COMPARATIVE STUDY OF NON-SPHERICAL GRAVITATIONAL COLLAPSE BETWEEN STRANGE QUARK MATTER AND HUSSAIN SPACE TIME

Lakshmi Madireddy[#1], K.D.Patil *[2], S.S.Zade [#3]

[#] Assistant Professor, Department of Applied Mathematics, RTM Nagpur University
B.D.College of Engineering, Sevagram, (M.S), India
[*]Professor, Department of Applied Mathematics, RTM Nagpur University
B.D.College of Engineering, Sevagram, (M.S), India
[#]Associate Professor, Department of Mathematics, RTM Nagpur University
J.B. College of Science, Wardha, (M.S), India

Abstract:
Earlier some of the authors have shown the nature and occurrence of the singularities in non spherical gravitational collapse of strange quark matter and Hussain space time. In the previous issues it was also shown that the naked singularities are formed. In the present work we explore the comparative study about the nature of naked singularities occurred in between both the space times. Implication of the same initial data to the non spherical gravitational collapse of strange quark matter and Hussain space times has been discussed.

Keywords – *Strange quark matter, Cosmic Censorship, Gravitational Collapse, Naked Singularity*
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I INTRODUCTION

Cosmic censorship conjecture is one of the most important issues in the active areas of research in general relativity and Astrophysics. In Cosmic censorship hypothesis, R.Penrose [23] has stated that all the singularities which are formed after the gravitational collapse cannot be visible by an observer. Even there are two versions for this hypothesis. One is weak version, in this it explains that the singularity obtained at the final state cannot be observed by an observer. In the strong version it explains that, the singularities are visible. From the past thirty years, there have been extensive studies on the nature of singularities. The study of gravitational collapse of spherically space times led to many examples [5-7, 12-17, 21]. The global behavior of radial null geodesics must be studied in full generality to study about the nature of the singularities.

Many scientists have given many proofs and disproves for this CCH. But till date no scientist or a researcher has given the exact proof for the above. Many have published the research papers on spherical symmetrical space time. In Einstein's field equations of general relativity the spherical symmetry is a characteristic feature of many solutions. We should also know about the non-spherical symmetry as well. In this manuscript we give a brief idea of the naked singularities observed in the non spherical space times and their comparisons..

One of the topics in Astrophysics is about the possibility of the discovery of strange stars. A quark star or a strange star is a hypothetical type of star composed a strange matter or a quark matter. Quark stars have not been directly observable but theoretical predictions based on their existence have been confirmed experimentally. When the neutron degenerates matter which makes up a neutron star is put under sufficient pressure due to the stars gravity, the individual neutron break down into their constituent quarks. Then strange quarks are formed from the above quarks with strange matter. The stars which are formed after this give the strange star or quark star [22]. The meaning of strange matter is just quark matter that contains here flavors of quarks up, down and strange. There is a critical pressure and the density, when the nuclear matter compressed beyond this density the protons and neutrons dissociate into quarks yielding the strange matter. Some of the research papers has given the strange quark matter appeared so far, by explaining the properties and formation of strange stars [1, 3, 4, 8, 20]. The strange quark matter is characterized by the equation $P = \dfrac{\rho - 4B}{3}$ (a), where ρ is the energy density, P is the thermodynamic pressure of the quark matter, and B is the difference between the energy densities of the perturbative and non-perturbative vacuum known as bag constant. The above equation gives obviously that the strange quark matter will satisfy the energy condition $\rho \geq P \geq 0$. [22]

It is generally believed that the most energetically favorable state of baryon matter consists of u, d and s quarks. There are two ways of strange quark matter, one is neutron stars into strange ones at ultra high densities and other is quarkhadron phase transition in the early universe. Several mechanisms have proposed about the formation of quark stars. Quark stars are expected during the collapse of the core of a massive star after the supernova explosion as a result of a first or second order phase transition, resulting in deconfined quark matter [3]. For the conversion of ordinary matter to strange quark matter the suitable environment is neutron star or a proto neutron star. In the study of strange quark star formation, dynamics and evolution the interesting problem is the collapse of the strange matter. The fundamental importance in general relativity is the theoretical understanding of the collapse.

The gravitational collapse of strange matter was studied by K.S.Cheng and T.Harko [9]. They also analyzed about the condition for the formation of naked singularities in the spherically symmetrical Vaidya space time.

