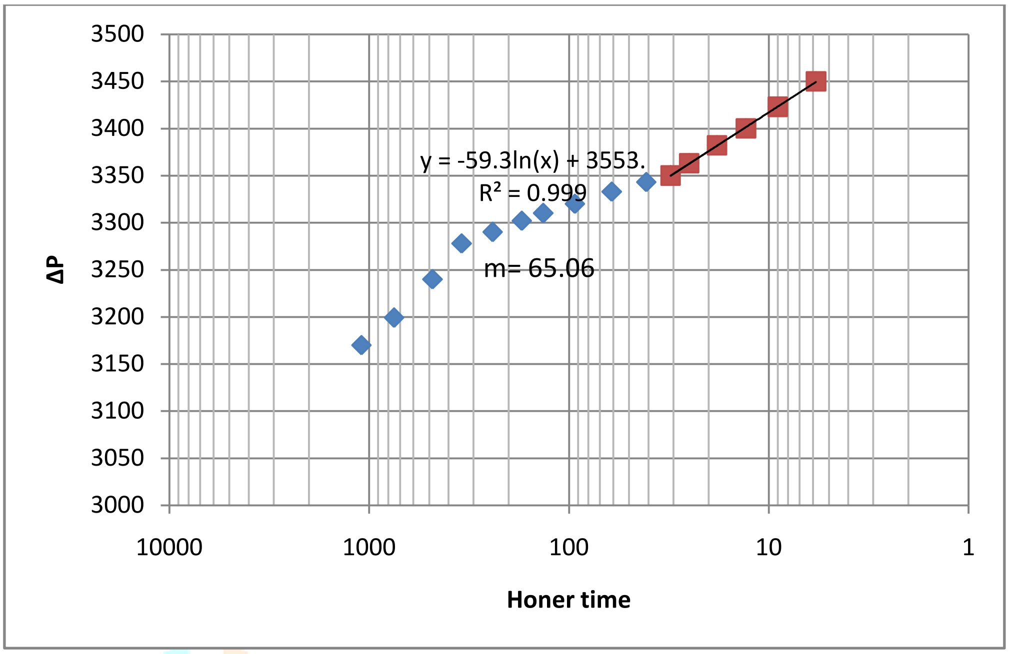

Figure 3.5: Horner Plot

The extrapolation of the straight line section to an infinite shut in time i.e. $\frac{tp+\Delta t}{\Delta t}$ =1 gives an estimates of Pi in a new reservoir, figure 3.7.

At infinite shut in time, Horner time $\frac{tp+\Delta t}{\Delta t}$ =1. Then the equation 3.11 becomes[$\Delta t \to \infty$]

P_{ws} = Pi - $\frac{162.6q B}{Kh}$ log(1)………………………………….3.15

P* = Pi……………………………………………………..3.16

So in practice, P* is approximately equal to Pi, only in a newly discovered well. For a reservoir with strong water drive P* will be close to average reservoir pressure i.e. P*$\approx Pi \approx Pavg$. For a bounded reservoir that is already producing under pseudo steady state, P* will always be higher than average reservoir pressure. In an old reservoir, the following relation always exists.

P_i>P*>Ṗ

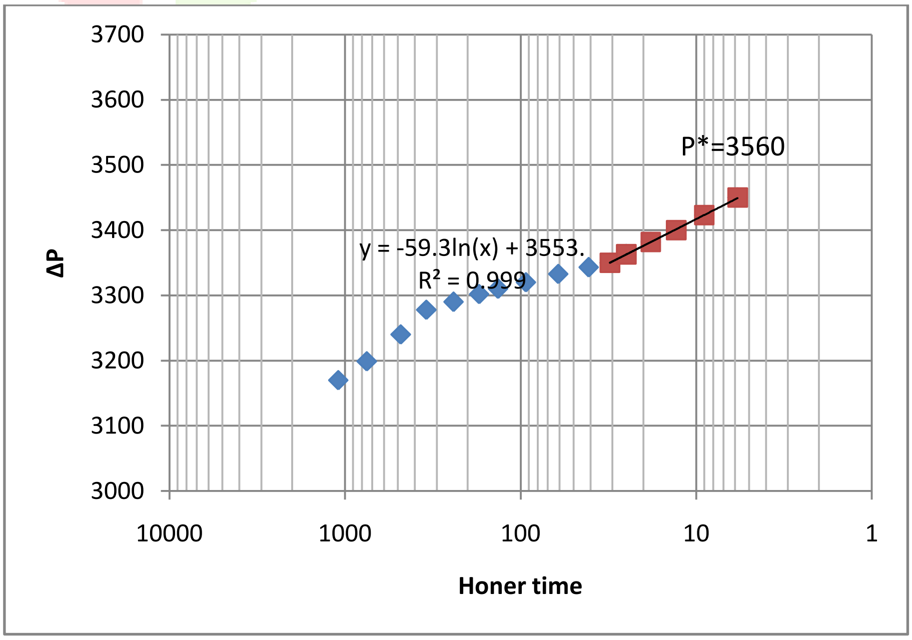

Figure 3.6: P* from Horner Plot

3.6.1.1: Equivalent Time:

In a conventional Horner Plot, it is assumed that the total production time (t_p) is much higher than the shut in period.

$t_p > \Delta t_{total}$...3.17

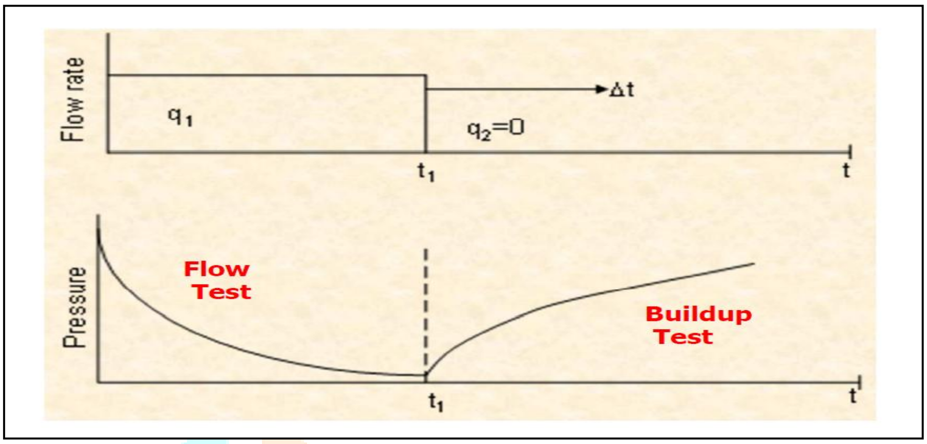

Figure 3.7: Buildup test

Prior to buildup, the well is allowed to flow (drawdown) at a constant rate until the stabilized pressure is achieved at t_p=0. But this is rarely the case, if ever. The pressure drop at the wellbore, prior to shut in is given by-

$P_{wf(\Delta t=0)} = P_i - \frac{162.6q\mu B}{Kh}\left[logtp + log\left(\frac{K}{\emptyset\mu Ctrw^2}\right) - 3.23 + 0.869S\right]$.........3.18

Subtracting equation 2.18 from equation 2.11-

$P_{ws} - P_{wf} = -\frac{162.6q\mu B}{Kh}\left[log\left(\frac{tp+\Delta t}{\Delta t}\right) - logtp - log\left(\frac{K}{\emptyset\mu Ctrw^2}\right) + 3.23 - 0.869S\right]$

$\Delta P = -m\left[log(tp+\Delta t) - log\Delta t + logtp - log\left(\frac{K}{\emptyset\mu Ctrw^2}\right) + 3.23 - 0.869S\right]$

$\Delta P = m\left[log(\Delta t.tp) - log(tp+\Delta t) + log\left(\frac{K}{\emptyset\mu Ctrw^2}\right) - 3.23 + 0.869S\right]$

$\Delta P = m log\left[\frac{tp+\Delta t}{\Delta t}\right] + log\left(\frac{K}{\emptyset\mu Ctrw^2}\right) - 3.23 + 0.869S$.....................3.19

Therefore, effective or equivalent shut in time is-

$\Delta te = \frac{tp}{\frac{tp+\Delta t}{\Delta t}} = \frac{tp.\Delta t}{tp+\Delta t}$ 3.20

$\Delta P = m\left[log(\Delta te) + log\left(\frac{K}{\emptyset\mu Ctrw^2}\right) - 3.23 + 0.869S\right]$.....................3.21

The plot of equation 2.21 ΔP= (Pws-Pwf) vs. Δte on a semi log, figure2.8, should yield a straight line with slope m, corresponding to infinite acting radial flow line. Then K is calculated once m is determined.

$K = \frac{162.6q\mu\boldsymbol{B}}{mh}$...3.22

The Δte is useful for radial flow in homogeneous infinite acting reservoir. It is not recommended in hydraulically fractured wells, where linear flow is dominant during the early time pressure data. ΔteShould not be used when multiphase flow is dominant.

3.6.1.2: Determination of Skin:

The skin factor is obtained from Horner Plot at Δt=1hour and P1hr, Figure 3.8.

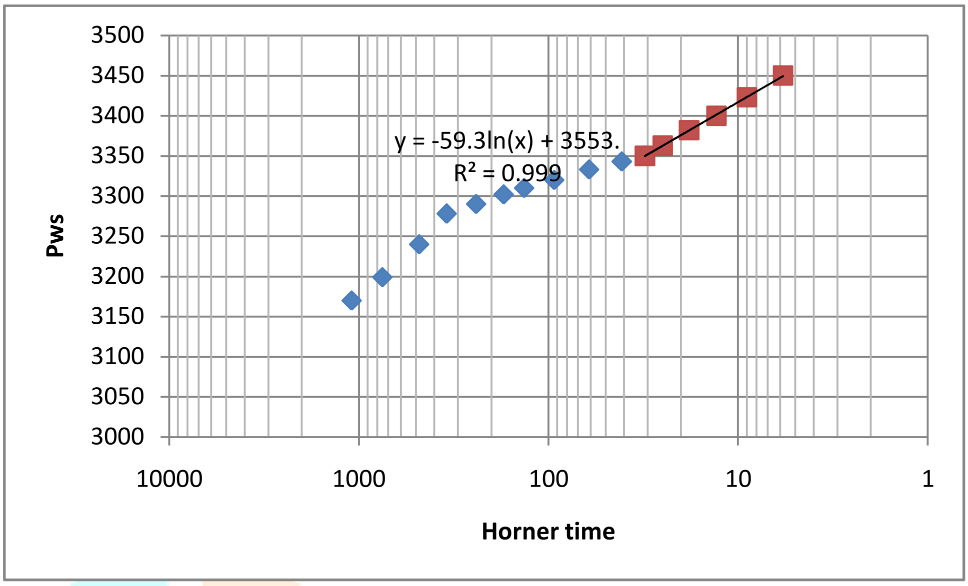

Figure 3.8: Horner Plot

$$S = 1.151\left[\frac{P1hr - Pwf\,(\Delta t=0)}{m}\ log\left(\frac{K}{\emptyset\mu Ctrw^{2}}\right) + 3.23\right] \ldots\ldots 3.23$$

And the skin factor preferable to plot of ΔP= (P_{wf}-P_{ws}) Vs Horner time $\frac{tp+\Delta t}{\Delta t}$ or Δte, at $\Delta t = 1\ hour$ and ΔP1hr, figure 3.10.

$$S = 1.151\left[\frac{\Delta P1hr}{m} - log\left(\frac{K}{\emptyset\mu Ctrw^{2}}\right) + 3.23\right] \ldots\ldots 3.24$$

3.6.2: MDH Plot:

Miller- Dyes- Hutchinson derived an equation in terms of P_{ws} and Δt.

$$\Delta P = P_{wf}(\Delta t = 0) - P_{ws} = \frac{162.6q\mu B}{Kh}\left[log\left(\frac{K}{\emptyset\mu Ctrw^{2}}\right) - 3.23 + 0.869S\right] \ldots\ldots 3.25$$

Thus a semi log plot of shut in pressure (P_{ws}) or ΔP Vs shut in time Δt is referred to as MDH plot and it should yield a straight line during infinite acting period of the test. The slope, m, is used to calculate the formation permeability.

$$K = \frac{162.6q\mu B}{mh} \ldots\ldots 3.26$$

3.7: Well near a sealing fault:

3.7.1: Horner Plot:

A plot of p_{ws} Vs Horner time will yield two straight lines. The first line(after wellbore storage effects) of slope m corresponds to infinite acting time, with the help of which slope(m), formation permeability(K) and skin factor (S) are calculated, figure 3.9.

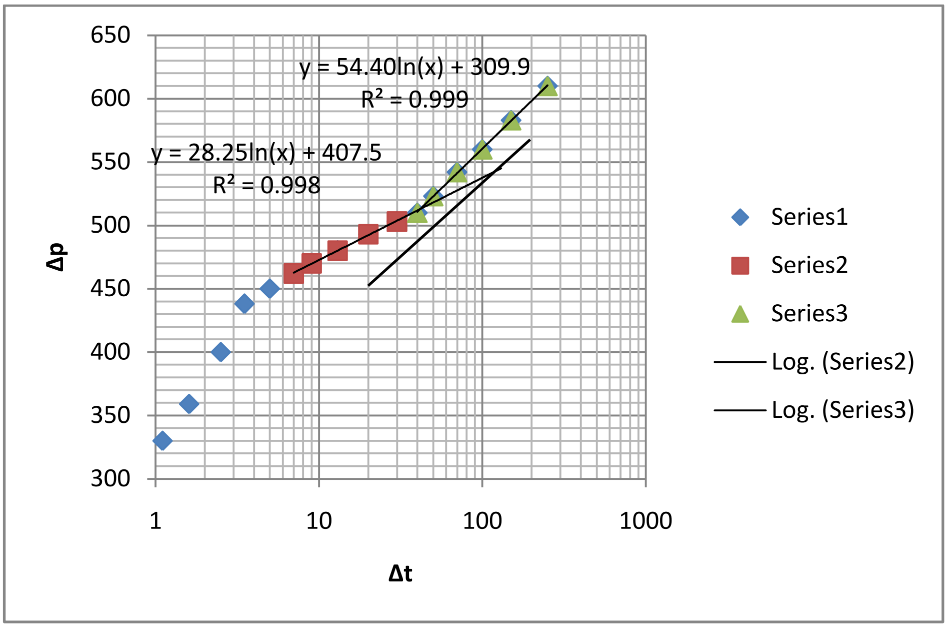

Figure.3.9: Intersection of two slopes.

The second straight line has slope of 2m and corresponds to effect of a sealing no flow boundary. The intersection of the two straight line will help to calculate the distance between the well and sealing fault.

$$d = 0.01217\sqrt{\left(\frac{k}{\emptyset\mu Ct}\right)\frac{tp}{\left(\frac{tp+\Delta t}{\Delta t}\right)x}} \quad \ldots\ldots 3.27$$

3.7.2: MDH Plot:

The intersection of straight lines of slope m1 and m2 are used to calculate the distance between the test well and no flow boundary.

$$d = 0.01217\sqrt{\left(\frac{k}{\emptyset\mu Ct}\right)(\Delta t)x} \quad \ldots\ldots 3.28$$

3.8: Pressure Derivative Analysis:

Pressure derivatives can exchange pressure signals and in general, have been shown to be more sensitive to disturbances in the reservoir [6]. This result in greater detail on a derivative graph than it is apparent on a pressure graph. To use pressure derivatives in well test analysis, it is necessary to develop design equations based on pressure derivatives for the system. One useful derivative is the semi-log pressure derivatives, which is defined as the derivative of well pressure with respect to the natural logarithm of time. In dimensionless form, the semi-log pressure derivation is-

$$\frac{dP_wD}{d\ln t_D} = t_D\frac{dP_wD}{dt_D} \quad \ldots\ldots 3.29$$

On a log-log graph of semi-log pressure derivatives versus time, the various transient flow regimes show characteristic slopes. The possible transient

3.9: Pressure drawdown test:

Pressure drawdown test is conducted by producing well, starting ideally with uniform pressure in the reservoir. Rate and pressure are recorded as functions of time, figure 3.10.

