PAIR SUM LABELING OF GRAPH OPERATIONS IN $\Gamma(Z_n)$

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ABSTRACT: In this chapter, we investigate the pair sum labeling behavior of the graphs $\Gamma(Z_{2p})\cup\Gamma(Z_{2q})$, $\Gamma(Z_9)\cup\Gamma(Z_{2p})$, $\Gamma(Z_8)\cup\Gamma(Z_{2p})$, $\Gamma(Z_6)\cup\Gamma(Z_{2p})$, $m\Gamma(Z_{p^2})$, $p\leq 5$, Comp $\Gamma(Z_8)\odot\Gamma(Z_4)$, $\Gamma(Z_6)\odot\Gamma(Z_4)$, $\Gamma(Z_6)\odot 2\Gamma(Z_4)$, $\Gamma(Z_8)\odot 2\Gamma(Z_4)$, $\Gamma(Z_9)\odot 2\Gamma(Z_4)$, $S(\Gamma(Z_{2p}))$, $S(\Gamma(Z_6)\odot\Gamma(Z_4))$, $S(\Gamma(Z_8)\odot\Gamma(Z_4))$ and $S(\Gamma(Z_8)\odot\Gamma(Z_4))$.

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1. INTRODUCTION

Let G be a (r, s) graph. An one to one map $f:V(G)\rightarrow\{\pm 1,\pm 2,\ldots\ldots\ldots,\pm r\}$ is called a pair sum labeling if the induced edge mapping, $f_e:E(G)\rightarrow Z-\{0\}$ defined by $f_e(uv)=f(u)+f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm l_1,\pm l_2,\ldots.,\pm l_{s/2}\}$ or $\{\pm l_1,\pm l_2,\ldots.,\pm l_{(s-1)/2}\}\cup\{l_{(s+1)/2}\}$ according as S is even or odd. A graph with a pair sum labeling defined on it is called pair sum graph. The pair sum labeling was introduced in [3] by R.Ponraj and et al. In [3], [4], [5] and [6] they study the pair sum labeling of cycle, path, star and some of their related graphs. Let R be a commutative ring and let Z(R) be its set of zero-divisors. We associate a graph $\Gamma(R)$ to R with vertices $\Gamma(R)^*=Z(R)-\{0\}$, the set of non-zero zero divisors of R and for distinct $u,v\in Z(R)^*$, the vertices u and v are adjacent if and only if $uv=0$. The zero divisor graph is very useful to find the algebraic structures and properties of rings. The idea of a zero divisor graph of a commutative ring was introduced by I.Beck in [2]. The first simplication of Beck's zero divisor graph was introduced by D.F.Anderson and P.S.Livingston [1]. Their motivation was to give a better illustration of the zero divisor structure of the ring. D.F.Anderson and P.S.Livinston, and others e.g., [7, 8, 9], investigate the interplay between the graph theoretic properties of $\Gamma(R)$ and the ring theoretic properties of R. Throught this paper, we consider the commutative ring R by Z_n and zero divisor graph $\Gamma(R)$ by $\Gamma(Z_n)$.

2. PAIR SUM LABELING OF UNION OF $\Gamma(Z_n)$

Theorem 2.1. $\Gamma(Z_{2p})\cup\Gamma(Z_{2q})$ is a pair sum graphs where p and q are different prime numbers.

Proof. Let $\{p,2,4,....,2(p-1)\}$ be the vertices of $\Gamma(Z_{2p})$. That is, the vertex set of $\Gamma(Z_{2p})$ is $\{u,u_1,u_2,....,u_{p-1}\}$ and the $E(\Gamma(Z_{2p}))=\{uu_i:1\le i\le p\}$. Let $\{q,2,4,....,2(q-1)\}$ be the vertices of $\Gamma(Z_{2q})$, That is, the vertex set of $\Gamma(Z_{2q})$ is $\{v,v_1,v_2,....,v_{q-1}\}$ and $E(\Gamma(Z_{2q}))=\{vv_i:1\le i\le q\}$, where p and q are distinct prime numbers.

Case (i): Let $p=q$.

Clearly from [6], $T\cup T$ is a pair sum tree, for any tree.

Case (ii): Without loss of generality, assume that $q>p$.

