

STATISTICAL ANALYSIS OF FUZZY MEMBERSHIP FUNCTION FOR QUALITY EVALUATION

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Abstract: Quality evaluation is an essential need for accepting cotton sweater product shaving desired quality specification. Human perceived sensation depends on physiological and physical properties of quality as well as demographic characters of consumers. In this paper, a statistical calculation method of fuzzy statistics for quality evaluation is proposed. It illustrates the cotton sweater product of quality evaluation such as colour, designs, quality and price etc. of a product with consumer's demographic characters on a hedonic rule.

Keywords: Fuzzy set, fuzzification, de-fuzzification, performance measures, hedonic scale, swatter.

I. Introduction

Fuzzy sets were introduced by Lotfi A Zadeh in 1965 to represent/manipulate data and information possessing non statistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems. Fuzzy number is an extension of the interval of confidence on uncertainty. A is a fuzzy set and x is a relevant object, the proposition x is a member of A is not necessarily either true or false, as required by two – valued logic, but it may be true only to some degree, the degree to which x is actually a member of A . It most common, but not required to express degrees of membership in fuzzy sets as well as degrees of truth of the associated propositions by numbers in the closed unit interval $[0,1]$. The extreme values in this interval, 0 and 1, then represent, respectively. The fuzzy evaluation considers the maximum presumption to be at zero. However, one must not confuse fuzzy numbers with random numbers. For example, instead of describing the weather today in terms of the exact percentage of cloud cover, we can just say that is sunny. While the latter description is vague and less specific, it is often more useful. In order for a term such as sunny to accomplish the desired introduction of vagueness, however, we cannot use it to mean precisely 0% cloud cover. Its meaning is not totally arbitrary, however; a cloud cover of 100% is not sunny, and neither, in fact, is a cloud cover of 80%. We can accept certain intermediate states, such as 10% or 20% of cloud cover, as sunny. But where do we draw the line? If, for example, any cloud cover of 25% or less is considered sunny, does this mean that a cloud cover of 20% is not? This is clearly unacceptable, since 1% of cloud cover hardly seems like a distinguishing characteristic between sunny and not sunny. Randomness and uncertainty are two very different and important concepts which can be used together but should not be confused. Fuzzy sets support a flexible sense of membership of elements to a set, while in set theory only an element either belongs to a set. In the fuzzy set theory, many degrees of membership between 0 and 1 are obtained. Let X is a nonempty set. A fuzzy set A in X is characterized by its membership function $A : X \rightarrow [0, 1]$ and $A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$. If people can use the membership function to express the degree of their feeling based on their own concept, then the result will be closer to their real thought.

Table 1: Response Comparison between Fuzzy Logic and Traditional Method

Panel member	Fuzzy Logic Evaluation		Traditional Method	
	Good	Poor	Good	Poor
F_1	72%	18%		–
F_2	88%	02%		–
F_3	44%	56%	–	
F_4	32%	68%	–	
F_5	49%	51%	–	
F_6	45%	55%	-	
F_7	95%	05%	-	
Score	70.4%	29.6%	39%	61%

Consider a fuzzy set as well as traditional method used to evaluate the taste of mango juice from seven evaluator's sensory evaluation of quality with the two methods (Table1).Based on traditional method, taste of mango juice is poor while based on fuzzy set taste is good. The fuzzy sensory evaluation of quality method is more reasonable as compared to traditional method. Fuzzy linguistic descriptions are formal representations of a system made through fuzzy. In the IF-THEN rules system, one encodes knowledge about a system in a statement from if a set of conditions is satisfied and then a set of consequents can be inferred. A collection of rules referring to a particular system is known as a fuzzy rule base for a system as a single scalar quantity. This conversion of a fuzzy set to single value is called de-fuzzification.