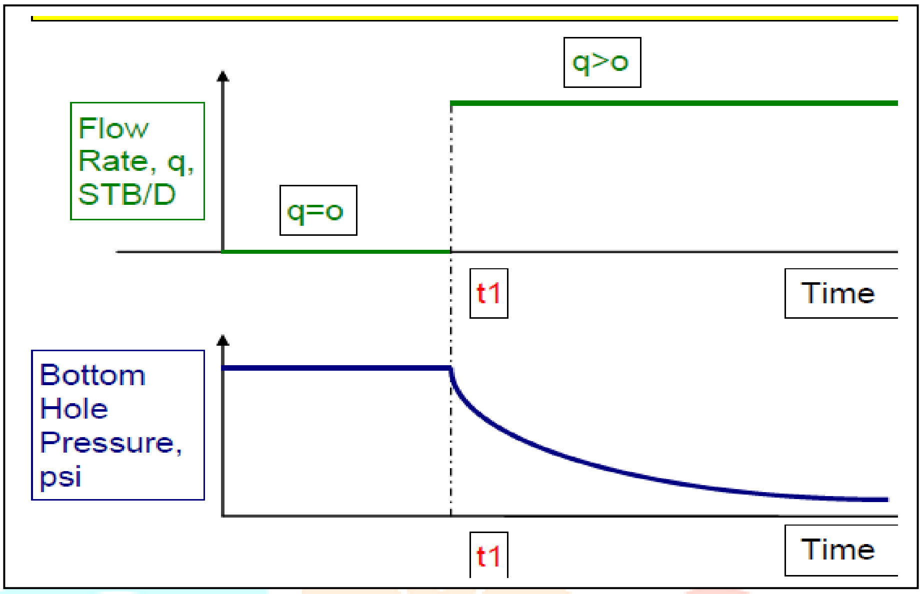

Figure.3.10: Schematic Diagram of constant rate drawdown testing

An idealized constant rate drawdown test is an infinite acting reservoir is modeled by-

P_{wf} = Pi + $162.6\frac{q\mu \boldsymbol{B}}{Kh}\left|\log\left(\frac{1688\, \emptyset \mu Ctrw\,2}{Kt}\right) - 0.869S\right|$…………………..3.30

The usual test has an ETR, an MTR and LTR.

3.9.1: Early Time Region:

The ETR usually is dominated by wellbore loading: the rate at which fluid is removed from the wellbore exceeds the rate at which fluid enters the wellbore until, finally, equilibrium is established. Until that time, the constant flow rate at the sand-face required by equation 3.29 is not achieved and the straight line plot of P_{wf}Vs log t suggested by equation 3.29 is not achieved.

Duration of wellbore unloading can be estimated by-

$t_{wbs} = \frac{(200000+12000S)Cs}{\frac{Kh}{\square}}$ ……………………………………….3.31

If the effective radius of the zone of altered permeability is unusually large (e.g.; in a hydraulically fractured well), the duration of ETR may depend on the time required for the radius of investigation to exceed the fracture half length.

3.9.2: Middle Time Region:

The MTR begins with the ETR ends (unless boundaries or important heterogeneities are unusually near the well). In the MTR, a plot of P_{wf}Vs log t is a straight line with slope m, given by figure 3.11.

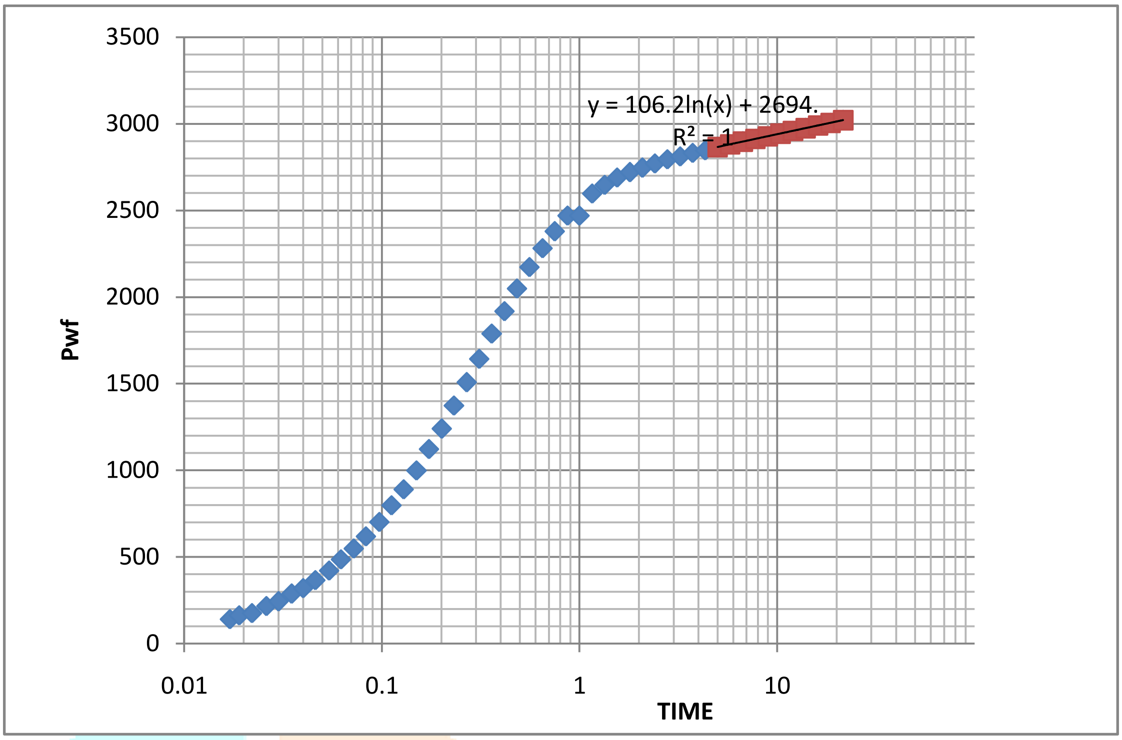

Figure 3.2: Plot of P_{wf}Vs log t

m = 162.6 $\frac{q\mu B}{Kh}$ ……………………………………………………………….3.32

Thus, effective formation permeability can be estimated from this slope.

K = 162.6 $\frac{q\mu B}{mh}$……………………………………………………………….3.33

After K is estimated from identified MTR, skin factor, S, can be determined. The usual equation results from solving equation3.1 for S. setting t=1hour and letting P_{wf} = P1hr be the pressure on the MTR line at 1 hour flow time, the result is –

S = 1.151$\left[\frac{(Pi-P1hr)}{m} - log\left(\frac{K}{\emptyset\mu Ctrw^{2}}\right) + 3.23\right]$……………………..….3.34

<u>3.9.3: Late Time Region:</u>

The LTR begins when the radius of investigation reaches a portion of the reservoir influenced by reservoir boundaries or massive heterogeneities. For a well centered in a square or circular drainage area, this occurs at a given time approximately by-

t_{tl}= $\frac{380\emptyset\mu CtA}{K}$ ………………………………………………………………3.35

Where, A is the drainage area of the tested well.

Thus, the typical constant rate drawdown test plot has the shape, figure 3.12.

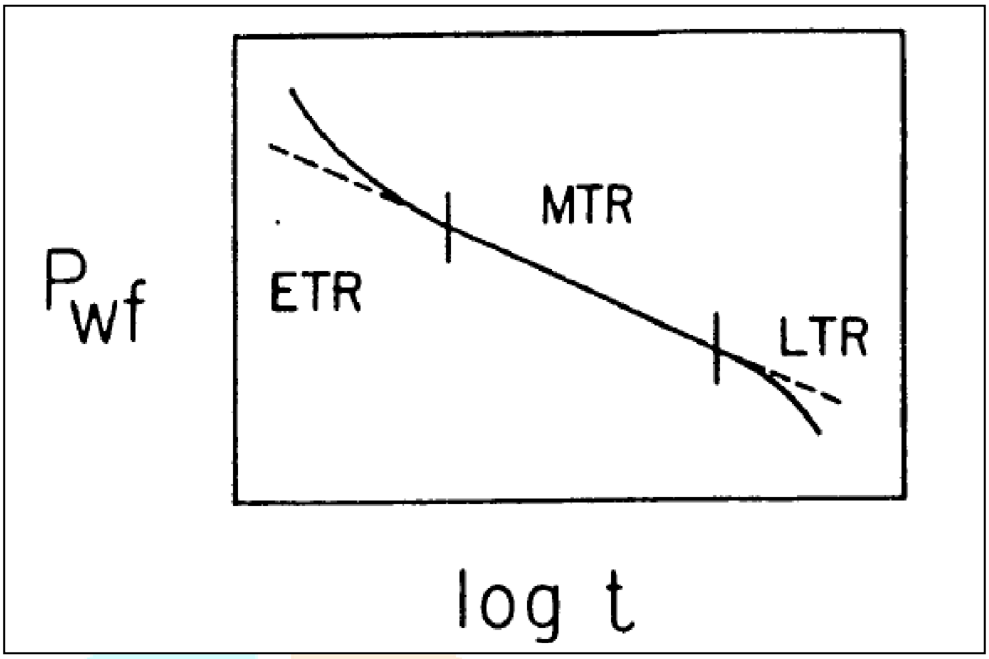

Figure: 3.12: Typical constant rate drawdown test graph

3.10: Determination of Pore Volume:

Another use of drawdown test is to estimate reservoir pore volume, are and even oil in place. This is possible when the radius of investigation reaches all boundaries during a test so that pseudo steady state flow is achieved.

The formulae used to calculate the drainage area, pore volume and oil in place are-

$A = \frac{0.23395\,qB}{\emptyset hCtm*}$ ………………………………………………………….3.35

$V_p = \frac{0.23395\,qB}{Ctm*}$………………………………………………………….3.36

$N = \frac{(1-Swi)Vp}{5.615\,Boi}$ ………………………………………………………….3.37

Where, m* is simply the slope of the straight line P_{wf}Vs t plot on ordinary Cartesian graph paper, Figure 3.13.

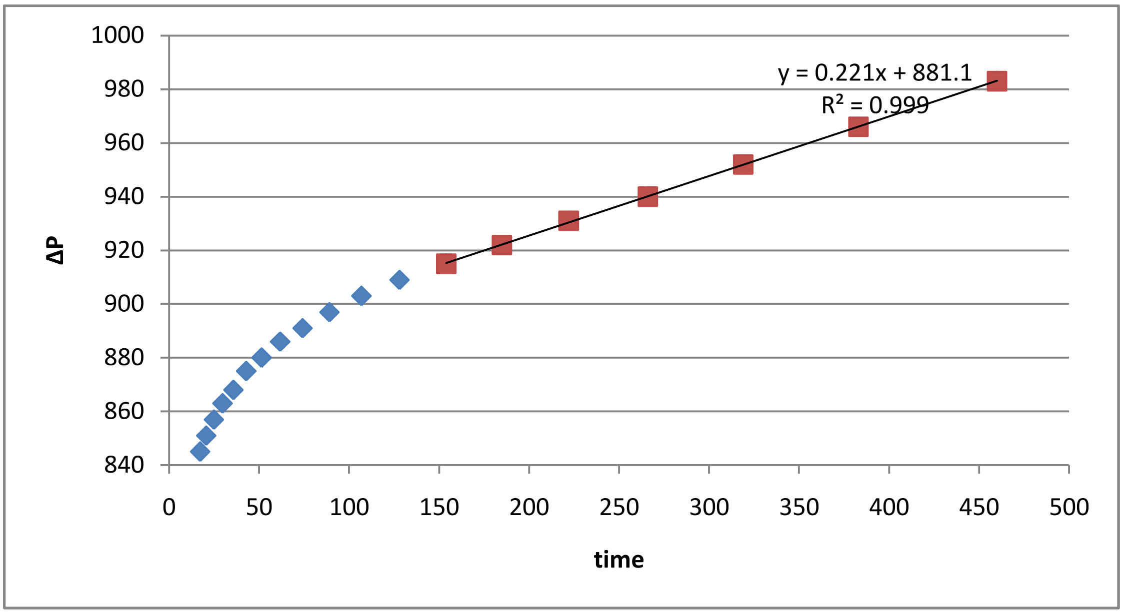

Figure: 3.13: Cartesian plot of P_{wf} Vs t to determine m*

3.11: Basic theory of gas flow in reservoirs:

Gas flow in infinite acting reservoir can be expressed by an equation that is similar for flow of slightly compressible liquids but here pseudo pressure (↓P) is used instead of pressure.

Pseudo pressure is used because flow of real compressible gases in the reservoir depends on physical properties such as viscosity, isothermal compressibility and deviation factor and they are strong functions of pressure.

The equation becomes-

$\downarrow(Pw_f) = \downarrow(P_i) + 50300\frac{P_{sc}}{T_{sc}}\frac{q_g T}{Kh}\left[1.151\log(\frac{1688\varphi\mu_i c_{it}\mu_w{}^2}{kt}) - (S + D|q_g|)\right]$...3.38

Pseudo pressure is defined as

$\downarrow(P) = 2\int_{P_B}^{-P}\frac{P}{\mu t}dp$...3.39

After modification, for stabilized flow,

$\downarrow(Pw_f) = \downarrow(\bar{P}) - 50300\frac{P_{sc}}{T_{sc}}\frac{q_g T}{Kh}\left[\ln\left(\frac{\mu_e}{\mu_w}\right) - 0.75 + S + D|q_g|\right)$.....3.40

Equation 3.38 & 3.39 provides the basis for gas well test analysis.

If P>3000 psi, equation becomes simpler in terms of P.

If P<2000 psi, equation becomes simpler in terms of P^2.

Equation 3.38 is generalized for any drawdown test for any drainage area pressure $(\overline{P})$ that may be much lower than initial pressure (P_i) after years of production.

$$\downarrow(Pw_f) = \downarrow(\bar{P}) + 50300\frac{P_{sc}}{T_{sc}}\frac{q_g T}{Kh}\left[1.151\log(\frac{1688\varphi\mu_{\bar{P}} c_{i\bar{P}}\mu_w{}^2}{kt}) - (S + D|q_g|)\right].3.41$$

For p<2000psia, μZ_g ≈constant for most gases. Where, P=$\bar{P}$

$\downarrow(P) = \frac{2}{\mu_{\bar{P}} Z_{\bar{P}g}}(\frac{P^2}{2} - \frac{P_B{}^2}{2})$

$$2\int_{P_B}^{-P}\frac{P}{\mu Z}dP = \frac{2}{\mu_{\bar{P}} Z_{\bar{P}} g}\left(\frac{P^2}{2}-\frac{{P_B}^2}{2}\right)+50300\frac{P_{sc}}{T_{sc}}\frac{q_g T}{kh}[1.151\log\left(\frac{1688\varphi\mu_{\bar{P}}C_{t\bar{P}}{\mu_w}^2}{kt}\right)-(S+D|q_g|)]$$

$${P_{wf}}^2=\bar{P}^2+1637\frac{q_g\mu_{\bar{P}}Z_{\bar{P}}gT}{Kh}[\log((1688\varphi\mu_P C_{t\bar{P}})/Kh)-(\frac{s+D|q_g|}{1.151})] \text{ ....3.42}$$

For stabilized flow

$${P_{wf}}^2=\bar{P}^2-1422\frac{q_g\mu_{\bar{P}}Z_{\bar{P}}gT}{Kh}[\ln\left(\frac{\mu_e}{\mu_w}\right)-0.75+S+D] \text{......3.43}$$

Equation 8.6 is the deliverability equation if P_{wf} is given, corresponding to a given pipeline or back pressure, the rate q_g at which the well will deliver gas can be estimated. However, some other parameters have to determine prior to estimating q_g.