Define,

$f(u)=1$

$f(u_i)=i+1,\ 1\le i\le n$

$f(v)=-1$

$f(v_i)=-(i+1),\ 1\le i\le n$

$f(v_{p+2i-1})=-(p+3+i),\ 1\le i\le \frac{q-p}{2},\ if\ q-p\ is\ even$

$f(v_{p+2i-1})=-(p+3+i),\ 1\le i\le \frac{q-p-1}{2},\ if\ q-p\ is\ odd$

$f(v_{p+2i})=p+i+1,\ 1\le i\le \frac{q-p}{2},\ if\ q-p\ is\ even$

$f(v_{p+2i})=p+i+1,\ 1\le i\le \frac{q-p-1}{2},\ if\ q-p\ is\ odd$

Thus the edge set,

$f_e(E(\Gamma(Z_{2p})\cup\Gamma(Z_{2q})))=\{\pm 3,\pm 5,\ ....,\pm(p+2)\}\cup\{\pm(p+3),...,\pm(p+q+3)/2\}\cup\{-(p+q+5)/2\}$

if $q-n$ is odd. Clearly the function is a pair sum labeling. Hence, for any distinct prime numbers p and q, $\Gamma(Z_{2p})\cup\Gamma(Z_{2q})$ is a pair sum labeling graph.

Theorem 2.2. For any graph $\Gamma(Z_n)$, the following holds:

(i) $\Gamma(Z_8)\cup\Gamma(Z_{2p})$ is a pair sum graph.

(ii) $\Gamma(Z_6)\cup\Gamma(Z_{2p})$ is a pair sum graph.

(iii) $\Gamma(Z_9)\cup\Gamma(Z_{2p})$ is a pair sum graph.

Proof. (i) To prove $\Gamma(Z_8)\cup\Gamma(Z_{2p})$ is a pair sum graph.

Let uvw be the path $\Gamma(Z_8)$. Since the vertex set of $\Gamma(Z_8)$ is $\{2,4,6\}$. Clearly, $\Gamma(Z_8)$ is isomorphic to P_3. Let $V(\Gamma(Z_{2p}))=\{v,v_i:1\le i\le p\}$ and $E(\Gamma(Z_{2p}))=\{vv_i:1\le i\le p\}$.

If $p=3$, then $\Gamma(Z_{2p})=\Gamma(Z_6)$. Clearly, we know that $\Gamma(Z_6)$ is isomorphic to $K_{1,2}$ or $\Gamma(Z_6)$ is isomorphic to P_3. Hence, the union of two paths (or) the union of trees is pair sum graph. That is $\Gamma(Z_8)\cup\Gamma(Z_{2p})$ is a pair sum graph.

(ii) To prove $\Gamma(Z_6)\cup\Gamma(Z_{2p})$ is a pair sum graph.

Let uvw be the path $\Gamma(Z_6)$. Since the vertex set of $\Gamma(Z_6)$ is $\{2,3,4\}$. Clearly, $\Gamma(Z_6)$ is isomorphic to P_5. Using the above proof (i) $\Gamma(Z_6)\cup\Gamma(Z_{2p})$ is a pair sum graph.

(iii) To prove $\Gamma(Z_9)\cup\Gamma(Z_{2p})$ is a pair sum graph.

Let uv be the path $\Gamma(Z_6)$. The vertex set of $\Gamma(Z_6)$ is $\{3,6\}$. Clearly, $\Gamma(Z_9)$ is a path of order 2. We know that the union of two trees is a pair sum labeling graph. Hence, $\Gamma(Z_9)\cup\Gamma(Z_{2p})$ is a pair sum labeling.

In general, for any path P_m in $\Gamma(Z_n)$, the vertex set of P_m is $\{u_1,u_2,....,u_m\}$. That is $u_1, u_2, \ldots, u_m$ be the path P_m. Let $V(\Gamma(Z_{2p}))=\{v,v_i :1\le i\le p\}$ and $E(\Gamma(Z_{2p}))=\{vv_i :1\le i\le p\}$.