This manuscript gives the extension of the non spherical gravitational collapse of strange quark matter [16] and also we compare this with non spherical gravitational collapse in Hussain space time.

This paper contains the Non spherical collapse of strange quark matter in section 2 then Non spherical collapse in Hussain space time in section 3. Section 4 gives about the occurrence of naked singularities and their comparison in two space times followed by concluding remarks in section 5.

II NON SPHERICAL COLLAPSE OF STRANGE QUARK MATTER

The line element describing the radial collapse of charged strange quark fluid in cylindrical, toroidal or planar space time can be written as [9,10,11,22]

$$ds^2 = -\left[\alpha^2 r^2 - \frac{qm(u,r)}{r}\right]du^2 + 2dudr + r^2\left(d\theta^2\, d\phi^2\right) \tag{1}$$

Where u is an advanced Eddington time coordinate, and α is defined by the relation $\alpha = \sqrt{-\wedge/3}$.

r is the radial coordinate whose range is $0 < r < \infty$, $m(u,r)$ is the mass function giving the gravitational mass inside the sphere of radius r.

The energy momentum tensor can be written in the form [20, 24]

$$T_{\mu\nu} = T_{\mu\nu}{}^{(n)} + T_{\mu\nu}{}^{(m)} + E_{\mu\nu} \tag{2}$$

and also $T_{\mu\nu}{}^{(n)} = \tau(u,r) l_\mu l_\nu$ is the component of matter which moves along the hyper surface u moves along with the constant.

The energy momentum tensor of the strange quark matter can be written in the form

$$T_{\mu\nu}{}^{(m)} = (P+\rho)(l_\nu n_\mu + l_\mu n_\nu) + Pg_{\mu\nu} \tag{3}$$

The electro magnetic combination is given by

$$E_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}\right) \tag{4}$$

We have consider the two null vectors n_ν, l_μ such that

$$l_\mu = \lambda_\mu{}^0,$$
$$n_\mu = \frac{1}{2}\left[\alpha^2 r^2 - \frac{qm(u,r)}{r}\right]\delta_\mu{}^0 - \delta_\mu{}^1, \tag{5}$$
$$l_\upsilon n^\upsilon = -1, \qquad l_\upsilon l^\upsilon = n_\upsilon n^\upsilon = 0$$

The energy momentum tensor of eq (2) is given by,

$$\rho = \frac{qm^{'} - 3\alpha^2 r^2}{8\pi r^2} - \frac{q^2 e^2(u)}{8\pi r^4} \tag{6}$$

$$p = \frac{6\alpha^2 r - qm^{''}}{16\pi r} - \frac{q^2 e^2(u)}{16\pi r^4} \tag{7}$$

$$\tau = \frac{q\dot{m}}{8\pi r^2} \tag{8}$$

The energy conditions for the above will be as follows:

1. The dominant energy conditions are
   $\mu \geq 0, \quad \rho \geq P \geq 0$ (9)

2. The weak and strong energy conditions are
   $\mu \geq 0, \quad P \geq 0, \quad \rho \geq 0$ (10)

By combining the equation (a) with equations (6) and (7) we get the following

$$3qm''r^2 + 2qm'r = 64\pi Br^3 + 24\alpha^2 r^3 - \frac{q^2 e^2(u)}{r} \tag{11}$$

On solving the above equation we get

$$qm(u,r) = qg(u) + qh(u)r^{1/3} + Ar^3 - \frac{q^2 e^2(u)}{4r} \tag{12}$$

Where h(u) and g(u) are the two arbitrary functions of u and $A = (8\pi B/3) + \alpha^2$.

By substituting the above mass function in equation (1) we get

$$ds^2 = -\left[\alpha^2 r^2 - \frac{qg(u)}{r} - \frac{qh(u)}{r^{2/3}} - Ar^2 + \frac{q^2 e^2(u)}{4r^2}\right]du^2 + 2dudr + r^2\left(d\theta^2\, d\phi^2\right) \tag{13}$$

By using the mass function we get the equations 6, 7, 8, as follows

$$\rho = \frac{1}{8\pi r^2}\left(\frac{1}{3}qh(u)r^{-2/3} + 8\pi Br^2 - \frac{3}{4r^2}q^2 e^2(u)\right) \tag{14}$$

$$P = \frac{1}{16\pi r}\left(\frac{2}{9}qh(u)r^{-5/3} - 16\pi Br - \frac{q^2 e^2(u)}{2r^3}\right) \tag{15}$$

$$\tau = \frac{1}{8\pi r^2}\left(q\dot{g}(u) + q\dot{h}(u)r^{1/3} - \frac{q^2 e(u)\dot{e}(u)}{2r}\right) \tag{16}$$

We always choose g(u) and h(u) in such a way that they follow the strong, weak and dominant energy conditions perfectly.