The well will flow at rate q_g until $\mu_i \geq \mu_e$ (stabilize flow), so the equation 3.43 becomes-

$$\bar{P}^2-{P_{wf}}^2=aq_g+bqg^2 \text{......................................3.44}$$

Where, a=1422 $\frac{\mu_{\bar{P}}Z_{\bar{P}}gT}{Kh}[\ln\left(\frac{\mu_e}{\mu_w}\right)-0.75+S]$.......................3.45

b=1422 $\frac{\mu_{\bar{P}}Z_{\bar{P}}gT}{Kh}D$

The well flowed for times $\mu_i \leq \mu_e$(transient flow). In this case we need to estimate Kh, S and D from transient test (build up and draw-down) for this equation 3.42 is used.

3.12: Gas well test:

Most gas well tests usually consist of at-least two flow rate periods this is because gas wells having low producing rate may have a flow rate dependent skin which can be identified by carrying out a second flow and build up period. This is the simplest form of deliverability test.

A deliverability test is run in gas wells to determine-

a) Absolute open flow potential (AOFP)

b) Rate dependent skin (non-Darcy skin).

AOFP is the theoretical flow rate at which the well would produce if the reservoir sand-face (P_{wf}) where at atmospheric pressure. This calculated rate is only of practical importance because the government of countries set the maximum rate at which the well can produce as a fraction of this flow rate.

Rate dependent skin is an additional pressure drop in the near wellbore which varies with the flow-rate, this will reflect on the well in flow performance.

3.13: Types of deliverability tests:

There are 3 types of deliverability tests.

a) Flow after flow tests (Rate on Rate or back pressure test).

b) Isochronal test.

c) Modified Isochronal test.

3.13.1: Flow after flow tests:

Flow after flow test comprises of a series of increasing flow rates, on increasing choke sizes beginning with an initial shut in condition, ideally at initial static reservoir pressure with no shut in periods between choke changes.

In other words we can say, in this method a well is allowed to flow at a selected constant rate until pressure gets stabilized i.e., pseudo-steady state is reached before change to next larger choke.

This test was designed for wells which stabilize rather quickly i.e. in less than about 3 hours from opening.

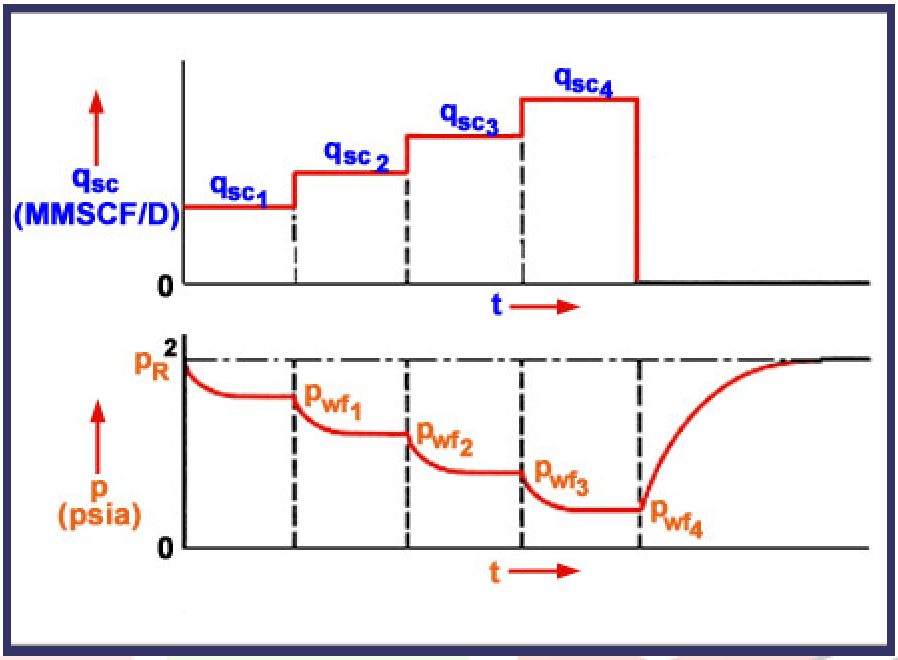

Figure 3.14: Schematic diagram of Flow after Flow test

This process is repeated for 3-4 rates. This type of flow occurs only in reservoirs of relatively high permeability thus short stabilization time.

Reservoirs of one Darcy permeability or more will probably exhibit this flow pattern. It is also a function of reservoir thickness and overall size (drainage radius available) and gas viscosity. Since pseudo steady state flow is achieved, the Raculins-Schillhardt equation used for analysis-

$q = c(\bar{P}^2 - {P_{wf}}^2)^n$ ……………………………………… 3.46

Where, q = Stabilized flow rate, (MSCF/day), (MMSCF/d)

C = Performance coefficient, a function of time for transient flow and for a given reservoir and gas. It becomes constant for pseudo-steady state flow.

$P_s or \bar{P}$ = Static reservoir pressure, Psia.

n = Flow exponent, varies between 0.5 and 1 and is constant with time.

P_{wf} =Stabilized flowing bottomhole pressure, Psia.

Fundamentally two different techniques are used to analyze these test data-Empirical method & Theoretical method.

Equation 3.38 is called as performance equation.

3.13.1(a): Empirical method:

It is a plot of $\Delta P^2 = (\bar{P}^2 - {P_{wf}}^2)$ Vs q_g on a log-log paper and is approximately a straight line for many wells in which pseudo-steady state is reached at each flow rate in flow-after flow test sequences.

The equation of the line for the plot is-

$$q_g = c(\bar{P}^2 - {P_{wf}}^2)^n$$

This plot is an empirical correlation of field data.

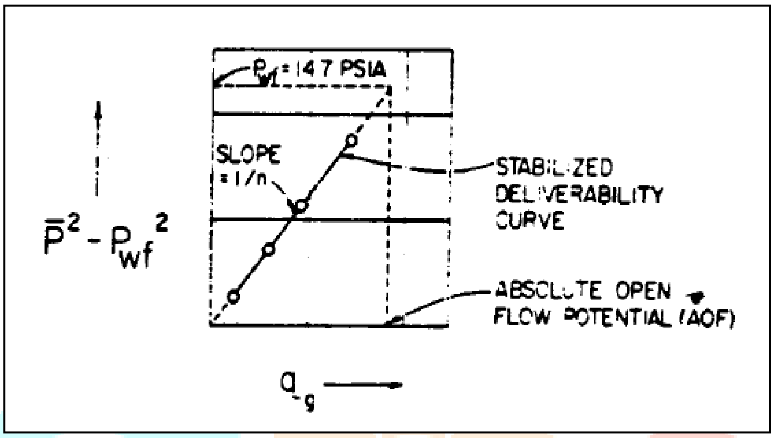

Figure 3.15: Empirical deliverability plot for flow –after-flow test.

The line is extrapolated beyond the region in which data are not obtained. The extrapolation is done to estimate the absolute open flow potential (AOFP).

It is the theoretical rate at well would produce if the flowing pressure (P_{wf}) where atmospheric. It may be necessary to extrapolate the curve far beyond the range of test data.

This AOFP can never be achieved in practice because of wellbore geometry. It is used practically to compare one wells performance to that of another well, and in setting allowable. A common allowable is 20% of AOFP.

The multi-flow period is followed by a pressure buildup, for at least the same time as total flow-time, to ascertain permeability and any formation damage, skin etc.

An AOFP determined for a very long extrapolation may be incorrect.

C and n taken in equation 3.46 are not constant. They depend on fluid properties that are pressure and time dependent. And this type of curve is used, periodic retesting of well will change the value of c and n.

3.13.1(a): Theoretical method:

The equation

$\bar{P}^2 - {P_{wf}}^2 = aq_g + {b_{q_g}}^2$ ………………………………3.47

Where, a and b are constants and can be determined from flow tests for at-least two rates in which q_g and corresponding values of P_{wf} are measured.

Where, $a = 1422 \frac{\mu_{\bar{P}} Z_{\bar{P}_g} T}{Kh} [\ln\left(\frac{\mu_e}{\mu_w}\right) - 0.75 + S]$

$$b = 1422\frac{\mu_{\bar{P}} Z_{\bar{P}g} T}{Kh} D$$

The equation 3.38 suggests that a plot between $(\frac{\bar{P}^2 - P_{wf}{}^2}{g})$ Vs q_g should be a straight line for pseudo-steady state flow with slope b and intercept a. This straight line has a sounder theoretical basis than empirical straight line. This plot should be possible to extrapolate the straight line to determine AOF with less error and correct deliverability estimates for changes in $\mu_{\bar{P}}, Z_{\bar{P}g}$ etc.

3.13.2: Isochronal tests:

Not all wells stabilize quickly enough to achieve pseudo-steady state flow on test. For those wells which do not stabilizes quickly, the RaculinsSchellhardt method should not be used. Isochronal (equal time) test procedure was later designed for use in such wells. This test measures the transient deliverability of a well in a lower transmissibility reservoir or we can say this test is for low permeability reservoir where it is frequently not possible to achieve ($r_i = r_e$)during the test.

Correct application requires that each and every flow period begins from an originally-static reservoir condition. Therefore intermediate shut-in periods must be of sufficient duration for pressure buildups to reach original static pressure at gauge depth.

In low or very low-transmissibility gas reservoirs, it may require days, or even weeks or for intermediate pressure build ups to reach original static reservoir pressure, even after relatively short flow periods.

Thus isochronal gas well test, does not really itself solve the problem of extended test time; because of the result of low transmissibility. But it is a way to deal with transient flow, though always the best way is not available.

For isotropic reservoir system, the pressure buildups are mirror image of the draw-downs except only one thing during image of the draw-downs except only one thing during flow period (draw-down) the reservoir fluid is removed from the reservoir. Thus, the draw-down and buildup systems are different in case of mass and volume removed, but does not depend on reservoir size/geometry.

But almost all reservoirs are fractured to some degree. Presence of fracture means the system is non-isotropic. Fractures have greater permeability than un-fractured rock of same reservoir. So, during pressure drawdown in such system are primarily imposed on high permeability fracture system. But pressure buildups involve the whole network of fractures as well.

Due to this permeability/transmissibility variation, this brings into play a time difference. For these reasons, drawdown and buildup times are usually unequal and so their curves are not mirror image of each other.

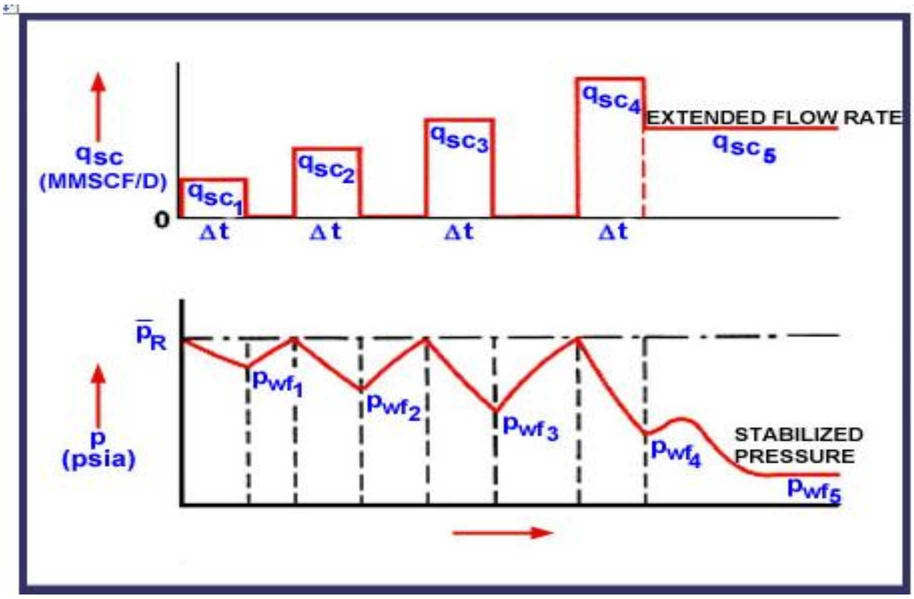

Figure 3.16: Schematic diagram of isochronal test

Flow times are equal (flow times are equal with different flow rates) and shut-in or build up periods are not equal, the well will be shut in until it reaches the static pressure).Build-up period increases. Also larger chokes produce larger transient flow-rates, but they remain transient i.e. they don't stabilize before shut-in.

Finally, pressure build-up curve recorded on isochronal shut-ins require special treatment to be analyzed for permeability because they were not preceded by stabilized or pseudo-steady state flow. Then also, the result will be some-what in error.

In summary, the isochronal test procedure does provide a way to handle transient flow of gas wells, but it fails to solve the problem of required extended test time brought into play by the inherently low transmissibility of some reservoirs.

3.13.3: Modified isochronal tests:

In 1959, Katz etal.suggested a modification to the isochronal test. They suggested that both the shut-in period and the flow period for each test could be of equal duration provided that the unstabilized shut-in pressureat the end of each test be used instead of the static reservoir pressure, $\bar{P}R$.

The objective of the modified isochronal test is to obtain the same data as in an isochronal test without using the sometimes lengthy shut-in periods required for pressure to stabilize completely before each flow test is run.

In the modified isochronal test, figure 3.17, shut-in periods of the same duration as the flow periods are used.

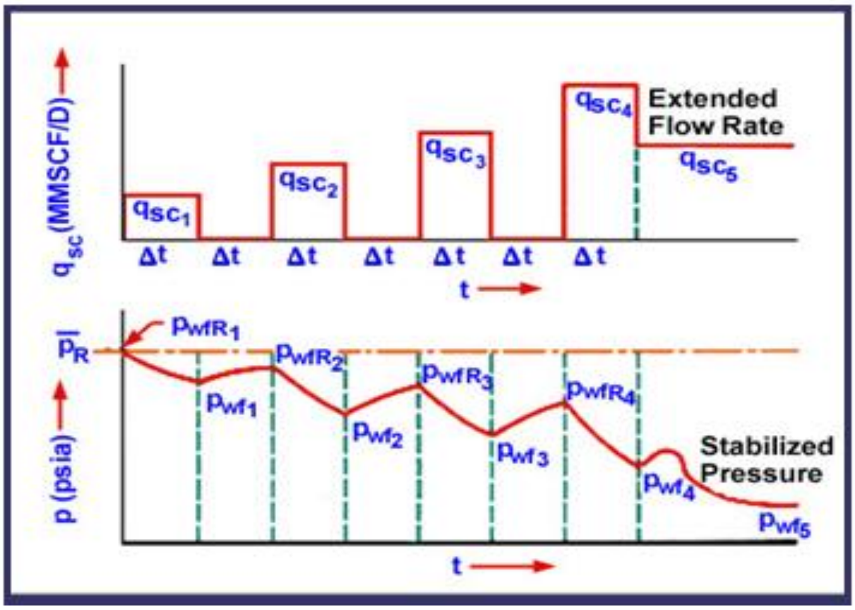

Figure 3.17: Schematic diagram of modified isochronal test.

MATERIAL BALANCE METHOD FOR OIL AND GAS WELLS:

4.1: Introduction

Material balance equation (MBE) is a basic reservoir engineering tool for interpreting and predicting reservoir performance. MBE is applied to estimate initial hydrocarbon volumes in place, predict future reservoir performance and predict ultimate hydrocarbon recovery under various types of primary driving mechanism.