Case (i): $m = p$

Define,

$f(u_i)=i, 1\le i\le m$

$f(v)=-1$

$f(v_i)=-2i, 1\le i\le m$

Hence, $f_e(E(P_m)\cup\Gamma(Z_{2p}))=\{\pm 3,\pm 5,...,\pm(2p-1)\}\cup\{-(2p+1)\}$

Case (ii): $p > m$

Define,

$f(u_i)=i, 1\le i\le m$

$f(v)=-1$

$f(v_i)=-2i, 1\le i\le m-1$

$f(v_{m+2i-1}) = 2m+i, 1\le i\le \frac{p-m+1}{2}, if\ p-m is odd.$

$f(v_{m+2i-1}) = 2m+i, 1\le i\le \frac{p-m}{2}, if\ p-m is even.$

$f(v_{m+2i-2}) = -(2m+i-2), 1\le i\le \frac{p-m+1}{2}, if\ p-m is odd.$

$f(v_{m+2i-2}) = -(2m+i-2), 1\le i\le \frac{p-m}{2}, if\ p-m is even.$

Here

$f_e(E(P_m)\cup\Gamma(Z_{2p}))=\{\pm 3,\pm 5,...,\pm(2m-1)\}\cup\left\{\pm 2m,\pm(2m-1),....,\frac{(3m+p-2)}{2}\right\}$ if $p-m$ is odd

$f_e(E(P_m)\cup\Gamma(Z_{2p}))=\{\pm 3,\pm 5,...,\pm(2m-1)\}\cup\left\{\pm 2m,\pm(2m+1),....,\frac{(3m+p-2)}{2}\right\}\cup\left\{\frac{(3m+n)}{2}\right\}$ if $n-m$ is even. Then, f is a pair sum labeling.

Theorem 2.3. If any prime $p\le 5$, then $m\Gamma(Z_{p^2})$ is a pair sum graph.

Theorem 2.4. If $p\ge 11$, then $m\Gamma(Z_{p^2})$ is not a pair sum graph.

Proof. We prove this by the method of contradiction. Suppose, $m\Gamma(Z_{p^2})$ is a pair sum graph. We know that, if $\Gamma(Z_n)$ is a (r,s) pair sum graph then $s\le 4r-2$, where r is the number of vertices and s is the number of edges in $\Gamma(Z_n)$. We know that $\Gamma(Z_{p^2})$ is isomorphic with K_{p-1}. Then, the number of edges in a complete graph K_{p-1} is $\frac{(p-1)(p-2)}{2}$. Then the total edges of m copies of $\Gamma(Z_{p^2})$ is $\frac{m(p-1)(p-2)}{2}$.

Then, $\frac{m(p-1)(p-2)}{2} \le 4(p-1)-2 = 4p-6$

$m(p-1)(p-2)-8p+12 \le 0$

$8m - m(p-1)(p-2)-12 \ge 0$

If $p=11$ and $m=5$, then $8m-m(p-1)(p-2)-12$ is a negative value. Clearly, $8m-m(p-1)(p-2)-12 \ge 0$, a contradiction.

Hence, If any prime $p \ge 11$, then $m\Gamma(Z_{p^2})$ is not a pair sum graph.

3. PAIR SUM LABELING OF CORONA OF TWO ZERO DIVISOR GRAPHS

In this section, we investigate the pair sum labeling behavior of some graphs obtained as a Corona of two standard graph in zero divisor graphs.

Theorem 3.1. (i) The comb $\Gamma(Z_6)\odot\Gamma(Z_4)$ is a pair sum graph.

(ii) $\Gamma(Z_8)\odot\Gamma(Z_4)$ is a pair sum graph.

(iii) $\Gamma(Z_9)\odot\Gamma(Z_4)$ is a pair sum graph.

Proof. (i) and (ii), We know that, $\Gamma(Z_6)\odot\Gamma(Z_4)$ and $\Gamma(Z_8)\odot\Gamma(Z_4)$ are same graphs. Since $\Gamma(Z_6)$ and $\Gamma(Z_8)$ are isomorphic to P_3.

Let P_3 be the path uvw and u and w are the pendent vertices, adjacent to v. Let $n=3=2m+1$, where $m=1$.

Define, $f: V(P_3 \odot \Gamma(Z_4)) \to \{\pm 1, \pm 2, ..., \pm(2m+1)\}$ by

$f(u_i) or\ f(w_i) = 2i; 1 \le i \le m$

$f(u_i) or\ f(w_i) = -(2m+4); i = m+1$

$f(u_i) or\ f(w_i) = 2i - 4(m+1); m+1 \le i \le 2m+1$

$f(v_i) = 2i-1; 1 \le i \le m$

$f(v_i) = 2m+8; i = m+1$

$f(v_i) = 2i-4m-3; m+2 \le i \le 2m+1$

Here, $f_e(E(P_3) \odot \Gamma(Z_4)) = \{\pm 3, \pm 6, \pm 7, ..., \pm(4m-2), \pm(4m-1)\} \cup \{\pm 4, \pm(4m-4)\}$

Then, f is a pair sum labeling.