We need to consider the radial null geodesics defined by $ds^2 = 0$ and also $\dot{\theta} = \dot{\phi} = 0$

From the above equation (13) we have the following:

$$\frac{dr}{du} = \frac{1}{2}\left(\alpha^2 r^2 - \frac{qg(u)}{r} - \frac{qh(u)}{r^{2/3}} - Ar^2 + \frac{q^2 e^2(u)}{4r^2}\right) \tag{17}$$

The above equation does not give an analytic solution. If we choose some approximate values then we get the above equation in terms of elementary function.

Let us choose some values in such a way that $q(g(u)) = \frac{\beta}{2}u$, $q(h(u)) = \frac{\lambda}{2}u^{2/3}$ and $q^2 e^2(u) = \gamma^2 u^2$

$$\frac{du}{dr} = \frac{2}{\alpha^2 r^2 - \frac{\beta u}{2r} - \frac{\lambda u^{2/3}}{2r^{2/3}} - Ar^2 + \frac{\gamma^2 u^2}{4r^2}} \tag{18}$$

$$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} X = \lim_{\substack{u\to 0\\ r\to 0}} \frac{u}{r}$$

$$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr} = \frac{2}{-\frac{\beta}{2}X_0 - \frac{\lambda}{2}X_0^{2/3} + \frac{\gamma^2}{4}X_0^2} \tag{19}$$

$$X_0 = \frac{8}{-2\beta X_0 - 2\lambda X_0^{2/3} + \gamma^2 X_0^2}$$

$$\gamma^2 X_0^3 - 2\lambda X_0^{5/3} - 2\beta X_0^2 - 8 = 0 \tag{20}$$

Put $X_0 = X^3$

Then equation 20 becomes $\gamma^2 X^9 - 2\lambda X^5 - 2\beta X^6 - 8 = 0$ (21)

III NON- SPHERICAL COLLAPSE IN HUSSAIN SPACE TIME

The metric of non spherical collapse of Hussain space time is considered as in equation 1. [10,11,17]
Where we take q m (u,r) as in the following:

$$qm(u,r) = qf(u) + qg(u)\ln r, \; k = \frac{1}{2}$$
$$qm(u,r) = qf(u) - \frac{g(u)}{(2k-1)r^{2k-1}}, \; k \neq \frac{1}{2} \quad (22)$$

Where u is an advanced Eddington time coordinate, and α is defined by the relation $\alpha = \sqrt{-\wedge/3}$.
From the references [1,3] we write the topology of two dimensional space:
Topology of toroidal model is given by $S \times S$, planar symmetrical model has $R \times R$ where as cylindrical symmetrical model has $R \times S$.
Ranges for the values of θ and ϕ are given as follows

(i) Toroidal : $0 \le \theta \le 2\pi, 0 \le \phi \le 2\pi$.
(ii) Planar: $-\infty < \theta < \infty, -\infty < \phi < \infty$
(iii) Cylindrical: $-\infty < \theta < \infty, 0 < \phi < 2\pi$

The parameter q has different values which are taken from Arnowitt-Deser-Misner (ADM) masses of the corresponding static black holes [15]. Depending upon the topology of the two dimensional space the parameter q takes the different values. In toroidal model m acts as the mass and q takes the value as $2/\pi$, for the planar symmetrical model m acts as the mass per unit area so it takes the value as $2/r^2$ and finally in the cylindrical symmetric model m acts as the mass per unit length that is it takes $4/\alpha$. By using equation 17 the metric in equation 1 becomes the asymptotically anti-de-sitter for k > 1/2 where as for k< 1/2 it will become Cosmological.