Development and operation of a gas reservoir depends upon the future performance of the reservoir. The predicted performance of a gas reservoir can be achieved if we recognize the sources of the energy and estimate the original gas in place.

In a normally pressured volumetric reservoir the gas is produced by the expansion.[3]. The original gas in place in a normally pressured (volumetric) reservoir may be estimated from production and pressure history using the Material balance technique. The extrapolation of the plot of pressure divided by gas deviation factor (P/Z) versus cumulative gas production to the point where P/Z equals zero yields original gas in place.

4.2: Material Balance For oil wells:

It was developed by Schilthuis in 1991. The simplest form of the material balance equation is-

Initial Volume = Volume remaining + Volume removed.

The expression for total pore volume is-

$$m = \frac{Initial\ volume\ of\ gas\ cap}{volume\ of\ oil\ initially\ in\ place} = \frac{GBgi}{NBoi}$$

Initial volume of gas cap, GB_{gi} = m× $NBoi$……………………………4.1

And the total volume of the hydrocarbon system is given by-

Initial oil volume+ Initial Gas Cap Volume = PV× $(1 - Swi)$

$NB_{oi} + GB_{gi}$ = PV× $(1 - Swi)$

NB_{oi} + m× $NBoi$ = PV× $(1 - Swi)$

PV = $\frac{NBoi + m \times NBoi}{(1-Swi)} = \frac{(m+1)NBoi}{(1-Swi)}$……………………………………………4.2

Where, Swi = initial water saturation

Treating the reservoir as container or tank, the reservoir initially looks like, Figure 4.1. Volumetric balance expressions can be derived to account for all volumetric changes which occur during the natural productive life of the reservoir.

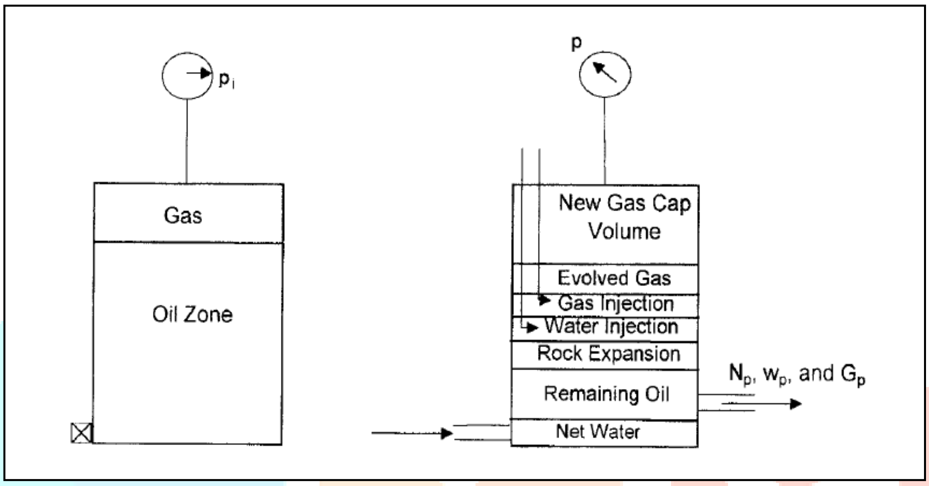

Figure: 4.1: Tank Model concept

The generalized form of material balance equation is –

Pore Volume occupied by the oil initially in place at P_i +

Pore Volume occupied by the gas in the gas cap at P_i =

Pore Volume by the remaining oil at P +

Pore Volume occupied by the gas cap in the gas cap at P +

Pore Volume occupied by the evolved solution gas +

Pore volume occupied by net water influx at P +

Change in Pore volume due to connate water expansion and pore volume reduction due to rock expansion +

Pore volume occupied by the injected gas at P +

Pore volume occupied by the injected water at P

Each term is determined separately from the hydrocarbon PVT and rock properties.

4.2.1: Pore volume occupied by the oil initially in place:

Volume occupied by initial oil in place = NB_{oi}……………………………4.3

4.2.2: Pore volume occupied by the gas in the gas cap:

Volume of gas cap = mNB_{oi}…………………………………………………4.4

4.2.3: Pore volume occupied by the remaining oil:

Volume of the remaining oil = (N-Np)B_o……………….……………4.5

4.2.4: Pore volume occupied by the gas cap at reservoir pressure, P:

As the reservoir pressure drops to a new level P, the gas in the gas cap expands and occupies a larger volume. Assuming no gas is produced from the gas cap during the pressure decline, and then the new volume of the gas cap is given by-

Volume of the gas cap at P = $\left[\frac{mNBoi}{Bgi}\right] Bg$………………………………4.6

4.2.5: Pore volume occupied by the evolved solution gas:

This volumetric term can be determined by applying material balance on the solution gas-

[Volume of gas initially in solution] = [volume of the evolved solution gas] +[volume of the gas produced] + [volume of the gas remaining in solution]

[Volume of the evolved solution gas] = [volume of gas initially in solution] – [volume of the gas produced] – [volume of gas remaining in solution]

[Volume of the evolved solution gas] = [NR_{si}-N_pR_p-(N-N_p) Rs]B_g……4.7

4.2.6: Pore Volume occupied by the net water influx:

Net water influx = W_e-W_pB_w………………………………………………4.8

4.2.7: Change in Pore Volume due to initial water and rock expansion:

In case of under saturated oil reservoir, the component that describing the reduction in the hydrocarbon pore volume due to expansion of initial(connate)water and the reservoir rock cannot be neglected. The magnitude of water and rock compressibility is same as the order of oil compressibility. But, these components however can be neglected for gas cap drive reservoir or when reservoir pressure drops below the bubble point pressure (where in case of gas cap, the gas expansion magnitude is more and in case of saturated reservoir the magnitude of the gas evolved from the solution is more and hence C_w&$C_f$can be neglected).

The compressibility coefficient C, describes the changes in volume (expansion) of the fluid or material (rock) with changing pressure which is given by the formula-

$C = \frac{-1}{V}\frac{\partial V}{\partial P}$

$\Delta V = C \times V \times \Delta P$……………………………………………………………4.9

Where, ΔV represents the net change or expansion of the material as a result of change in pressure, ΔP.

Therefore, the reduction in the pore volume due to expansion of the connate water in the oil zone is given by-

Connate water expansion = $C_w \times (PV.Swi)\Delta P$……………………………4.10

Putting the value of PV from equation 4.2 in equation 4.10 -

Connate water expansion = $\left(\frac{(m+1)NBoi}{(1-Swi)}\right)\Delta P.Swi \times C_w$……………………4.11

Similarly, the reduction in the pore volume due to expansion of the reservoir rock is given by-

Change in pore volume due to rock expansion = $\left(\frac{(m+1)NBoi}{(1-Swi)}\right)\Delta P \times Cf$………4.12

Change in total pore volume = $\left(\frac{(m+1)NBoi}{(1-Swi)}\right)\Delta P.Swi \times Cw + \left(\frac{(m+1)NBoi}{(1-Swi)}\right)\Delta P \times Cf$

$= \left(\frac{(m+1)NBoi}{(1-Swi)}\right)\Delta P \times (Cf + Swi \times Cw)$……………4.13

$= \left(\frac{(m+1)NBoi}{(1-Swi)}\right)\Delta P \times (Cf + Swi \times Cw)$……………4.13

4.2.8: Pore volume occupied by the injection gas and water:

Assume that G_{inj} volume of gas and W_{inj} volume of water have been injected for pressure maintenance. Therefore, the total pore volume occupied by the two injected fluids are-

Total volume = $G_{inj}.B_{ginj} + W_{inj}B_w$……………………………………4.14

Where, G_{inj} = cumulative gas injected, scf

B_{ginj} = injected gas formation volume factor, bbl/scf

W_{inj} = cumulative water injected, STB

B_w = water formation volume factor, bbl/STB

Combining above equations

$NB_{oi} + mNB_{oi} = (N-N_p)B_o + \left[\frac{mNBoi}{Bgi}\right]Bg + [NR_{si}-N_pR_p-(N-N_p)R_s]B_g + W_e-W_pB_w + \left(\frac{(m+1)NBoi}{(1-Swi)}\right)\Delta P \times (Cf + Swi \times Cw) + Ginj.Bginj + Winj.Bw$

$$N = \frac{Np\,[Bo+(Rp-Rs)Bg]-We+WpBw-Ginj.Binj-Winj.Bw}{(Bo-Boi)+(Rsi-Rs)Bg+mBoi\left[\frac{Bg}{Bgi}-1\right]+\frac{(m+1)}{1-Swi}\Delta P\times(Cf+Swi\times Cw)} \quad ..4.15$$

This is the Material balance equation.

In a more convenient way, the MBE can also be written with the concept of total (2 phase) formation volume factor, B_t.

$B_t = B_o+(R_{si}-R_s)B_{gi}$…………………………………………………………..4.16

The equation becomes for no gas and water injection-

$$N = \frac{Np\,[Bt+(Rp-Rsi)Bg]-We+WpBw}{(Bt-Bti)+mBti\left[\frac{Bg}{Bgi}-1\right]+\frac{Bti\,(m+1)}{1-Swi}\Delta P\times(Cf+Swi\times Cw)} \quad ……………4.17$$

Considering $Np[Bt+(Rp-R_{si})B_g] = A$

$$N = \frac{A - We + WpBw}{(Bt - Bti) + mBti\left[\frac{Bg}{Bgi} - 1\right] + \frac{Bti\,(m+1)}{1 - Swi}\Delta P \times (Cf + Swi \times \mathrm{Cw})}$$

$$N(\mathrm{Bt} - \mathrm{Bti}) + N\,mBti\left[\frac{Bg}{Bgi} - 1\right] + N\frac{Bti\,(m+1)}{1 - Swi}\Delta P \times (Cf + Swi \times \mathrm{Cw}) + W_e + W_p B_w = A$$

$$\frac{N(\mathrm{Bt} - \mathrm{Bti})}{A} + \frac{N\,mBti\left[\frac{Bg}{Bgi} - 1\right]}{A} + \frac{N\frac{Bti\,(m+1)}{1 - Swi}\Delta P \times (Cf + Swi \times \mathrm{Cw})}{A} + \frac{We + WpBw}{A} = 1 \ldots\ldots\ldots\ldots\ldots\ldots 4.18$$

DDI + SDI + EDI + WDI = 1

Where, DDI = depletion drive index

SDI = segregation (gas cap) drive index

EDI = expansion (rock and liquid) depletion index

WDI = water drive index

4.2.9: Depletion Drive:

It is the oil recovery mechanism wherein the production of the oil from its reservoir rock is achieved by the expansion of the original oil volume because of the evolution of the dissolved gas. The driving mechanism is mathematically represented by the term-

$$DDI = \frac{N(\mathrm{Bt} - \mathrm{Bti})}{A}$$

4.2.10: Gas Cap Drive:

It is the oil recovery mechanism wherein the displacement of oil from the formation is accomplished by the expansion of the original free gas cap. The driving mechanism is mathematically represented by the term-

$$SDI = \frac{N\,mBti\left[\frac{Bg}{Bgi} - 1\right]}{A}$$

4.2.11: Water Drive:

It is the oil recovery mechanism wherein the displacement of the oil is accomplished by the net encroachment of water into the oil zone.

$$WDI = \frac{We + WpBw}{A}$$

4.2.12: Expansion Drive:

For, under saturated oil reservoirs with no water influx, the principle source of energy is a result of the rock and fluid expansion.

$$EDI = \frac{N\frac{Bti\,(m+1)}{1 - Swi}\Delta P \times (Cf + Swi \times \mathrm{Cw})}{A}$$

The contribution of the rock and fluid expansion to the oil recovery is too small and hence it is negligible and can be ignored.

Cole(1969) pointed out that as the sum of the driving indexes is equal to one, if the magnitude of one of the index term is reduced, then one or both of the remaining terms must be correspondingly increased. Effective water drive will result in maximum recovery from a reservoir. Therefore if possible the reservoir should be operated to yield maximum water drive index and minimum values for the depletion-drive index and gas cap drive index. Maximum utilization of the most effective drive should be taken, if a water drive is weak to provide effective displacement then the energy of the gas cap should be utilized for displacement then the energy of the gas cap should be utilized for displacement. But the depletion drive index should be maintained as low as possible at all times, as this is normally the most inefficient driving force available.

Equation of drive indexes can be solved at any time to determine the magnitude of the various drive indexes. The force displacing the oil and gas from the reservoir changes from time to time and this reason the drive indexes equation should be solved periodically to determine whether there has been any change in the driving indexes. Change in the fluid withdrawal rate(production rate) are primarily responsible for changes in the driving indexes. When the reservoir has a weak water drive but fairly large gas cap, then the displacing energy is the gas and a gas cap index is more. Theoretically, recovery by gas cap drive is independent of the producing rate, as the gas is readily expansible. But low vertical permeability could limit the rate of expansion of the gas cap, in such cases the gas cap drive index would be rate sensitive.

Gas coming into producing well, will also reduce the effectiveness of the gas cap expansion due to production of free gas. Gas coning is a rate sensitive phenomenon, the higher the producing rates, the greater the amount of coning. An important factor in determining the effectiveness of a gas cap drive is the degree of conservation of the gas cap gas. But this is practically not possible because of royalty owners or lease agreements to completely eliminate gas cap gas production. When free gas is being produced, the gas cap drive index can often be increased by shutting in high gas-oil ratio wells.

4.3: Basic assumptions in MBE:

MBE calculation is based on changed in reservoir conditions over discrete periods of time during the production history. Assumptions are-

4.3.1: Constant Temperature:

Volume changes in the reservoir are assumed to occur without any temperature changes. If temperature changes occur, they are usually significantly small to be ignored without significant error.

4.3.2: Pressure equilibrium:

All parts of the reservoir has the same pressure and therefore fluids properties throughout are constant. Minor changes in the vicinity of the wellbore can be ignored.

4.3.3: Constant reservoir volume:

Reservoir volume is assumed to be constant except for those conditions of rock and water expansion or water influx that are specifically considered in the equation

4.3.4: Reliable production data:

All production data should be recorded with respect to the same time period. If possible, gas cap and solution gas production records should be maintained separately.

There are essentially three types of production data that must be recorded in order to use the MBE in performing reliable reservoir calculations. These are oil production data, gas production data and water production data.

4.4: MBE as an equation of straight line:

The MBE consists of a group of terms of which it is comprised of-

Np [Bo+ (Rp-Rs) Bg] - represents the reservoir volume of cumulative oil and gas produced.

We-Wp.Bw- refer to net water influx that is retained in the reservoir.

Ginj.Bginj+Winj.Bw – Pressure maintenance term represents cumulative fluid injection in the reservoir.

$mBoi\left(\frac{Bg}{Bgi}-1\right)$- represents the net expansion of the gas cap that occurs with the production of Np stock tank barrels of oil.