Fuzzy reasoning is an effective analysing method for sensory evaluation. There are many methods for fuzzification. Helledoorn and Thomas (1993) have specified five criteria which to measure the methods. These criteria will be repeated here for the benefit of the reader who also ponders the question just given in terms of the advantages and disadvantages of the various methods. The first criterion is *continuity*. Second, a criterion is called *dis-ambiguity*. The third criterion is called *plausibility*. To be plausible, Z^* should lie approximately in the middle of the support region of C_k and have a high degree of membership in C_k . The centroid method equation Z^* does not exhibit plausibility in some situations when it lies in a region of the output that has a low degree of membership. The fourth criterion is that of *computational simplicity* and the fifth criterion is called the *weighting* method. Sargunamary(2009) et al. has studied fuzzy statistical measures for linguistic variables associated with traditional ranks through rectangular membership function. Shu-Meei and Berlin Wu (2008) introduce a fuzzification method for fuzzy statistical measures. They propose an integrated fuzzy evaluation procedure to measure the intellectual capital. In this method linguistic variables are uniformly used for fuzzification. Lee et al. have used IF-THEN rules on fuzzy reasoning application for sensory evaluation of sausages. Evaluators are asked to rate the contribution level of each attribute like texture, taste, order and appearance of a product to describe the overall preference as one out of five alternatives i.e., very important, important, moderate, slight and very slight. The preference level of each attribute for sausage samples is also simultaneously evaluated on a hedonic scale going from excellent, good, fair, poor and very poor. The results of the sensory evaluations are converted in to fuzzy sets. The fuzzy sets for the contribution of weight and preference of the attributes are composed to infer the overall preference of sausage by fuzzy reasoning. Martinez has proposed linguistic decision analysis to sensory evaluation. He uses 2-tuple representation model for managing the uncertainty and vagueness of the information in sensory evaluation of human senses (sight, taste, touch, smell and hearing). His frame work is based on a decision analysis scheme. Hough et al. have determined the consumer acceptance limits of ultra-heat the sensory properties such as taste, flavour and consumer acceptance limits of ultra-heat treated milk using survival analysis. Jaeger et al. develop model systems for testing the sensory properties such as taste, flavour and consumer acceptance of new fruit cultivators. Sinija(2011) et al has studied ranking of the quality of instant green tea powder and granule samples using triangular fuzzy membership distribution. The ranking method cannot be helpful for comparing similar qualities involving large sample size. Sune(2002) et al. makes a comparative study of sensory attribute used by children and experts to evaluate e chocolate. They have surprisingly found that some of the attributes most cited by children are not those better explained by experts. Their study finds the semantic gaps and differences between children and adult experts in rating sensory attributes. It is essential to understand these differences which may be explained by demographic characters like age, environment, social status, etc. In this paper, mathematical modelling of fuzzy statistics for quality evaluation using a hedonic scale is proposed and thereby a desired consistency table is constructed for obtaining coefficient of association between physical intensity and evaluator's demographic character like age, gender, social status or environment. An example of cotton sweater is illustrated the computation feasibility of the fuzzy statistics quality measures on the hedonic rule.

II. Statistical Models for Quality Evaluation

Huda Habib (2015) has carried out the Bamboo textile is one of the oldest materials, which have been came under the spot light and has become greatly available selection over the last few years to be used in sweater, due to its good properties of absorption. It has been studied the effect of some production parameters on the properties of cotton sweater such as the quality of terry fabrics to meet the required properties at the lowest cost. The author research has nine experimental samples of cotton sweater fabrics, which differ in the levels of the pile length and pile density were used. The selection of these parameters is because of their effect on the absorption properties of the sweater fabric. Results were compared according to standard specification *ASTM D5433* to evaluate the quality of sweater fabrics which produced by bamboo yarns with the previous variables. Results indicated that all the tested samples not only meet the requirements of the standard values of specifications for the properties of durability but also better than them, such as tensile strength in the warp and weft direction and pile withdraw. Also the results achieved acceptable values of the of absorption properties such as vertical wicking of water, spreading of water in horizontal direction, and absorption rate. In addition to that, the hand properties, such as softness, smoothness and drape ability, were compatible with standard specification and the end use. This confirms that every sample which produced in this search achieved all requirements of the standard specification of sweater and will be suitable for the end use and aftercare processing.