(iii) We know that $\Gamma(Z_9)\odot\Gamma(Z_4)$ is isomorphic to $P_2 \odot K_1$.

Let $n=2m=2$. Then define $f: V(\Gamma(Z_9)\odot\Gamma(Z_4)) \to \{\pm 1, \pm 2\}$ by $n \equiv 2(\mathrm{mod}\ 4)$

$f(u_i) = 2i,\ if\ i\ is\ odd\ and\ i \le m-1$

$f(u_i) = 2i-1,\ if\ i\ is\ even\ and\ i \le m$

$f(u_i) = -(2m+1),\ if\ i = m$

$f(u_i) = 2m+3,\ if\ i = m+1$

$f(u_i) = 2i-1-4m,\ if\ i\ is\ odd\ m+1 < i \le 2m$

$f(u_i) = 2i-(4m+2),\ if\ i\ is\ even\ m+1 < i \le 2m$

$f(v_i) = 2i-1,\ if\ i\ is\ odd\ and\ i \le m$

$f(v_i) = 2i,\ if\ i\ is\ even\ and\ i \le m$

$f(v_i) = 2i-1-4m,\ if\ i\ is\ even\ and\ m < i \le 2m$

$f(v_i) = 2i-(4m+2),\ if\ i\ is\ odd\ m < i \le 2m$

Here, $f_e(E(\Gamma(Z_9)\Theta\Gamma(Z_4)) = \{\pm 3, \pm 5, ..., \pm(4m-5)\} \cup \{\pm 2, \pm 4, \pm 6\}$.

Then, f is a pair sum labeling. Therefore $\Gamma(Z_6)\Theta\Gamma(Z_4)$, $\Gamma(Z_8)\Theta\Gamma(Z_4)$ and $\Gamma(Z_9)\Theta\Gamma(Z_4)$ are pair sum graphs.

Theorem 3.2. The graph $\Gamma(Z_n)\Theta 2\Gamma(Z_4)$ is a pair sum graph, where $n = 6$, 8 and 9.

Proof. We know that $\Gamma(Z_6) \cong \Gamma(Z_8) \cong P_3$. Let v_i and w_i be the pendent vertices adjacent to u_i for $1 \le i \le n$.

Case (i) $n = 3\,[odd]$

Define,

$$f(u_i) = 3i - 1;\ 1 \le i \le \frac{n-1}{2}$$

$$f(u_i) = \frac{1-3n}{2};\ i = \frac{n+1}{2}$$

$$f(u_i) = 3i - (3n+2);\ \frac{n+3}{2} \le i \le n$$

$$f(v_i) = 3i\ ;\ 1 \le i \le \frac{n-1}{2}$$

$$f(v_i) = \frac{3n+5}{2};\ i = \frac{n+1}{2}$$

$$f(v_i) = 3i - (3n+3);\ \frac{n+3}{2} \le i \le n$$

$$f(w_i) = 3n - 5;\ i = 1$$

$$f(w_i) = 3i - 2\ ;\ 2 \le i \le \frac{n-1}{2}$$

$$f(w_i) = \frac{3(n+1)}{2};\ i = \frac{n+1}{2}$$

$$f(w_i) = 3i - (3n+1);\ \frac{n+3}{2} \le i \le n$$

Here, $f_e(E(\Gamma(Z_n)\Theta 2\Gamma(Z_4)) = \{\pm 2, \pm 3, ..., \pm(3n-4)\} \cup \{\pm(3n-3)\}$.

Then, f is a pair sum labeling.

Case (ii): We know that $\Gamma(Z_9) \cong P_2$. Clearly, u and v are pendent vertices in P_2.

Let $n = 2$ is even.

Since, $P_2\Theta 2\Gamma(Z_4) = B_{2,2}$. Let $V(B_{2,2}) = \{u, v, u_i, v_j : 1 \le i \le 2, 1 \le j \le 2\}$ and $E(B_{2,2}) = \{uv, uu_i, vv_j : 1 \le i \le 2, 1 \le j \le 2\}$.