There are three unknowns the original oil in place"N", the cumulative water influx 'We' and the original size of the gas cap as compared to the oil zone size"m". A methodology has been used for determining the above three unknowns. Havlena&Odeh (1963) expressed the equation of material balance in the form-

$N_p [B_o + (R_p - R_s) B_g] + W_p.B_w = N[(B_o - B_{oi}) + (R_{si} - R_s)B_g] + mNB_{oi}\left(\frac{Bg}{Bgi}-1\right) + NBoi\frac{(m+1)}{1-Swi}\Delta P \times (Cf + Swi \times Cw) + We + W_{inj}.B_w + G_{inj}B_{ginj}$…………………..4.19

Havlena&Odeh expressed the above equation in a condensed form as-

$F = N[E_o + mE_g + E_{fw}] + (W_e + W_{inj}.B_w + G_{inj}.B_{ginj})$…………………..4.20

For the purpose of simplicity, assuming that no pressure maintenance by gas or water injection then the above relationship can be simplified and written as-

$F = N[E_o + mE_g + E_{fw}] + We$…………….…………………………….4.21

The terms F, E_o, E_g and E_{fw} are defined by-

F represents the underground withdrawal and given by –

$F = N_p[B_o + (R_p - R_s)B_g] + W_pB_w$……………………………………….4.22

In terms of two phase formation volume factor Bt, the underground withdrawal, F, can be written as-

$F = N_p [B_t + (R_p - R_{si}) Bg\} + W_pB_w$……………………………………….4.23

Eo is the expansion of oil and its originally dissolved gas is expressed in terms of-

$Eo = (B_o - B_{oi}) + (R_{si} - R_s)B_g$ ……………………………………………4.24

In terms of B_t-

$E_o=B_t-B_{ti}$……………………………………………………………4.25

E_g is the expansion of the gas cap gas and is given by-

$E_g=B_{oi}\left[\left(\frac{Bg}{Bgi}\right)-1\right]$…………………………………………………………4.26

In terms of two phase formation volume factor, $B_{oi} = B_{ti}$

$E_g = B_{ti}\left[\left(\frac{Bg}{Bgi}\right)-1\right]$………………………………………………………..4.27

E_{fw} represents the expansion of the initial water and reduction of pore volume is given by-

$E_{fw} = B_{oi}\frac{(m+1)}{1-Swi}\Delta P \times (Cf + Swi \times Cw)$……………………………….4.28

Havlena&Odeh examined several cases of varying reservoir types with equation 4.21 and pointed out that the relationship can be rearranged into the form of a straight line.

4.5: Straight Line solution method to MBE:

The significance of the straight line approach is that the sequence of plotting is important and if the plotted data deviates from this straight line there is some reason for it. The significance observation will provide the engineer with valuable information that can be used in determining the following unknowns- initial oil in place,"N", size of the gas cap, "m" , water influx, "We" and driving mechanism.

4.5.1: Volumetric Under saturated Oil Reservoir:

Assuming no water or gas injection the linear form of MBE equation is-

$F = N[E_o + mE_g + E_{fw}] + We$…………………………………………….4.29

For a volumetric and undersaturated reservoir, the condition associated with driving mechanism are-

We = 0, since the reservoir is volumetric.

m = 0, since the reservoir is undersaturated.

Rs = Rsi = Rp, since all produced gas is dissolved in oil.

Equation 4.29 becomes-

$F = N[E_o + 0 + E_{fw}] + 0$

$F = N [E_o + E_{fw}]$

$N = \frac{F}{[Eo + Efw]}$………………………………………………………………4.30

From equation 4.22,

$F = N_p[B_o+(R_p-R_s)B_g]+W_pB_w$

$F = N_p[B_o+0]+W_pB_w$

$F = N_pB_o+W_pB_w$……………………………………………………………4.31

And equation 4.28 becomes,

$E_{fw} = B_{oi}\frac{(m+1)}{1-Swi}\Delta P \times (Cf + Swi \times Cw)$

$E_{fw} = \frac{Boi}{1-Swi}\Delta P \times (Cf + Swi \times Cw)$……………………………………..4.32

When a new filed is discovered, one of the first tasks of the reservoir engineer is to determine if the reservoir can be classified as volumetric reservoir i.e. We = 0.

The classical approach of addressing this problem is to assemble all the necessary data (i.e. production, pressure and PVT data) that are required to evaluate Eo.The term $\frac{F}{[Eo + Efw]}$ for each pressure and time observation is plotted Vs cumulative production, Np, or time, figure 4.2.

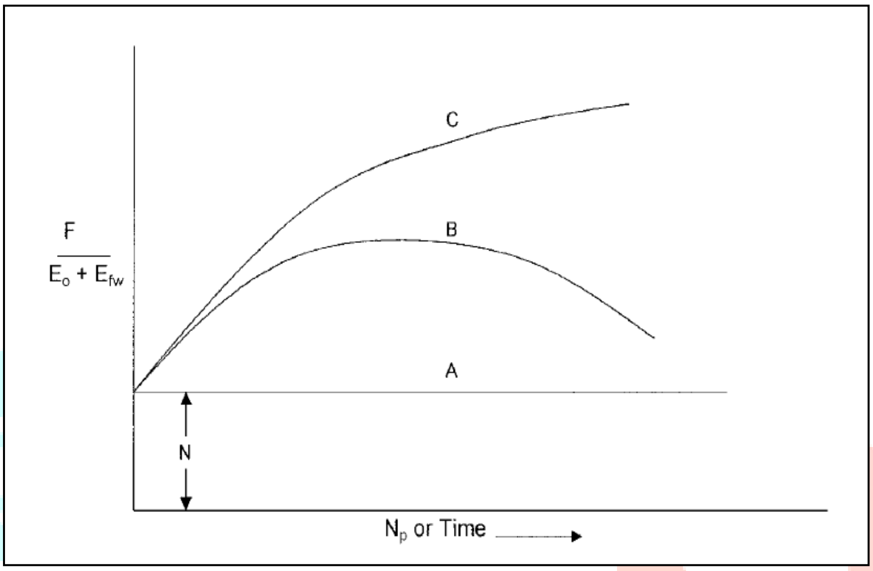

Figure 4.2: $\frac{F}{[Eo + Efw]}$ Vs Np or time plot for different reservoirs.

All the calculated points of F/ (E_o + E_{fw}) lie on a horizontal straight line. Line A in the plot implies that the reservoir can be classified as a volumetric reservoir. This defines a purely depletion drive reservoir whose energy derives solely from the expansion of the rock, connate water and the oil. Furthermore, ordinate value of the plateau (x=0) determines the initial oil in place, N.

Alternately, the curves B and C indicate that the reservoir has been energized by water influx, abnormal pore compaction or a combination of these two. Curve C might be for a strong water drive field in which the aquifer is displacing an infinite acting behavior where b represents an aquifer whose outer boundary has been felt and the aquifer is depleting in unison with the reservoir itself.

Dake(1994) point out that in water drive reservoirs, the shape of the curve i.e. $\frac{F}{[Eo + Efw]}$ Vs time is highly dependent. For instance, if the reservoir is producing at higher than the water influx rate i.e. the calculated value of $\frac{F}{[Eo + Efw]}$ will dip downward revealing a lack of energizing by the aquifer, whereas, if the rate is decreased, the reverse happens and the points are elevated. The equation 4.30 can be used to verify the characteristic of the reservoir driving mechanism and to determine the initial oil in place.

A plot of the underground withdrawal F Vs the expansion term (E_o+E_{fw}) should result in a straight line going through the origin with N. It should be noted that the origin is a must point, thus one has a fixed point to guide the straight line plot as shown in figure 4.3.

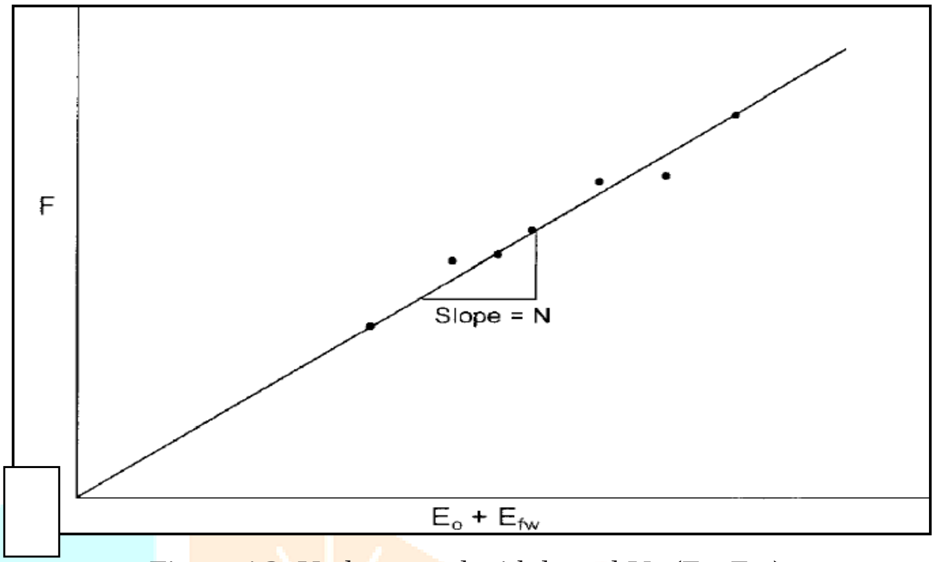

Figure 4.3: Underground withdrawal Vs (E_o+E_{fw}).

The linear plot of underground withdrawal F Vs (E_o+E_{fw}) indicates that the field is producing under volumetric performance i.e. no water influx and strictly by pressure depletion and fluid expansion. On the other hand, a nonlinear plot indicates that the reservoir should be characterized as a water drive.

4.5.2: Volumetric saturated oil reservoir:

An oil reservoir that originally exists at its bubble point pressure is called as saturated oil reservoir. The main driving mechanism for this type of reservoir is due to the liberation and expansion of the solution gas as the pressure drops below the bubble- point pressure. The only unknown in a volumetric saturated oil reservoir is the initial oil in place, N.

Assuming the water and rock expansion term $E_{wf}$is negligible in comparison with the expansion of solution gas.

F = N.E_o……………………………………………………………4.33

The plot of underground withdrawal (F) evaluated by using the actual reservoir production data as a function of fluid expansion term Eo, should result in a straight line going through the origin with a slope of N. If the plot terms out to be nonlinear then there is an unsuspected water influx into the reservoir helping to maintain the pressure.

4.5.3: Gas cap drive reservoirs:

For a reservoir in which the expansion of the gas cap gas is the predominant driving mechanism and assuming that the natural water influx is negligible (We=0) and the effect of water and pore compressibility (E_{fw}= 0) is negligible. In such condition the Havlena –Odeh material balance is given by-

$F = N[E_o + mE_g]$ ………………………………………………………….4.34

There are 3 possible unknowns in the equation 4.34.

4.5.3.1: N is unknown, m is known:

The plot of F Vs $[E_o + mE_g]$ on a Cartesian scale would produce a straight line through the origin with a slope N as shown in figure 4.4.

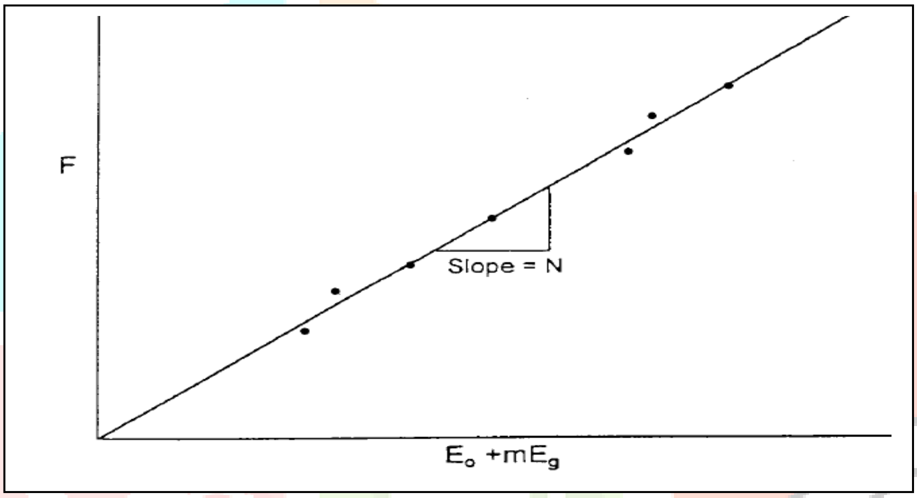

Figure 4.4: F Vs $[E_o + mE_g]$

4.5.3.2: m is unknown, N is known:

Rearranging equation 4.34,

$\frac{F}{N} - Eo = mEg$……………………………………………………4.35

The plot of $\frac{F}{N} - Eo$ Vs Eg would produce a straight line with a slope of m as shown in figure 4.5. One of the advantages of this particular arrangement is that the straight line must pass through origin which therefore acts as a control point.

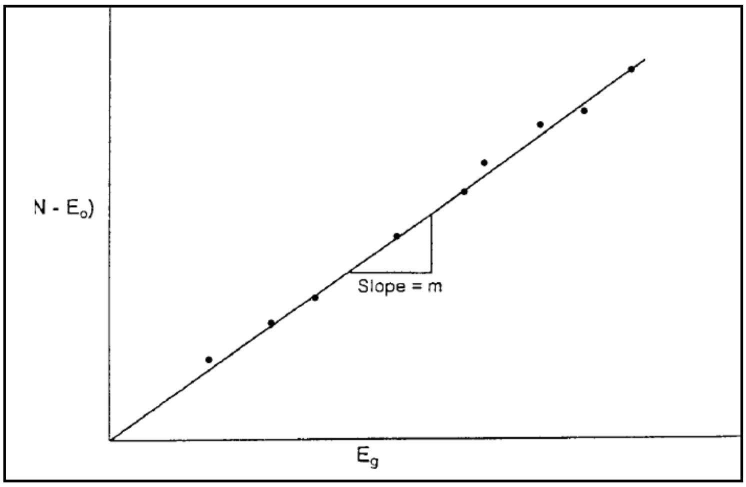

Figure 4.5: Plot of [$\frac{F}{N} - Eo$] VsEg

4.5.3.3: Both N and m are unknown:

$\frac{F}{Eo} = N + mN\frac{Eg}{Eo}$……………………………………………………4.36

The plot of $\frac{F}{Eo}$Vs $\frac{Eg}{Eo}$ should be linear with intercept N and slope mN and m as slope/intercept, as shown in figure 4.6.

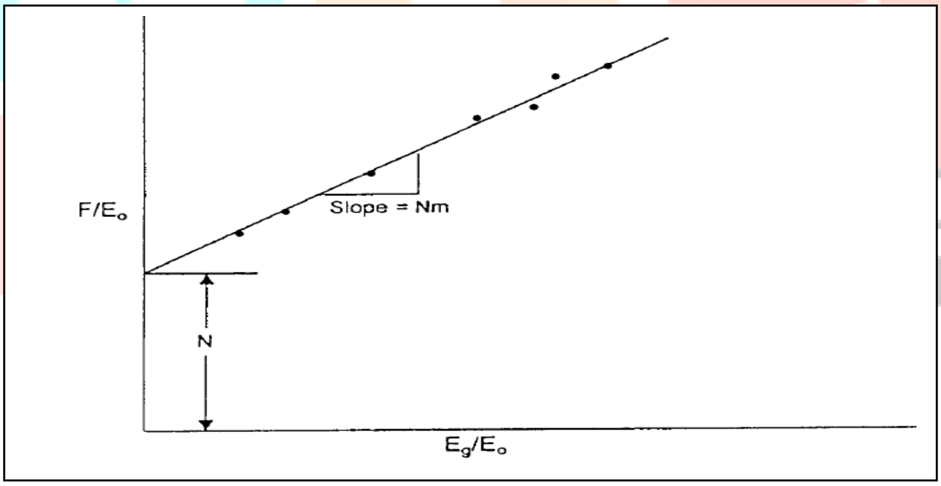

Figure: 4.6. Plot of $\frac{F}{Eo}$Vs $\frac{Eg}{Eo}$

4.5.4: Water Drive Reservoir:

The full MBE can be expressed as-

F = N[E_o +mE_g+E_{fw}] +We

Dake pointed out that E_{fw} for water drive reservoir can be neglected. This is not only because of water and pore compressibilityare small but also because a water influx helps to maintain the reservoir pressure and the ΔP appearing in the E_{fw} term is reduced.