Luis Martinez (2007) evaluation is a process that analyses elements in order to achieve different objectives such as quality inspection, marketing and other fields in industrial companies. This paper focuses on sensory evaluation where the evaluated items are assessed by a panel of experts according to the knowledge acquired via human senses. In these evaluation processes the information provided by the experts implies uncertainty, vagueness and imprecision. The use of the Fuzzy Linguistic Approach (32) has provided successful results modelling such a type of information. In sensory evaluation it may happen that the panel of experts have more or less degree knowledge of about the evaluated items or indicators. So, it seems suitable that each expert could express their preferences

in different linguistic term sets based on their own knowledge. In this paper, we present a sensory evaluation model that manages multi-granular linguistic evaluation framework based on a decision analysis scheme. This model will be applied to the sensory evaluation process of Olive Oil.

Renato Coppi et al. (2005) have been developed in which a coalition of Fuzzy Sets Theory and Statistics has been established with different purposes. These namely are: (i) To introduce new data analysis problems in which the objective involves either fuzzy relationships or fuzzy terms; (ii) To establish well-formalized models for elements combining randomness and fuzziness; (iii) To develop *uni-variate* and *multivariate* statistical methodologies to handle fuzzy-valued data; and (iv) To incorporate fuzzy sets to help in solving traditional statistical problems with non-fuzzy data. In spite of a growing literature concerning the development and application of fuzzy techniques in statistical analysis, the need is felt for a more systematic insight into the potentialities of cross fertilization between Statistics and Fuzzy Logic. In line with the synergistic spirit of Soft Computing, some instances of the existing research activities on the topic are recalled. Particular attention is paid to summarize the papers gathered in this Special issue, ranging from the position paper on the theoretical management of uncertainty by the *father* of Fuzzy Logic to a wide diversity of topics concerning foundational/methodological/applied aspects of the integration of Fuzzy Sets and Statistics.

Hrehova Stella-VagaskaAlena (2012) has carried out the possibility of using artificial intelligence elements in order to evaluate the quality of a manufacturing process. There are described selected indexes of a production process quality evaluation based on statistical process control (SPC), their interpretation and evaluation by means of fuzzy sets, which enable us to work with inaccurate, incomplete or vague information about a monitored and reviewed phenomenon. There are described possibilities of using program system *MATLAB* and its toolboxes *SIMULINK* and Fuzzy Logic to evaluate quality of the manufacturing process based on fuzzy principles.

Cotton sweater evaluation is based on quality. Quality can't be directly measured in ratio scales. After the measurement of nominal or ordinal scales, it can be converted into quantitative measure using fuzzy logic, since traditional method helps to select only one option. In fuzzy logic, many possible values are considered for reducing the uncertainty of human thoughts. Further, quality evaluation of cotton sweater depends not only on physical and physiological properties of cotton sweater but also consumers' demographic characters. Differential equations are of great importance in science and engineering as many physical laws and relations appear mathematically in the form of differential equations. Assessment of the quality of a product (cotton sweater) can be classified into two categories viz. acceptability and non-acceptability depending on several attributes. This kind of classification is realistic since perceived sensation is a fuzzy concept. In the fuzzy concept of sensory evaluation of cotton sweater, judgment fuzzy membership function is required to determine the appropriate measures.

III. Fuzzy Statistics Analysis on Quality Evaluation of Cotton Sweaters

In this section soft computing method is proposed for calculating fuzzy response of quality attributes.

Step 1: Let U be the universal set, $L = \{L_1, L_2, \dots, L_k\}$ be a set of k linguistic variables on U and U_{ij} be the degrees of membership of i^{th} evaluator in j^{th} linguistic variable.