Define, $f : V(B_{2,2}) \to \{\pm 1, \pm 2, ..., \pm(2n+2)\}$ by

$$f(u) = -1$$

$$f(v) = 2$$

$$f(u_i) = -2i; 1 \le i \le n$$

$$f(v_i) = 2i - 1; 1 \le i \le n, \text{ where } n = 2.$$

Thus, $f_e(E(B_{2,2})) = \{\pm 3, \pm 5, ..., \pm(2n+1)\} \cup \{1\}$ and have $B_{2,2}$ is a pair sum graph. That is $\Gamma(Z_9)\Theta 2\Gamma(Z_4)$ is a pair sum graph.

4. PAIR SUM LABELING ON SUBDIVISION GRAPHS IN $\Gamma(Z_n)$

Hence, We investigate the pair sum labeling behavior of graphs obtained as the subdivision of some standard graphs in $\Gamma(Z_n)$.

Theorem 4.1. $S(\Gamma(Z_{2p}))$ is a pair sum graph.

Proof. Let $V(S(\Gamma(Z_{2p}))) = \{u, u_i, v_i : 1 \le i \le p-1\}$ and $E(S(\Gamma(Z_{2p}))) = \{uu_i, u_iv_i : 1 \le i \le p-1\}$.

Define $f : S(\Gamma(Z_{2p})) \to \{\pm 1, \pm 2, ..., \pm(2p-1)\}$ by

$f(u) = -1$

$f(u_i) = i+1\,; i = 1,2,...,n$

$f(v_i) = -(2i+1)\,; i = 1,2,...,n$

Here $f_e(E(S(\Gamma(Z_{2p})))) \to \{\pm 1, \pm 2, ..., \pm(p-1)\}$

Then f gives a pair sum labeling for $S(\Gamma(Z_{2p}))$.

Theorem 4.2. $S(\Gamma(Z_n) \odot \Gamma(Z_4))$ is a pair sum graph, where $n = 6, 8\, and\ 9$.

Proof. Let V($S(\Gamma(Z_n) \odot \Gamma(Z_4))$) = $\{u_i : 1 \le i \le 2n-1\} \cup \{w_i, v_i : 1 \le i \le n\}$.

Let E($S(\Gamma(Z_n) \odot \Gamma(Z_4))$) = $\{uu_{i+1} : 1 \le i \le 2n-2\} \cup \{u_{2i-1}w_i : 1 \le i \le n\} \cup \{v_iw_i : 1 \le i \le n\}$.

Since, $\Gamma(Z_6)$ and $\Gamma(Z_8)$ is path with length 2, and $\Gamma(Z_9)$ is a path with length 1.

Case (i): n is even $[\Gamma(Z_9) \cong P_2]$

Since we know that, any path is a pair sum graph. So, the Subdivision of $\Gamma(Z_9) \odot \Gamma(Z_4)$ is a pair sum graph.

Case (ii): $n = 3$ is odd $[\Gamma(Z_6) \cong \Gamma(Z_8) \cong P_3]$

Define, $f : V(S(P_3 \odot \Gamma(Z_4))) \to \{\pm 1, \pm 2, ..., \pm(4n-1)\}$ by

$f(u_{(n+1)/2}) = 1$

$f(u_{(n-1)/2}) = 8$

$f(u_{(n+3)/2}) = -8$

$f(u_{(n-1)/2}) = 8$

$$f(u_{(n-1)/2-2i}) = -10i+1,\ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1$$

$$f(u_{(n+1)/2-2i}) = 5i+5,\ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1$$

$$f(u_{(n+3)/2+2i}) = -10i-1,\ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1$$

$$f(u_{(n+1)/2+2i}) = -(5i+5),\ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1$$

$f(w_{(n+1)/2}) = -2$

$f(w_{(n-1)/2}) = -5$

$$f(w_{(n-1)/2-i}) = 5i+7,\ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1$$

$$f(w_{(n+3)/2+i}) = -(5i+7),\ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1$$

$f(v_{(n+1)/2}) = 3$

$f\left(v_{(n-1)/2}\right)=9$

$f\left(v_{(n+3)/2}\right)=-9$

$f\left(v_{(n-1)/2-i}\right)=5i+8,\ 1\le i\le\left\lfloor\frac{n}{2}\right\rfloor-1$

$f\left(v_{(n+3)/2+i}\right)=-(5i+8),\ 1\le i\le\left\lfloor\frac{n}{2}\right\rfloor-1$

Here $f_e(E(S(P_3\odot\Gamma(Z_4))))=\{\pm1,\pm2,...,\pm5\}$

Then f is pair sum labeling.