F = N[E_o +mE_g] +We…………………………………………… 4.37

The reservoir doesnot have initial gas cap i.e.Eg = 0

F = NE_o +We…………………………………………………….4.38

Equation 4.38 is rearranged as

$\frac{F}{Eo} = N + \frac{We}{Eo}$……………………………………………………4.39

Several water influx models have been described like Pot aquifer model, Steady state model (Schilthuis model) and unsteady state model (Van- Everdinger- Hurst model).

4.5: Material Balance for Gas wells:

Material balance methods are developed in terms of cumulative fluid production and changes in reservoir pressure and therefore requires accurate measurements of both quantities.

Unlike volumetric methods which can be used early in a reservoir life, material balance method can't be applied until after some development and production. However, an advantage of material balance method is that they estimate only gas volumes that are in pressure communication with and that may be initially recovered by the producing well. Conversely, volumetric estimates are based on the total gas volume in place, part of which may not be recoverable with the existing wells because of un-identified reservoir heterogeneities.

4.5.1 Volumetric dry-gas reservoirs:

As started, volumetric reservoirs are completely enclosed and receive no external energy from other sources, such as aquifers. If rock and connate water expansions are negligible sources of internal energy then the dominant drive mechanism is gas expansion as reservoir pressure decreases. Comparison of typical values of gas and liquid compressibility's shows that gases can be as much as 100 or even 1000 times more compressible than relatively incompressible liquids, so simple gas expansion is a very efficient drive mechanism often allowing up to 90% of in-place gas to be recovered.

Assuming a constant reservoir pv over the producing life of the reservoir, we can drive a material balance equation by equating the reservoir pore volume occupied by the gas at initial conditions to that occupied by the gas at some later conditions following gas production and the associated pressure reduction . Referring to the tank type model, Figure:, the material balance equation can be written as

$GB_{gi} = (G - G_p)B_g$ ………………………………………4.40

Where,

GB_{gi} =Reservoir volume occupied by the gas at initial reservoir pressure (bbl)

$(G - G_p)B_g$ =Reservoir volume occupied by the gasafter gas production, at a pressure below the initial reservoir pressure (bbl).

Equation 4.40 can be re-write as

$G_p = G(1 - \frac{B_{gi}}{B_g})$…………………………..4.41

If we substitute the ratio of the gas FVF evaluated at initial and later conditions, $\frac{B_{gi}}{B_g} = \frac{\frac{Z_i}{P_i}}{\frac{Z}{P}}$ in equation 4.41 and the equation becomes

$$G_p = G\left(1 - \frac{\frac{Z_i}{P_i}}{\frac{Z}{P}}\right)$$

$$= G(1 - \frac{Z_i P}{Z P_i})\text{.................................}4.42$$

Where the gas recovery factor is $(1 - Z_i P/Z P_i)$

Further, we can rewrite equation 4.42 as

$$\frac{P}{Z} = \frac{P_i}{Z_i}\left(1 - \frac{G_P}{G}\right) = \frac{P_i}{Z_i} - \frac{P_i}{Z_i G} G_p\text{..................}4.43$$

Similar to van Everdingen et al's and Havlena and Odeh's graphical analysis techniques, the form of equation 5.4 suggests that if the reservoir is volumetric, a plot of p/z vs. G_p will be a straight line, from which we can estimate both original gas in place and gas reserves at some abandonment conditions

4.5.2: Gas Compressibility:

Gas compressibility can be defined as follows

$$C_g = \frac{1}{P} - \frac{1}{Z}\left(\frac{\partial Z}{\partial P}\right)\text{..............................}4.44$$

As reservoir pressure is decreased, gas will expand in the pores of the reservoir rock. In normally pressured reservoir, the gas compressibility will be in the order of 50 micro sips and will increase as reservoir pressure decreases.

Using the above definition of gas compressibility, as pressure is increased; the gas compressibility will get smaller and smaller.

4.5.3: P/Z in normally pressured dry-gas reservoir:

The gas material balance equation for a volumetric dry-gas reservoir states that the original gas in place is equal to reservoir with-drawl divided by gas expansion this relation can be expressed mathematically as

$$G = \left(\frac{G_P B_g}{B_g - B_{gi}}\right)\text{..}4.45$$

The gas formation volume factor can be defined as the volume of gas at reservoir condition required to produce one scf of gas at standard temperature and pressure.

$$B_g = \frac{P_{sc} Z T}{T_{sc} P}\text{...................................}4.46$$

By use of the definition of B_g and the gas material balance equation it can be shown that equation 4.45 can be expressed as

$$\frac{P}{Z} = \frac{P_i}{Z_i} - \frac{P_i}{Z_i G} G_p\text{........................}4.47$$

The above P/Z relationship considers only gas expansion and is often used to extrapolate to the original gas in place.

Equation 4.4.7 states that in a volumetric gas reservoir a plot versus cumulative gas production should yield a straight line. The intercept and slope are as follows.

$b = \frac{P_i}{Z_i}$……………………………4.48

$m = -(\frac{P_i}{Z_i G})$…………………………4.48

The p/z relationship is used to obtain original gas in place by extrapolating to a reservoir pressure of zero. At a pressure of zero it can be seen from equation 5.8 that

$G = G_P$……………………….4.49

The p/z curve is a graphical solution of the gas material balance equation and is used by reservoir engineers to obtain an estimation of the original gas in place.

DECLINE CURVE ANALYSIS FOR GAS WELLS

5.1: Introduction:

Decline curves are one of the most extensively used forms of data analysis employed in evaluating gas reserves and predicting future production. The decline curve analysis technique is based on the assumption that past production trends their controlling factors will continue in the future and, therefore can be extrapolated and described by a mathematical expression

The methods of extrapolating a trend for the purpose of estimating future performance must satisfy the condition that the factors that caused changes in past performance, for example decline in the flow rate will operate in the same way in the future. These decline curves are characterized by 3 factors.

i. Initial production rate or the rate at some particular time.

ii. Curvature of the decline.

iii. Rate of decline.

Decline curve analysis technique consists of conventional Arps (1945) method, classical Fetkovich (1980) type curve matching method, Palacia and Blasingama (1993) and Agarwal et.al (1998) type curve matching method and FMB (1998) reservoir engineering methods [5]

5.1 Arps decline curve analysis:

Wells with a long production history and producing at constant BHFP, Arps (1945) derived 3 types of production decline with the help of rate versus time curve relationship which includes exponential decline, hyperbolic decline and harmonic decline.

Arps empirical rate/time decline equation is the most conventional decline curve analysis

$q(t) = \frac{q_t}{(1+bD_it)^{Y_b}}$……………………………..5.1

Where, D_i= Initial decline rate ($days^{-1}$)

b= Arps decline curve exponent

q_t =Gas flow rate at time t, MMSCF/day

q_i= Initial gas flow rate, MMSCF/day

t= Time, days.

The three different forms of decline are based on the value of the decline exponents b. these three forms of decline exponential, harmonic and hyperbolic have different shape on Cartesian and semi-log graphs of gas production rate versus time and gas production rate versus cumulative gas production.

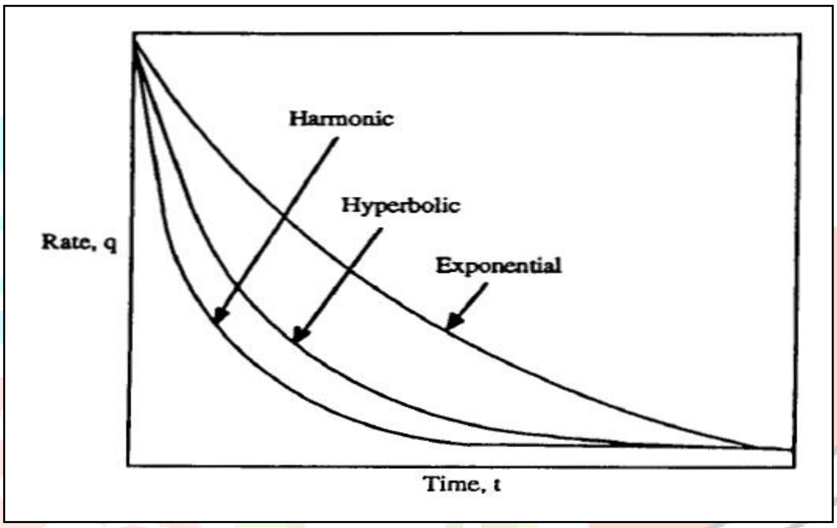

Figure 5.1: Decline curve shapes for a Cartesian plot of rate Vs time

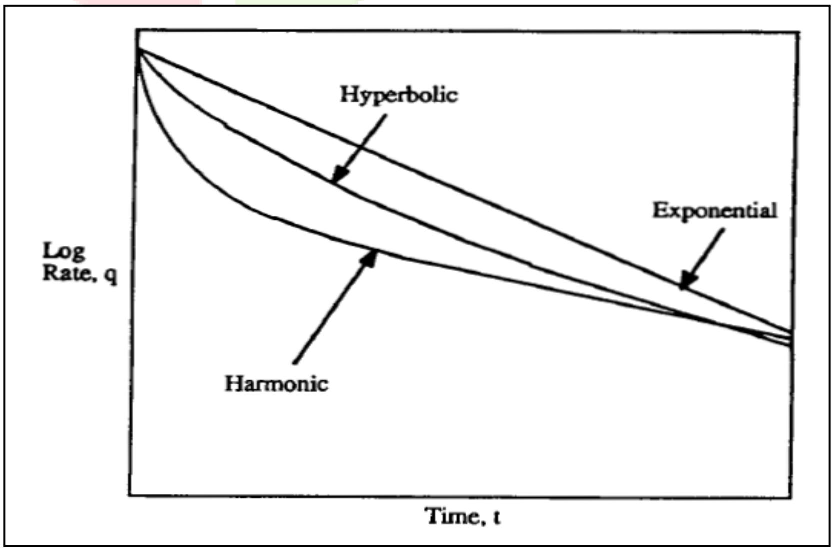

Figure 5.2: Decline curve shapes for a semi-log plot of rate Vs time

Consequently, these curve shapes can help identify the type of decline for a well and, if the trend is linear, extrapolate the trend graphically or mathematically to some future points.

Figure 5.1 & 5.2 show typical responses for exponential, hyperbolic and harmonic declines. Because of their characteristic shapes, these plots can be used as a diagnostic tool to determine the type of decline curve before any calculations are made.

5.1.1: Exponential decline:

Exponential decline, sometimes called as constant percentage decline, is characterized by a decrease in production rate per unit time that is proportional to the production rate. The exponential decline equation can be derived from equation 5.1 when b=0 as-

$q(t) = \frac{q_i}{e^{D_i t}} = q_i e^{-D_i t}$…………………..5.2

Taking natural logarithm (ln) of both sides of equation 5.2 becomes-

$\log[q(t)] = \log(q_i) + \log(e^{-D_i t})$…………………………5.3

Which, after rearranging gives-

$\log[q(t)] = \log q_i - D_i t$…………………………………………5.4

Because the natural logarithm is related to the base 10(log) by ln(x)=2.303log(x), so we can re-write equation 5.4 in terms of the log function as-

$\log[q(t)] = \log q_i - \frac{D_i t}{2.303}$………………………………5.5

The equation 5.5 suggests that a plot of log gas flow rate q(t) Vs t will be a straight line with a slope $\frac{D_i}{2.303}$ and an intercept $\log(q_i)$. After calculating the initial decline rate and the initial gas flow rate, we can use equation 6.5 to extrapolate the production trend into the future to some economic limit. From this extrapolation, we can estimate gas reserves and the time at which the economic limit will be reached.

The curve of rate versus cumulative production for exponential decline will be linear on a Cartesian graph, as indicated by the following derivation if we integrate equation 5.2 from initial time to time "t", we obtain-

$Q(t) = \int_0^t q(t)dt = \int_0^t q_i e^{-D_t} dt$…………………………………..5.6

The cumulative gas production is-

$G_p(t) = \left(-\frac{q_i}{D_i} e^{-D_i t}\right)_0^t$ ………………………………………………5.7

Rearranging yields-

$G_p(t) = -\frac{1}{D_i}(q_i e^{-D_i t}) + \frac{q_i}{D_i}$ ……………………………………5.8

Combining equation 5.2 and 5.8 we can write the cumulative production relation in terms of rate-

$G_p(t) = -\frac{1}{D_i} q(t) + \frac{q_i}{D_i}$………………………………........5.9

Rearranging and solving for production rate-

$q(t) = -G_p(t)D_i + q_i$ …………………………………..5.10

Equation 5.10 suggests that a plot of q(t) Vs$G_p(t)$ will yield a straight line of slope $-D_i$ and intercept q_i,

5.1.2: Harmonic decline:

When b=1, the decline is said to be harmonic, and the general decline equation given by equation (1) reduces to

$q(t) = q_{i/(1+D_i t)}$ ……………………………………..5.11

Taking logarithms on both sides equation 6.11 yields-

$\log q(t) = \log q_i - \log(1 + D_i t)$ ………………………5.12

The form of equation 5.12 suggests that q(t) is a linear function of (1+$D_i t$) on log-log graph paper and will exhibit a straight line with slope-1 and an intercept of log (q_i). To predict future performance of wells exhibiting harmonic decline behavior we must assume values of D_i until a plot of $\log[q(t)] vs \log(1 + D_i t)$ is a straight line with slope of -1.

To use rate Vs cumulative production plot for harmonic decline, we must integrate equation 5.11 with respect to time to obtain a relationship for cumulative production.

$G_p(t) = \int_0^t q(t)dt = \int_0^t \frac{q_i}{(1+D_i t)} dt$……………………………5.13

$G_p(t) = \frac{q_i}{D_i} \ln(1 + D_i t)$= 2.303$\frac{q_i}{D_i} \log(1 + D_i t)$………5.14

Substituting the value of log(1+$D_i t$) from equation 5.12 in equation 5.14-

$G_p(t) = 2.303 \frac{q_i}{D_i} [\log q_i - \log q(t)]$……..5.15

In terms of production rate

$\log q(t) = \log(q_i) - \frac{D_i}{2.303 q_i} G_p(t)$…………………………5.16

The form of equation 5.16 suggests that a plot of $\log q(t) vs G_p(t)$ will be linear with a slope of $-\frac{D_i}{2.303 q_i}$ and an intercept of $\log q_i$

5.1.3: Hyperbolic Decline:

When 0<b<1, the decline is hyperbolic, and the rate behavior is described by-

$$q(t) = \frac{q_i}{(1 + bD_i t)^{\frac{1}{b}})}$$

Taking the logarithm on both sides of equation 5.1 and rearranging yields-

$\log q(t) = \log(q_i) - \frac{1}{b} \log(1 + bD_i t)$ 5.17

The form of equation 5.17 suggests that , if rate versus time data can be modeled with the hyperbolic equation, then a log-log plot of q(t) Vs (1+b$D_i t$) will exhibit a straight line with slope of 1/b and an intercept of logq_i).

The cumulative production versus time relationship is obtained by integrating equation 5.1-

$$G_p(t) = \int_0^t q(t)dt = \int_0^t \frac{q_i}{(1+bD_it)^{\frac{1}{b}}}dt \text{ ....5.18}$$

After integrating and rearranging-

$$G_p(t) = \frac{q_i}{D_i(b-1)}[(1+bD_it)^{\frac{(1-b)}{(-b)}} - 1]\text{...........................5.19}$$

If we substitute $q_i = {q_i}^b {q_i}^{1-b}$ into equation 5.19 and then rearranging we can write-

$$G_p(t) = \frac{q_i b}{D_i(b-1)}\{[q_i(1+bD_it)^{-\frac{1}{b}}]^{1-b} - {q_i}^{1-b}\}\text{...................5.20}$$

Substituting the value of $q_i(1+bD_it)$ from equation 5.1 into equation 5.20 yields an expression for cumulative gas production in terms of gas flow rate during hyperbolic decline-

$$G_p(t) = \frac{q_i b}{D_i(b-1)}[q(t)^{1-b} - {q_i}^{1-b}]\text{................................5.21}$$

Hyperbolic decline never has a simple straight line relationship for either rate Vs time or rate Vs cumulative production plots on any co-ordinate system.