Step 2: Determine the evaluators and linguistic variables of the hedonic scale. Contact the evaluators and get their traditional hedonic scores for the linguistic variables. Let L_j , $j=1$ to k be the linguistic variables with the traditional score R_j on the universal set. Form a desired consistency table with the evaluators' scores of the attributes and physiological differences like age, gender, social status or environment which is considered for finding the coefficient of association between them. Let the evaluators' score for the linguistic variables of n panel members be $X = \{R_k, R_2, R_5, \dots, R_1, R_6\}$. Rewrite the traditional scores in the increasing order, that is $X = \{R_1, R_2, \dots, R_k\}$.

Step 3: The degrees of membership functions of the linguistic variables are:

$$\mu L_j(x_j) = [k(1-r)x_j + y_0]^{1/(1-r)} \quad -1 \leq r \leq 1 \text{ and } x_j \geq 0$$

$$\text{and } \mu L_j(x_j) = y_0 e^{kx} \quad \text{where } L_j = \{(L_j, \mu L_j(x_j)), x_j \in X\}$$

Step 4: For converting the traditional scores into degrees of membership function values, one can divide the interval $[0, R_k + 1]$ into desired partitions with homogenous intervals based on the linguistic variables scores, say $U = \{[0, R_1], [R_1, R_2], \dots, [R_k, R_k + 1]\}$. The medians of each interval are m_1, m_2, \dots, m_{k+1} respectively.

Step 5: For $k=5$, the fuzzy entity of the linguistic variables L_1, L_2, L_3, L_4 and L_5 of memberships are:

L_1	Poor	1
L_2	Satisfied	2
L_3	Good	3
L_4	Very Good	4
L_5	Excellent	5

Step 6: when the evaluators' traditional score of L_j falls in the middle of medians of the intervals, then the fuzzy entity values of scores are $m_{ij} = \int_{m_j}^j \mu L_j(x) dx$ and $m_{ij+1} = \int_j^{m_{j+1}} \mu L_j(x) dx$. Thus the fuzzy quality evaluator entity values of the score are:

$$F_1 = \{m_{11}, m_{12}, 0, 0, 0\}$$

Step 7: For obtaining the total degree of membership sensory score equal to 1, individual fuzzy sensory scores are divided by total score $\sum_{j=1}^5 m_{ij}$ for $i = 1, 2, 3, \dots, n$

$$\frac{m_{ij}}{\sum_{j=1}^5 m_{ij}} = M_{ij} \text{ and } \sum_{j=1}^5 M_{ij} = 1, i = 1, 2, \dots, n$$

Table 3: Fuzzy Response for an Attribute

Panel member	Poor $L_1=1$	Satisfied $L_2=2$	Good $L_3=3$	Very Good $L_4=4$	Excellent $L_5=5$
F_1	M_{11}	M_{12}	0	0	0
F_2	0	M_{22}	M_{23}	0	0
⋮	⋮	⋮	⋮	⋮	⋮
F_n	0	0	0	M_{n4}	M_{n5}

Step 8: The fuzzy quality responses of the evaluators for an attribute are obtained in Table 3.

If fuzzy response is the highest, then $m_{i4} = m_{i5}$ and $m_{i5} = m_{i6}$ for all $i = 1, 2, \dots, n$. when the maximum score of the hedonic scale are 5.

Step 9: The fuzzy statistics quality measure such as mean $E[X]$ and variance $V[X]$ can be obtained by the method proposed below:

$$E[X] = \frac{(M_{11} + M_{21} + \dots + M_{n1})/n}{1} + \frac{(M_{12} + M_{22} + \dots + M_{n2})/n}{2} + \dots + \frac{(M_{15} + M_{25} + \dots + M_{n5})/n}{5}$$

$$E[X^2] = \frac{(\sum_{i=1}^n M_{i1}^2)/n}{1} + \frac{(\sum_{i=1}^n M_{i2}^2)/n}{2} + \dots + \frac{(\sum_{i=1}^n M_{i5}^2)/n}{5}$$

$$V[X] = (EX^2 - (EX)^2) \text{ and Standard Deviation } \sigma = \sqrt{V[X]}$$

The Standard Error (SE) of the fuzzy statistics mean = $\frac{\sigma}{\sqrt{n}}$ where n = number of respondents. It reveals the uncertainty of the

fuzzy statistical mean. The estimate of the fuzzy statistical mean varies in the limits $E[X] \pm \frac{\sigma}{\sqrt{n}}$.