So, let us choose the value of k as 2/3 and here we take $qf(u) = \frac{\beta}{2}u$, $qg(u) = \frac{\lambda}{2}u^{4/3}$ then equation 1 becomes

$$ds^2 - \left[\alpha^2 r^2 - \frac{\beta u}{2r} + \frac{3\lambda u^{4/3}}{2r^{4/3}}\right]du^2 + 2dudr + r^2\left(d\theta^2 d\phi^2\right) \quad (23)$$

$$\frac{du}{dr} = \frac{2}{\alpha^2 r^2 - \frac{\beta u}{2r} + \frac{3\lambda u^{4/3}}{2r^{4/3}}} \quad (24)$$

$$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} X = im_{\substack{u\to 0\\ r\to 0}} \frac{u}{r}$$

$$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr} = \frac{2}{-\frac{\beta}{2}X_0 + \frac{3}{2}\lambda X_0^{4/3}} \quad (25)$$

$$\frac{3}{2}\lambda X_0^{7/3} - \frac{\beta}{2}X_0^2 = 2 \quad (26)$$

$$3\lambda X_0^{7/3} - \beta X_0^6 - 4 = 0 \quad (27)$$

Put $X_0 = X^3$

Then equation 27 becomes $3\lambda X^7 - \beta X^6 - 4 = 0$ (28)

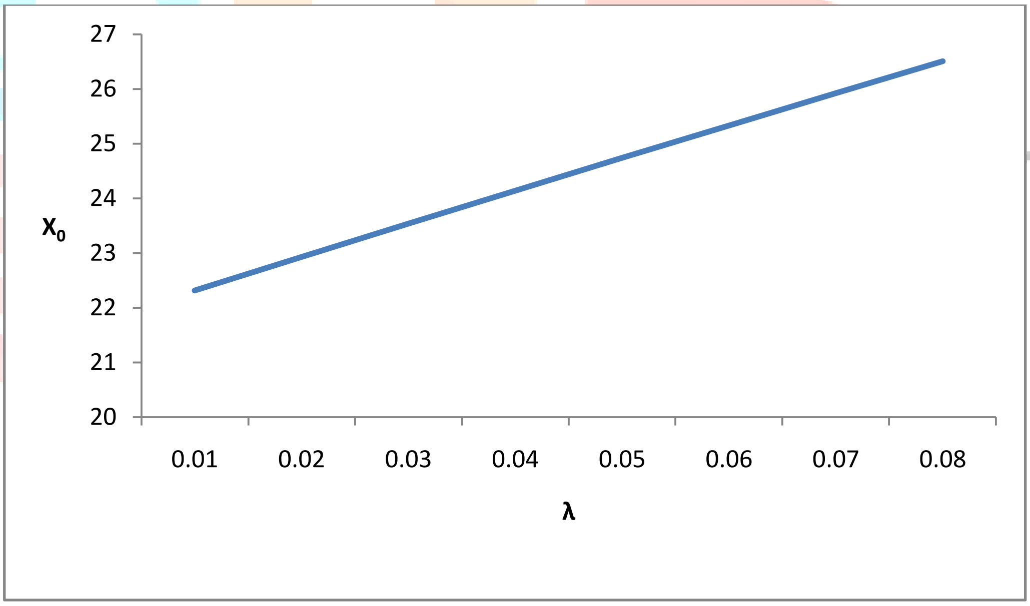
IV COMPARITIVE STUDY OF THE ABOVE TWO SPACE TIMES AND THEIR NATURE OF SINGULARITIES:

From equation 21 of Non spherical gravitational collapse of strange quark matter, we give the different values for β, γ, λ then different values of X_0 arises. If at least one of the roots obtained is positive then the singularity is naked. Otherwise it will be a black hole. Now, here we also find the different values of X_0 in non spherical gravitational collapse of Hussain space time. If at least one the roots obtained in Hussain space time are also positive, then finally we compare these two space times.

From Eq. 21 by keeping the values $\beta = 0.1$ and $\gamma = 0.1$ as fixed values we find the values of X_0 by taking different values of λ.