5. PAIR SUM LABELING OF PATH AND CYCLE RELATED TO ZERO DIVISOR GRAPH

In this chapter, we prove that $\Gamma(Z_9)\times\Gamma(Z_9)$, $C_n\times\Gamma(Z_9)$ and $[C_m,P_n]$ are pair sum graphs.

Theorem 5.1. The graph $\Gamma(Z_9)\times\Gamma(Z_9)$ is a pair sum graph.

Theorem 5.2. The graph $C_n\times\Gamma(Z_9)$ is a pair sum graph, if n is even.

Proof. Let $V(C_n\times\Gamma(Z_9))=\{u_i,v_i:1\le i\le n\}$ and

$E(C_n\times\Gamma(Z_9))=\{u_iu_{i+1},v_iv_{i+1}:1\le i\le n-1\}\cup\{u_iv_i:1\le i\le n\}\cup\{u_nu_1,v_nv_1\}$.

Define $f:V(C_n\times\Gamma(Z_9))\to\{\pm1,\pm2,...,\pm n\}$ as follows

Case (i): $n=4m+2$.

Define , $f(u_i)=i;1\le i\le 2m+1$

$f(u_{2m+1+i})=-i;1\le i\le 2m+1$

$f(v_i)=8m-2i+6;1\le i\le 2m+1$

$f(v_{2m+1+i})=-8m+2i-6;1\le i\le 2m+1$

Here $f_e(E(C_n\times\Gamma(Z_9)))=\{\pm3,\pm5,...,\pm(4m+1)\}\cup\{\pm2m\}\cup\{\pm(6m+5),\pm(6m+6),...,\pm(8m+5)\}$.

Case (ii): $n=4m$.

$f(u_i)=i;1\le i\le 2m-1$

$f(u_{2m+i})=-i;1\le i\le 2m-1$

$f(u_{4m})=-(2m+1)$

$f(v_{2m+1-i})=8m-2i+2;1\le i\le 2m$

$f(v_{2m+i})=-(4m+2i);1\le i\le 2m$

Here $f_e(E(C_n\times\Gamma(Z_9)))$

$=\{\pm3,\pm5,...,\pm(4m-3)\}\cup\{\pm2m,\pm4m\}\cup\{\pm(4m+3),\pm(4m+6),...,\pm(10m-3)\}\cup\{\pm(10m+1)\}$

Clearly, f is a pair sum labeling.

Theorem 5.3. The graph $[C_m,\Gamma(Z_6)]$ is a pair sum graph.

Proof. Let the first copy of the cycle C_m be $u_1u_2,...,u_mu_1$ and second copy of cycle C_m be $v_1v_2,...,v_nv_1$. Let $\Gamma(Z_6)$ be path P_3, $w_1w_2w_3$. Let $V([C_m,\Gamma(Z_6)])=V(C_m)\cup V(C_m)\cup V(\Gamma(Z_6))$ and $E([C_m,\Gamma(Z_6)])=E(C_m)\cup E(C_m)\cup E(\Gamma(Z_6))\cup\{u_1w_1,w_3v_1\}$.

Define $f:V([C_m,\Gamma(Z_6)])\to\{\pm1,\pm2,...,\pm3m\}$ by

$f(w_1)=2$

$f(w_2)=1$

$f(w_3)=-4$

$f(v_i) = -3m - i + 1, 1 \le i \le m$

$f(u_i) = 2m + 2 + i, 1 \le i \le m - 2$

$f(u_{m-1}) = 2m + 1$

$f(u_m) = 2m + 2$

Here,

$$f_e(E([C_m, \Gamma(Z_6)])) = \{\pm 3, \pm 5, ..., \pm n\} \cup \{\pm (4m+3), \pm (4m+5), ..., \pm (6m-1)\} \cup \left\{\pm (5m+1), \left(5m + \frac{7}{2}\right)\right\}$$

Then, Clearly f is a pair sum labeling.

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