IV. Application of Fuzzy Quality Evaluation of Swatter Products

In this chapter, the planned method is applied for the fuzzy evaluation of cotton sweater. A sweater (North American English) is a knitted garment intended to cover the torso and arms. A sweater is either a pullover or a cardigan, distinguished in that cardigans open at the front while pullovers do not. In British English, a pullover may also be called a jumper or jersey. There is no phrase equivalent to *sweater* covering both pullovers and cardigans. Sweaters are worn by adults and children of all genders; often over a shirt, blouse, T-shirt, or other top, but sometimes next to the skin. Sweaters were traditionally made from wool, but can now be made of cotton,

synthetic fibers, or any combination thereof. Sweaters are maintained by washing or dry cleaning, and the use of a lint roller or pill razor. There are different models of sweaters namely; *XXS* (81-86 cms), *XS* (86-91 cms), *S*(91-96 cms), *M* (96-101), *L* (101-106 cms), *XL* (106-111 cms) and *XXL* (111-116 cms).

In our study, we are taken the standard cotton swatters come in different dimensions, depending on the manufacturer, but they typically range in size from 42 inches by 44 inches to 111 inches by 116 inches. A sample survey method is used to collect the physical intensity form the five point hedonic scale for quality evaluation. The public aged are 60 or less than and considered from a citizen and senior citizen having a total strength of 740 and 460 respectively. There are 410 male and 320 females in the selected citizens and 240 males and 220 females in the selected senior citizens peoples. The total population size is 1200 and there are four stratum sizes $N_1 = 410, N_2 = 320, N_3 = 240, N_4 = 220$. A sample size of $n = 35$ is selected on the basis of square root of the population size (N). Using the method of proportion allocation of sample size to stratum, that is $n_i \propto N_i$ or $n_i = (n/N) \times N_i = 1, 2, 3$ and 4th stratum sample size are $n_1 = 12, n_2 = 10, n_3 = 7$, and $n_4 = 6$, Such that $n_1 + n_2 + n_3 + n_4 = 35$. In this method a sample of 35 evaluators of which 12 males and 10 females aged less than are equal to 60 and 7 males and 6 females aged more than 60 are selected by simple random sampling method for the mathematical computation of fuzzy statistics on quality evaluation of Cotton sweater. They are selected based on good health, non-smokers, non-beetle leaf chewers, interested in quality evaluation, ability to concentrate, learn and familiarity with Cotton sweater. They are trained quality evaluation procedure in terms of identification of quality attributes, score sheet and method of scoring before the actual test. Evaluation are asked to rate the physical intensity of quality attributes of cotton on the five point hedonic scale running from excellent, very good, good, satisfied and poor. The assigned scores of the attributes on hedonic scale are 5,4,3,2 and 1 for excellent, very good, good, satisfied and poor respectively. A and B be the attributes, Where A stands for the physical intensity scores greater than median as the scale and B stands for the panellists age which may be greater than 35. α and β are the complimentary attributes of A and B respectively. The 2×2 consistency table each attributes i.e., colour, design, quality, and price of the product is formed and then *Yule's Co-efficient* of associations calculated. Table 4 shows the mathematical models of perceived sensation of the quality attributes based on the *Yule's Co-efficient* of association.