Table.1. Values of X_0 for different values of λ

λ	X_0
0.01	22.3167
0.02	22.9295
0.03	23.5375
0.04	24.4141
0.05	24.7391
0.06	25.3324
0.07	25.9212
0.08	26.5052



Graph.1. Graph of the Values of X_0 against the values of λ

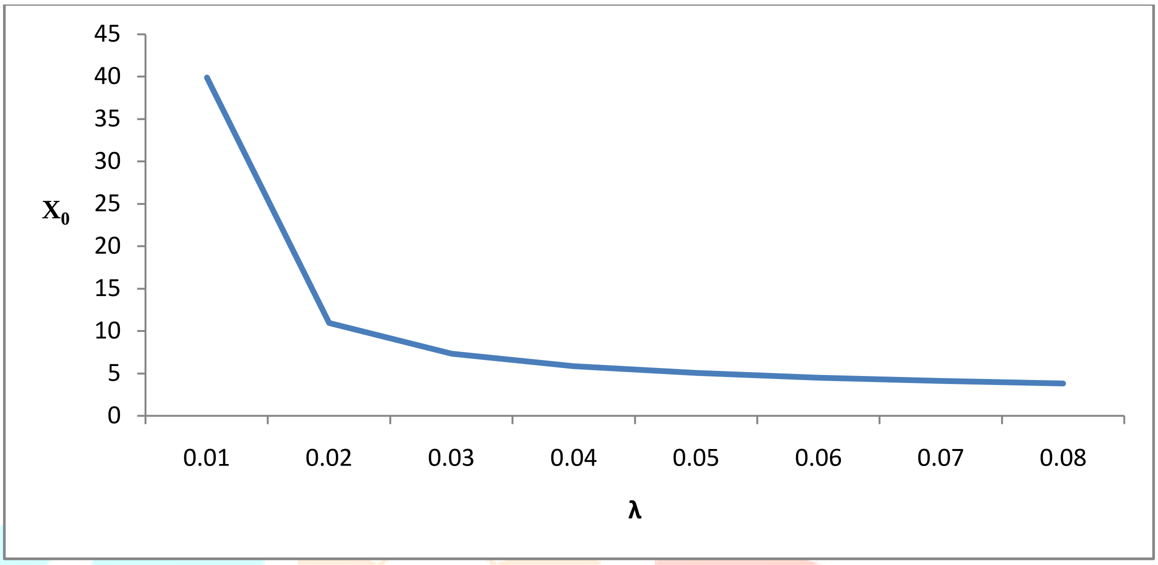
In the above graph the values of β and γ are taken as fixed values. We had given the different values for λ. By increasing the values of λ the values of X_0 are increasing. We also observed that the equation has at least one positive root. So, we can conclude that the singularity is naked.

Now from equation 28 of Hussain space time we keep the value of β as fixed, we differs the value of λ to get different values of X_0. Let us take the value of $\beta = 0.1$

Table.2. Values of X_0 for different values of λ

λ	X_0
0.01	39.8997
0.02	10.9622
0.03	7.3165
0.04	5.8626

0.05	5.0437
0.06	4.5032
0.07	4.1128
0.08	3.8140



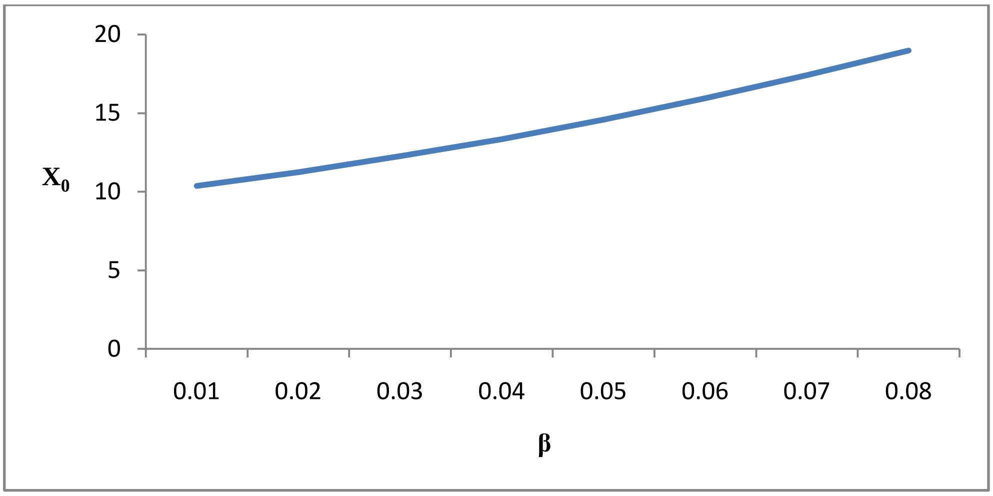
Graph.2. Graph of the Values of X_0 against the values of λ