CASE STUDIES

6.1: Case 1

A pressure drawdown test in a new oil well is strongly influenced by skin and wellbore storage. The measured pressure data as a function of time are listed in Table 7.1. Other known reservoir and well data are:

q = 2500 BPD   ∅ = 21%   μ = 0.92 cp   B = 1.21 rb/STB

r_w= 0.40 ft P_i = 6009 Psi   h = 23 ft   Ct = 8.71×10^{-6} Psi^{-1}

Table.7.1: Pressure Drawdown test data, conducted for the well are given below:

Time(hrs)	P_{wf}(psi)	Time(hrs)	P_{wf}(psi)
0.017	5868	0.87	3539
0.019	5846	1	3465
0.022	5833	1.16	3412
0.026	5792	1.34	3362
0.03	5765	1.55	3319
0.035	5721	1.8	3289
0.04	5688	2.08	3263
0.046	5643	2.41	3238
0.054	5587	2.79	3214
0.062	5522	3.23	3198
0.072	5460	3.74	3177
0.083	5390	4.32	3162
0.097	5306	5	3143.3
0.112	5211	5.79	3127.7
0.129	5118	6.71	3112
0.15	5010	7.76	3096.6
0.173	4886	8.98	3081.1
0.201	4769	10.4	3065.5

0.232	4635	12.03	3050
0.269	4501	13.93	3034.4
0.311	4366	16.12	3018.9
0.36	4220	18.66	3003.3
0.417	4090	21.6	2987.8
0.482	3960		
0.558	3836		
0.65	3727		
0.75	3630		

Calculate permeability, skin factor and wellbore storage coefficient.

Solution:

Table 7.2: Calculation of ΔP = (Pi-Pwf) for the given drawdown test data.

Time (hrs)	Pwf (psi)	ΔP (psi)	Time (hrs)	Pwf (psi)	ΔP (psi)
0.017	5868	141	3.74	3177	2832
0.019	5846	16	4.32	3162	2847
0.022	5833	176	5	3143.3	2865.7
0.026	5792	217	5.79	3127.7	2881.3
0.03	5765	244	6.71	3112	2897
0.035	5721	288	7.76	3096.6	2912.4
0.04	5688	321	8.98	3081.1	2927.9
0.046	5643	366	10.4	3065.5	2943.5
0.054	5587	422	12.03	3050	2959
0.062	5522	487	13.93	3034.4	2974.6
0.072	5460	549	16.12	3018.9	2990.1
0.083	5390	619	18.66	3003.3	3005.7
0.097	5306	703	21.6	2987.8	3021.2
0.112	5211	798			
0.129	5118	891			
0.15	5010	999			
0.173	4886	1123			
0.201	4769	1240			
0.232	4635	1374			
0.269	4501	1508			
0.311	4366	1643			
0.36	4220	1789			
0.417	4090	1919			
0.482	3960	2049			
0.558	3836	2173			
0.65	3727	2282			
0.75	3630	2379			
0.87	3539	2470			
1	3465	2470			
1.16	3412	2597			
1.4	3362	2647			

1.55	3319	2690
1.8	3289	2720
2.08	3263	2746
2.41	3238	2771
2.79	3214	2795
3.23	3198	2811

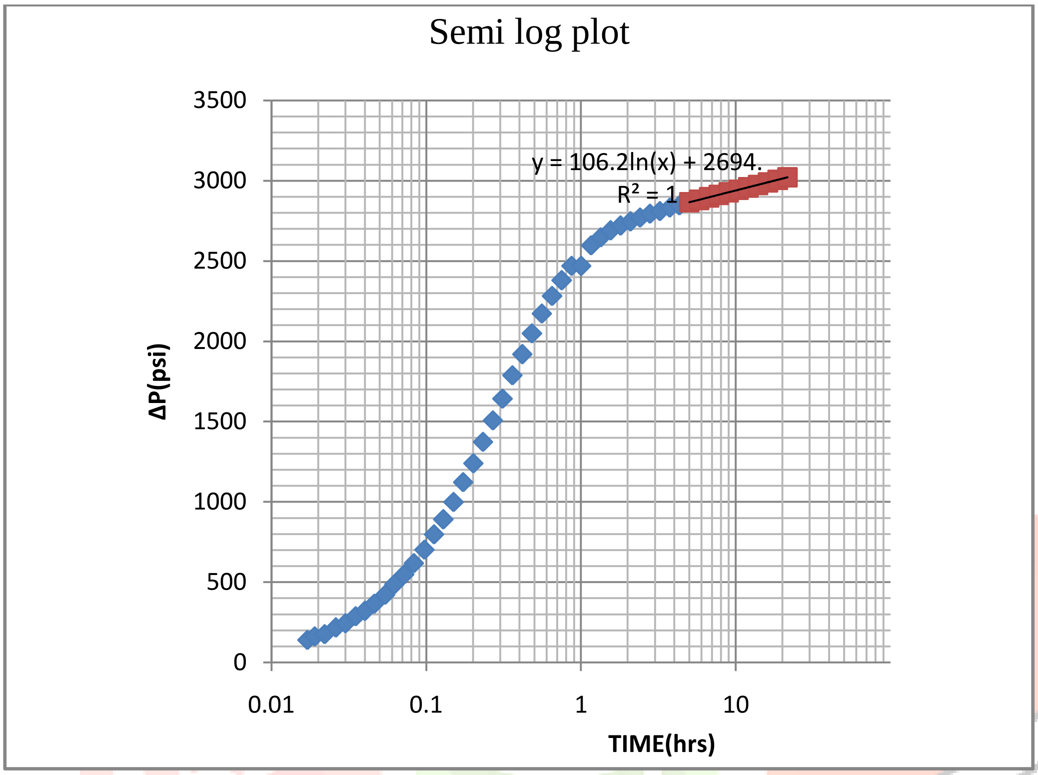

Figure. 7.1: Semi-log plot of ΔP = (Pi-Pwf) Vs flowing time (t)

If we plot semi log of ΔP Vs Time, then from the curve fit straight line equation-

Slope, m = 106.28×2.303

= 224.7628

The formation permeability, K, is-

$K = 162.6\,\frac{q\mu B}{mh}$

= 80.3823 mD

Skin factor can be calculated from-

$S = 1.151\left[\frac{\Delta P1hr}{m} - log\left(\frac{K}{\Phi\mu Ctrw^{2}}\right) + 3.23\right]$

$\Delta P1hr$ can be obtained by substituting t=1 in the curve fit function-

ΔP_{1hr}= 106.28*2.303log (1) + 2694.7

S = 6.637914

As S is positive, the additional pressure drop, ΔPs is given by-

ΔPs = 0.87mS

= 1411.87 Psi

The pressure change at the end of the test, ΔPe = 3021.2 Psi

The flow efficiency factor is given by-

$$F.E = 1-\frac{\Delta Ps}{\Delta Pe}$$

$= 0.532677 = 53.2677\%$

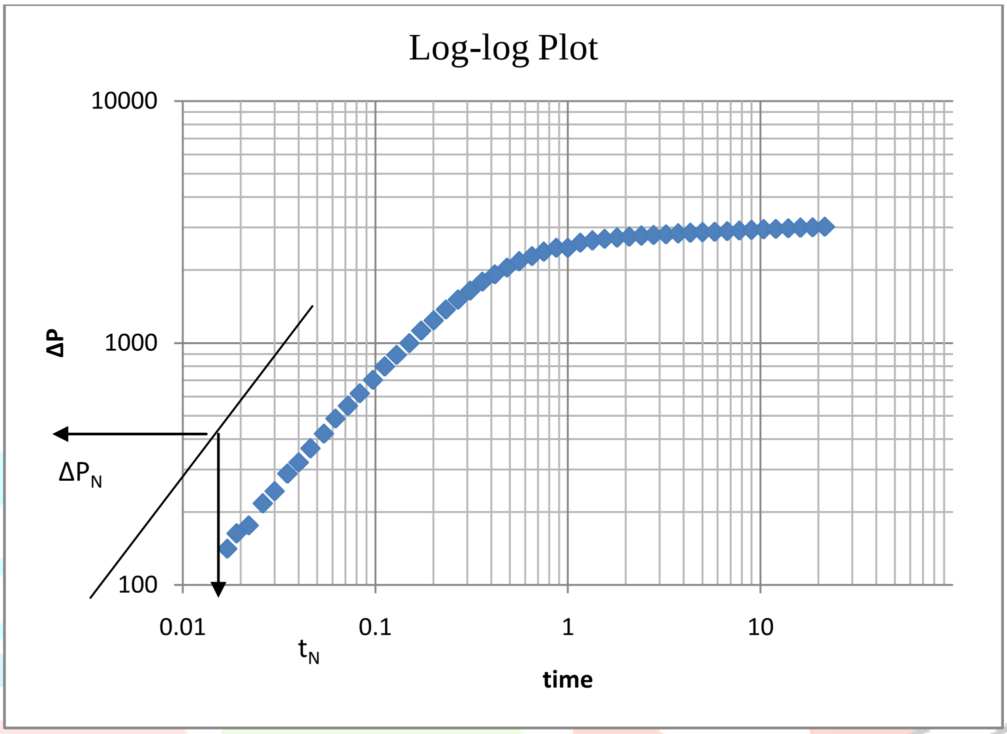

Figure: 7.2. Plot of ΔP Vs time on a log-log graph

From ΔP Vs time on log- log graph with unit slope, m_1=1

ΔP_N = 487 psi

t_N= 0.062 hr

Therefore, the wellbore storage coefficient can be calculated by-

$$C = \left(\frac{qB}{24}\right)\frac{tN}{\Delta PN}$$

= 0.016046 bbl/psi

From the semi log plot, the starting time of the radial flow line can be estimated by-

$$t_{SIAL} = \frac{(200000 + 12000\,S)\mu C}{Kh}$$

= 2.233053 hrs

6.2: Case 2:

A long constant-rate pressure-drawdown test (RLT) was run on an oil well. The test data are given in the table 7.3. The following are the well parameters-

P_i = 4412 psia h = 69 ftS_{wi} = 0.24 Ø = 3.9 % µ = 0.8 cp

q = 250 STB/D B = 1.136 bbl/STB r_w= 0.198 ft C_t = 17x10-6 psi-1

Table 7.3: RLT data are given below:

t(hrs)	P_{wf}(psi)	t(hrs)	P_{wf}(psi)
0	4412	51.5	3532
0.01	4400	61.8	3526
0.02	4389	74.2	3521
0.04	4366	89.1	3515
0.075	4333	107	3509
0.115	4288	128	3503
0.19	4217	154	3497
0.29	4120	185	3490
0.4	4052	222	3481
0.5	3994	266	3472
0.6	3953	319	3460
0.7	3912	383	3446
0.8	3884	460	3429
0.9	3855		
1	3834		
2	3680		
2.79	3653		
4	3625		
4.82	3616		
5.78	3607.5		
6.94	3600		
8.5	3593		
14.4	3573		
17.3	3567		
20.7	3561		
24.9	3555		
29.8	3549		
35.8	3544		
43	3537		

Analyze this test, i.e. find: K,S,Cs,FE,A,Vp and N.

Solution:

Table. 7.4: ΔP = (Pi-Pwf) values for the given RLT data

Time (hrs)	Pwf (psi)	ΔP (psi)	Time (hrs)	Pwf (psi)	ΔP (psi)
0.017	5868	141	3.74	3177	2832
0.019	5846	16	4.32	3162	2847
0.022	5833	176	5	3143.3	2865.7
0.026	5792	217	5.79	3127.7	2881.3
0.03	5765	244	6.71	3112	2897
0.035	5721	288	7.76	3096.6	2912.4
0.04	5688	321	8.98	3081.1	2927.9
0.046	5643	366	10.4	3065.5	2943.5
0.054	5587	422	12.03	3050	2959
0.062	5522	487	13.93	3034.4	2974.6

0.072	5460	549	16.12	3018.9	2990.1
0.083	5390	619	18.66	3003.3	3005.7
0.097	5306	703	21.6	2987.8	3021.2
0.112	5211	798			
0.129	5118	891			
0.15	5010	999			
0.173	4886	1123			
0.201	4769	1240			
0.232	4635	1374			
0.269	4501	1508			
0.311	4366	1643			
0.36	4220	1789			
0.417	4090	1919			
0.482	3960	2049			
0.558	3836	2173			
0.65	3727	2282			
0.75	3630	2379			
0.87	3539	2470			
1	3465	2470			
1.16	3412	2597			
1.4	3362	2647			
1.55	3319	2690			
1.8	3289	2720			
2.08	3263	2746			
2.41	3238	2771			
2.79	3214	2795			
3.23	3198	2811			

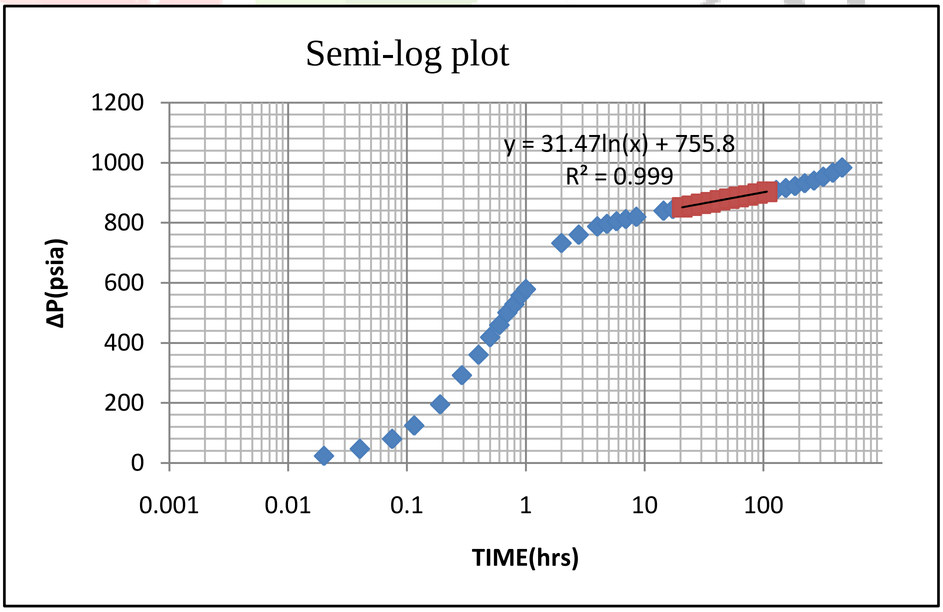

Figure: 7.3: Semi-log plot of ΔP = (P_i-P_{wf}) Vs flowing time (t)

Identifying the infinite acting straight line from semi-log plot of ΔP Vs time, the slope is-

m = 31.474× 2.303

= 72.48462 psi/cycle

The formation permeability is-

$K = 162.6\,\frac{q\mu B}{mh}$

= 7.386418 mD

Skin factor can be calculated from-

$S = 1.151\left[\frac{\Delta P1hr}{m} - log\left(\frac{K}{\Phi\mu Ctrw^{2}}\right) + 3.23\right]$