Table 4: Co-efficient of Association with the Mathematical Models

Attributes	Yule's co-efficient	Model
Colour	0.18	$y = 0.78x^{1.22}$
Design	0.12	$y = 0.86x^{1.14}$
Quality	0.09	$y = 0.90x^{1.09}$
Price	0.16	$y = 0.81x^{1.19}$
Over all	0.14	$y = 0.84x^{1.16}$

The interval $[0, 6]$ is chosen since the range of the scale is 1 to 5. The $[0, 6]$ interval is further divided into different equal length sub-intervals such as $[1,1], [1,2], [2,3], [4,5]$, and $[5,6]$. The median of these intervals are found as 0.5, 1.5, 2.5, 3.5, 4.5, and 5.5 respectively. If the score of hedonic scale is 1, then 1 is included between 0.5 and 1.5. The fuzzy response is very low that is $L_1=1$ for

color which is $m_{11} = \int_{0.5}^{1.22} 0.78x^{1.22} dx = 0.2759$ and $m_{12} = \int_1^{1.5} 0.78x^{1.22} dx = 0.5129$ for other score i.e., 2, 3, 4 and 5.

Table 5: Fuzzy Response for Traditional Scores

Traditional score	Poor	Satisfaction	Good	Very Good	Excellent	Fuzzy Response
	$L_1=1$	$L_2=2$	$L_3=3$	$L_4=4$	$L_5=5$	
1	0.2759	0.5129	0	0	0	0.7888
	0.3498	0.6502	0	0	0	0
2	0	0.7726	1.0495	0	0	1.8221
	0	0.4241	0.5759	0	0	0
3	0	0	1.3403	1.6432	0	2.9835
	0	0	0.4493	0.5507	0	0
4	0	0	0	1.9565	2.2791	4.2356
	0	0	0	0.4619	0.5381	0
5	0	0	0	2.6103	2.9492	5.5595
	0	0	0	0.4695	0.5305	0

Table 6: Fuzzy Response for Color of the Cotton sweater

Panel Member	Traditional score	$L_1=1$	$L_2=2$	$L_3=3$	$L_4=4$	$L_5=5$
F1	2	0	0.4241	0.5759	0	0
F2	4	0	0	0	0.4619	0.5381

<i>F3</i>	1	0.3498	0.6502	0	0	0
<i>F4</i>	5	0	0	0	0.4695	0.5305
<i>F5</i>	3	0	0	0.4493	0.5507	0

Table 7: Fuzzy Statistics Quality Measures

Attributes	Mean	SD	SE	Mean Limits	<i>r</i>
Color	0.0517	0.2661	0.0449	0.0517 ± 0.2661	0.18
Design	0.0530	0.1493	0.0252	0.0530 ± 0.0252	0.12
Quality	0.0521	0.1516	0.0256	0.0521 ± 0.0256	0.09
Price	0.0519	0.01536	0.0259	0.0519 ± 0.0259	0.16
Over all	0.0520	0.1513	0.0256	0.0520 ± 0.0256	0.14

The attributes expect for design which implies that there is a disassociation between the age groups and physical intensity for color quality and price whereas in case of design there is almost zero positive association. The overall estimated mean of the perceived sensation 6.52% with 5% level of significance which is less than the desired level of 75% as defined earlier. Hence, the positive association of the quality attributes of Cotton sweater is adversely influenced by the age of evaluators.

V. CONCLUSION

Fuzzy statistics quality evaluation is based on structure of non-digital set theory and rules. The evaluation of a product lies in its method to handle human thought and recognition. The advantage is its ability to deal with unclear systems and its use of linguistic variables. An accurate quantitative model is required to determine appropriate statistical performance measures for quality evaluation. The discussed procedure differs from the traditional assessment method and establishes the membership functions of evaluator's index to capture better results which will be closer to their real thoughts about the quality attributes. Further, this method of quality evaluation depends on the mathematical and statistics principles and does not involve complicated iterations. The fuzzy statistical measures obtained from the mathematical computations methods using fuzzy logic should be more reasonable than the traditional method of analysis. The mathematical calculation methods of fuzzy statistics for quality evaluation increases the amount of information from the evaluators and discusses the quality of factors such as color, design, quality, price etc. of a product with consumers demographic characters on a hedonic scale.

Acknowledgement

A. Santhakumaran and C.V KavithaAbirani (2016) have discussed the mathematical computation of fuzzy statistics for sensory evaluation. The authors greatly appreciate the editors and the referees for their valuable and helpful comments and suggestions regarding earlier version of the paper.

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