In the above graph the values of β are fixed. By increasing the values of λ the values of X_0 are decreasing. We also observed that the positive root was obtained. So, we can conclude that the singularity is naked. So now by comparing the Graphs 1 and 2 we conclude that the values of X_0 are increasing in graph 1 where as in Graph 2 the values of X_0 are decreasing with the increase in the same values of λ by taking the same value β as 0.1. Hence from both the space times we observe that the naked singularities are formed and also with the increase of λ value X_0 values are increasing in Strange quark matter where as X_0 values are decreasing in Hussain space time. So, nature of both the graphs is in reverse in the non spherical gravitational collapse of strange quark matter and Hussain space time

Now again from equation 21 by taking γ , λ as constant we differ the values of β . Let us take λ = 0.01 and $\gamma = 0.1$ as constant. We take different values for β

Table.3. Values of X_0 for different values of β

β	X_0
0.01	10.3645
0.02	11.2334
0.03	12.2497
0.04	13.339
0.05	14.5804
0.06	15.9424
0.07	17.4102
0.08	18.9722



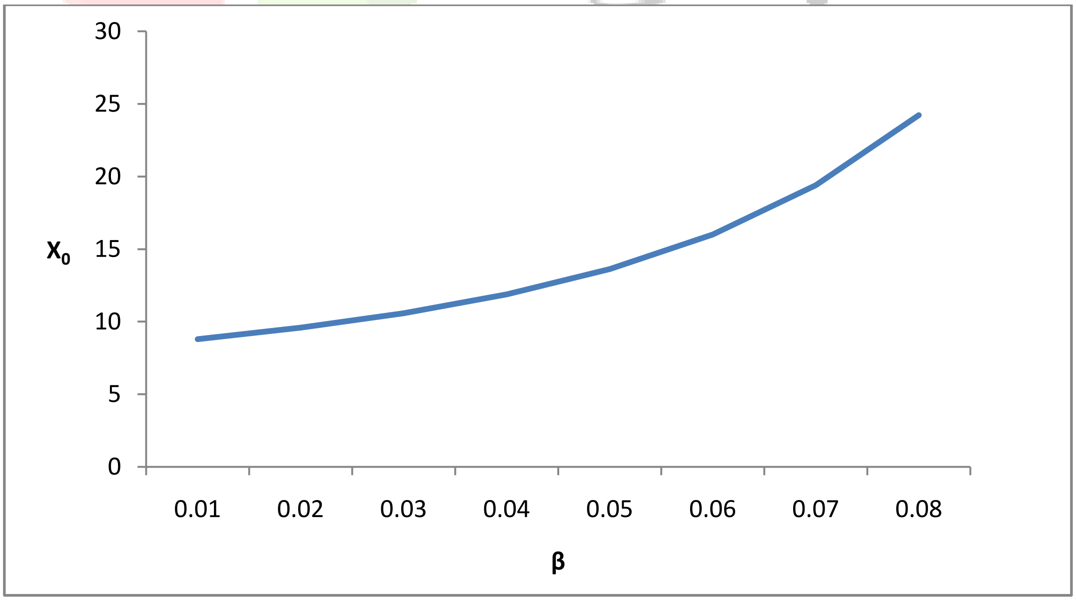
Graph.3. Graph of the Values of X_0 against the values of β

In the above graph the values of λ and γ are taken as fixed values. We had given the different values for β. By increasing the values of β the values of X_0 are increasing. We also observed that the equation has at least one positive root. So, we can conclude that the singularity is naked.

Now from equation 28 of Hussain space time we keep the value of λ as fixed values we differs the value of β to get different values of X_0. Let the value of $\lambda = 0.01$

Table.4. Values of X_0 for different values of β

β	X_0
0.01	8.780
0.02	9.5685
0.03	10.5655
0.04	11.8634
0.05	13.6040
0.06	16.0087
0.07	19.4066
0.08	24.2330