$\Delta P1hr$ can be obtained by substituting t=1 in the curve fit function

ΔP_{1hr}= 31.474ln (1) + 755.88

= 755.88 psi

S = 5.878897

As S is positive, the additional pressure drop, ΔPs is given by-

ΔPs = 0.87mS

= 370.7328 psi

The pressure change at the end of the test,ΔPe = 983 psi

The flow efficiency factor is given by-

$F.E = 1-\frac{\Delta Ps}{\Delta Pe}$

= 0.622856 = 62.2856%

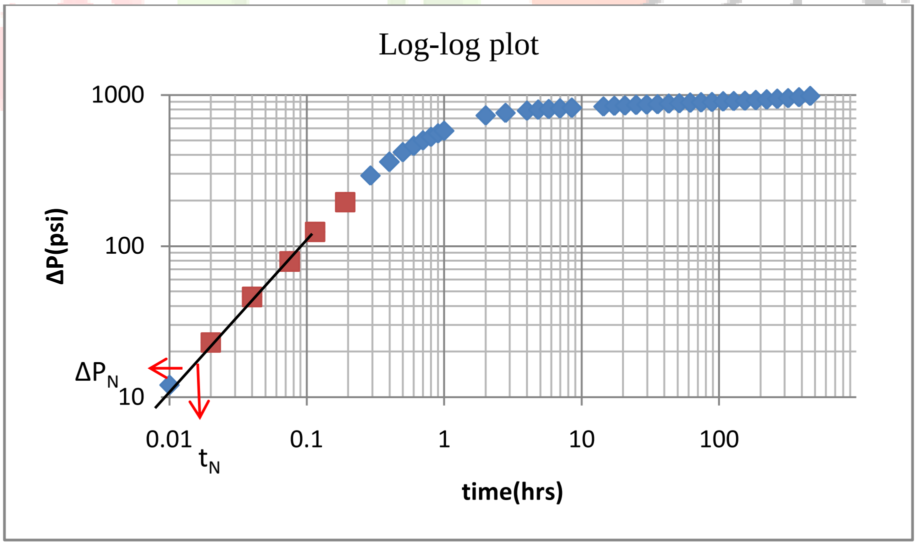

Figure: 7.4. Plot of ΔP Vs time on a log-log graph

From ΔP Vs time on log- log graph with unit slope, m_1=1

ΔP_N = 23 psi

t_N = 0.02 hrs

Therefore, the wellbore storage coefficient can be calculated by-

$C = \left(\frac{qB}{24}\right)\frac{\text{tN}}{\Delta\text{PN}}$

$= 0.01029$ bbl/psi

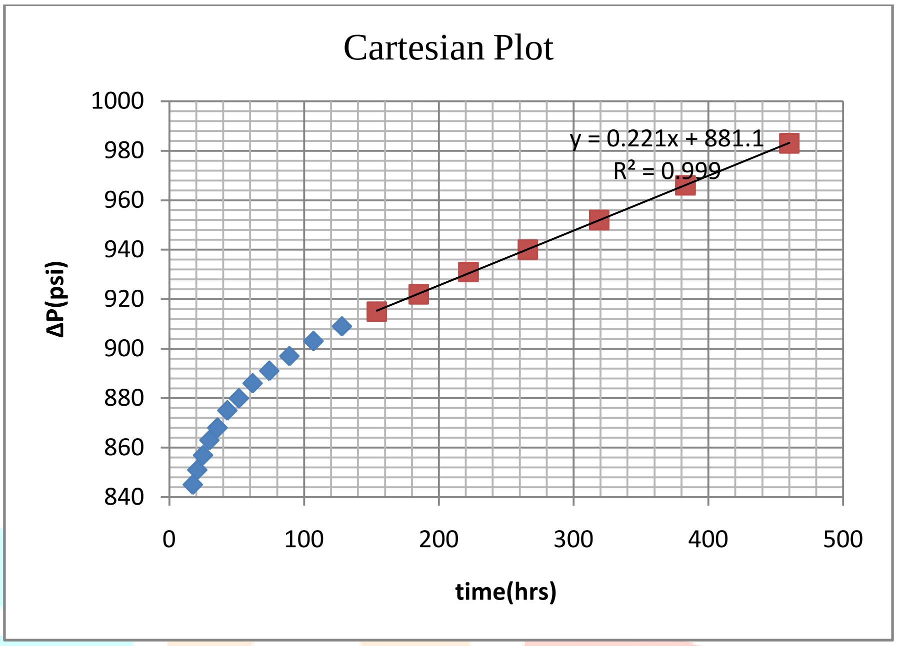

Figure: 7.5: Cartesian graph of ΔP Vs flowing time (t)

By plotting ΔP Vs time on a Cartesian graph, the pseudo steady state straight line is identified on the late time portion of the curve. The slope m* is calculated from the linear curve fit equation of this straight line.

m* = 0.2216 Psi/hr

The drainage area can be calculated by-

$A = \frac{0.23395\,qB}{\emptyset hCtm*}$

$= 6554039$ ft^2

The pore volume of the reservoir portion being drained by the test well is-

$Vp = \frac{0.23395\,qB}{Ctm*}$

$= 17636919$ ft^3

The initial oil in place contained in the drainage area is-

$N = \frac{(1-Swi)Vp}{5.615\,Boi}$

$= 2101023$ STB

6.3: Case 3

The discovery well of an oil reservoir above the bubble point (under-saturated black oil reservoir) was tested at a constant rate of 385 STB/D and produced a cumulative volume of 2780 STB of oil. The well was then shut-in for a buildup test, and bottom-hole pressure was recorded for 100 hours as shown in Table 7.5.

Other rock and fluid properties are given below.

r_w = 0.25 ft h = 36 ftØ = 13 % B = 1.67 bbl/STB
μ = 0.75 cp Ct = 11x10-6 psi^{-1}

1. Determine permeability, skin factor, Flow Efficiency, and WBS coefficient.

2. Estimate initial reservoir pressure.

Table 7.5: Buildup test data of an oil reservoir

Shut in time Δt(hr)	Pws(psi)	ΔP(psi)	Shut in timeΔt(hr)	Pws(psi)	ΔP(psi)
0	3534		12	4363	829
0.01	3544	10	16	4373	839
0.02	3554	20	20	4379	845
0.04	3574	40	24	4386	852
0.06	3594	60	30	4393	859
0.08	3614	80	40	4398	864
0.1	3634	100	50	4402	868
0.15	3680	146	60	4405	871
0.2	3723	189	72	4407	873
0.3	3800	266	83	4409	875
0.4	3866	332	100	4410	876
0.5	3920	386			
1	4103	569			
2	4250	716			
4	4320	786			
6	4340	806			
7	4345	811			
8	4350	816			

Solution:

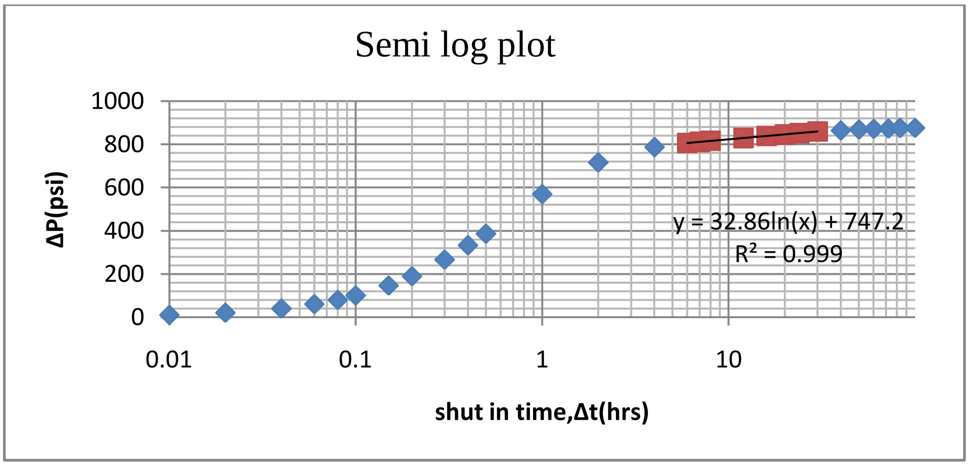

Figure 7.6: Semi-log plot of ΔP Vs shut in time (Δt)

From the curve fit equation, the slope,m = 32.868*2.303

= 75.695

Formation permeability is,

$K = 162.6 \frac{q\mu B}{mh}$

= 28.77327 mD

Skin factor can be calculated from-

$S = 1.151\left[\frac{\Delta P1hr}{m} - log\left(\frac{K}{\Phi\mu Ctrw^{2}}\right) + 3.23\right]$

$\Delta P1hr$ can be obtained by substituting t=1 in the curve fit function

ΔP_{1hr}= 32.868*2.303*log (1) + 747.29

= 747.29 Psi

S = 5.14459

The additional pressure drop due to skin (S>0) is:

ΔPs = 0.87mS

= 338.7952 psi

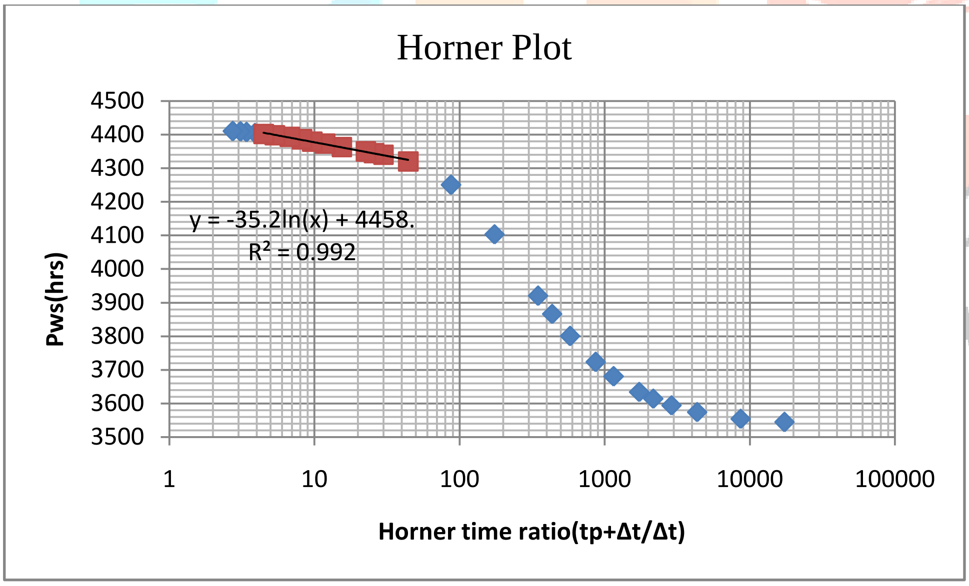

Figure 7.7: Horner plot to determine P*

From Horner Plot, P* is determined from the curve fit straight line equation as-

P* = -35.29*2.303*log(1) + 4458.6

= 4458.6 Psi

Since, the given well is a discovery well, P* = P_i = 4458.6 Psi

From the given data, $P_{wf(\Delta t=0)}$= 3534 Psi

Then the flow efficiency is given by –

$F.E = 1 - \frac{\Delta Ps}{P* - Pwf(\Delta t=0)}$

= 0.633576 = 63.3576 %

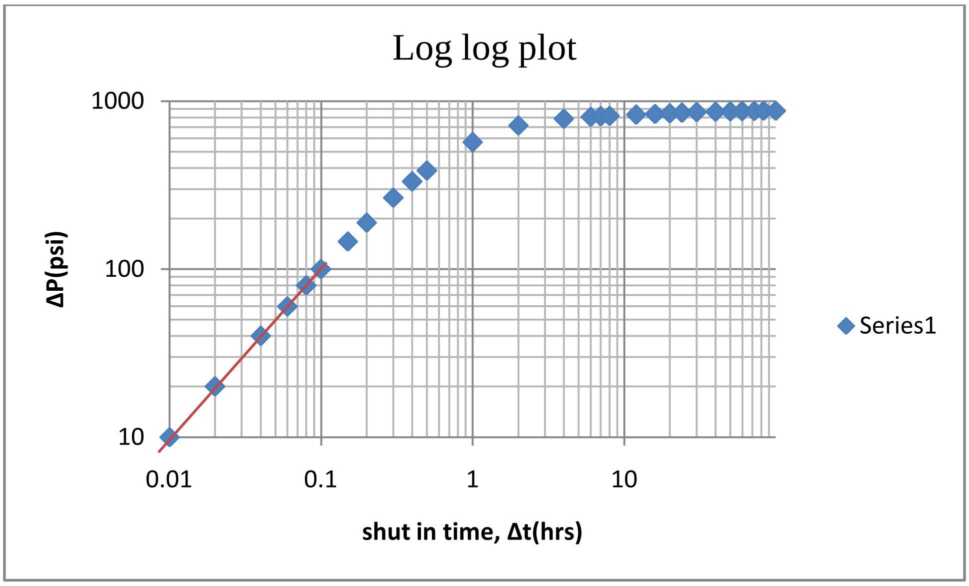

Figure. 7.8: Plot of ΔP Vs shut in time (Δt) on log- log graph

From the log- log plot of ΔP Vs shut in time, selecting a point N (0.01,10) on the unit slope line,

At t_N = 0.01 and ΔP_N = 10, the wellbore co-efficient is-

$$C = \left(\frac{q \times B}{24}\right)\frac{tN}{\Delta PN}$$

= 0.02679 bbl/psi

RESULTS AND DISCUSSION

7.1: Results:

Case 1:

The value of formation permeability is 80.3823md and the skin factor is 6.637914. Due to skin (s > 0), there is an additional pressure drop of 1411.87 psi. The flow efficiency is of 53.2677%.

From the early-time unit slope the wellbore storage co-efficient is estimated to be 0.016046 bbl/psi and the starting time of the radial flow (MTR) is also calculated and the value is 2.23053 hrs.

Case 2:

The calculated formation permeability is 7.386418md and the skin factor is 5.878897. Due to skin (s > 0), there is an additional pressure drop of 370.7328psia. The flow efficiency is of 62.2856%.

From the early-time unit slope the wellbore storage co-efficient is estimated to be 0.016046bbl/psi. With the Cartesian plot of ***Δ***P vs time, the calculated value of drainage area, pore volume of the reservoir and the initial oil in place are 6554039ft^2, 17636919ft^3 and 2101023stb.

7.2: Discussion

Case 1:

The formation permeability is of 80.3823md which indicates that the well has average permeability. The skin factor obtained is positive that means the some nearest to the wellbore is likely to be damaged. Since, the skin is positive, there is an additional pressure drop of 1411.87psia. The well is damaged and producing with only 53.2677%. The well is of course a good candidate for stimulation.

Case 2:

The formation permeability is of only 7.386418md which indicates that the well has poor permeability. The skin factor obtained is positive that means the Zone nearest to the wellbore is likely to be damaged. Since, the skin is positive, there is an additional pressure drop of 370.7328psia. The value of flow efficiency is 0.6228566 which implies the well is damaged and it is a good candidate for stimulation.

Case 3:

The formation permeability is of only 28.77327md which indicates that the well has poor permeability. The skin factor obtained is positive that means the zone nearest to the wellbore is likely to be damaged. Since the skin is positive there is an additional pressure drop of 338.7952psia

The value of flow efficiency is 0.633757 (<1) which implies the well is damaged and it is a good candidate for stimulation. As it is a discovery well, P* will be the initial reservoir.

CONCLUSION

Identification of different flow regimes that occur during build-up or drawdown test is very important for estimating the reservoir parameters.

Diagnostic plot helps in differentiating these flow regions like early time region, middle time region and late time region.

Horner time ratio yields accurate estimates of flow efficiency, skin factor and accurate estimate of average pressure (P^*) or initial reservoir pressure (P_i) can also be obtained.

The use of material balance can yield significant insight into reservoir characterization along with the original oil and gas in place estimation in different reservoir with different drives.

Simple exponential equations are used to predict the initial gas flowrate, initial decline rate and original gas in place.

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