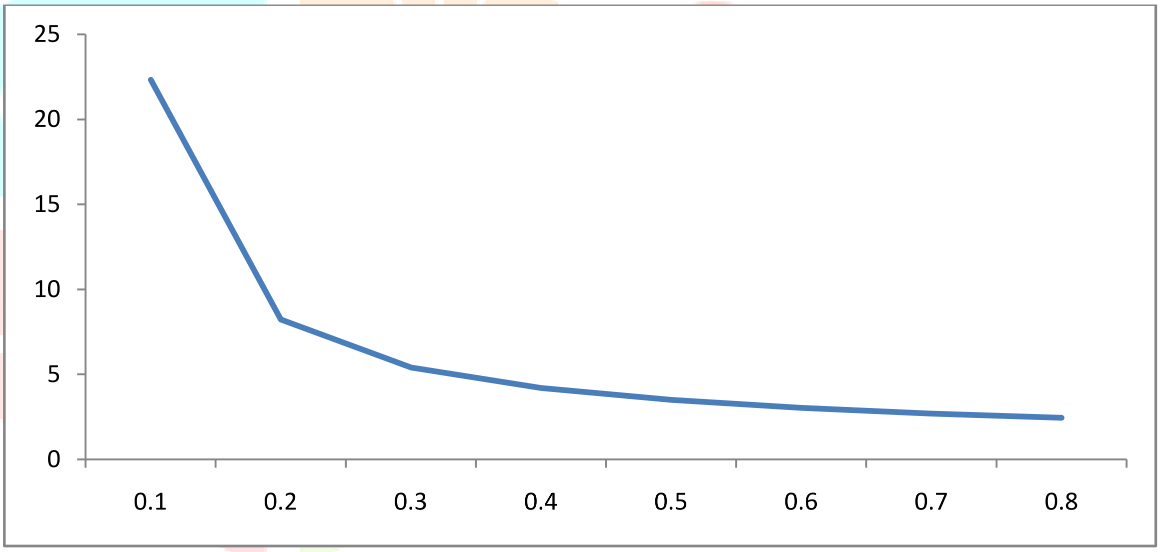


Graph.4. Graph of the Values of X_0 against the values of β

By observing the above tables 3 and 4 we can observe that there exists at least one positive root for the two equations of above space times. So, the singularities obtained are naked. Now by comparison we say that the values of X_0 are increasing by increasing the same values of β in the above two graphs 3 and 4. So, here by keeping same λ as fixed value we have taken the increasing same values of β then we got the increasing values for X_0 in both the space times. So, nature of both the graphs is same in the non spherical gravitational collapse of strange quark matter and Hussain space time.
Let us take equation 21 again for different values of γ by taking the β and λ as fixed values.
Now by taking the values $\beta = 0.1$ and $\lambda = 0.01$
So the following table gives the idea about the X_0 for different values of γ

Table.5. Values of X_0 for different values of γ

γ	X_0
0.1	22.3167
0.2	8.2130
0.3	5.3987
0.4	4.1839
0.5	3.4860
0.6	3.0240
0.7	2.6914
0.8	2.4382



Graph.5. Graph of the Values of X_0 against the values of γ

From the above graph we observe that by increasing the values of γ the values of X_0 are decreasing. At least one of the roots obtained is positive so the singularity is naked.
Now by comparing the graph 5 of Strange quark matter and graph 2 of Hussain space time we observe that the values of X_0 are decreasing by increasing the values of γ in graph 5 where as the values of X_0 are decreasing here also in graph 2 by increasing the values of λ. If we again compare the graph 5 of Strange quark matter with graph 4 of Hussain space time here we observe that the values of X_0 are decreasing by increasing the values of γ in graph 5 as in the graph 4 the values of X_0 are increasing by increasing the values of β.

V CONCLUSION

The aim of this manuscript is to expand the work done in the Non Spherical gravitational collapse of strange quark matter [22]. In this paper we have shown the nature of radial null geodesics obtained in non spherical space times of strange quark matter and also in Hussain space time are naked. The possible occurrence of naked singularity has been shown that the singularity is visible by the outside observer. So the existence of positive root from the above equations is a counter example to the strong version of Cosmic Censorship hypothesis. We also have compared these two space times by giving different values for γ , λ , β . By comparing we observed that the values of X_0 are increasing in strange quark matter where as in Hussain space time the values of

X_0 are decreasing with the increase in the same values of λ by taking β as same fixed value for the two space times. Similarly we also compared again by increasing the same values of β where we got the values of X_0 are increasing in both the space times with a fixed value λ . Finally in both the space times we got the singularities are naked and the nature of their graphs is different for fixed value of β, where as the nature of the graphs is same for fixed value of λ .

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