Aboodh Transform Technique for Newton's Law

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Abstract: The purpose of this paper is to discuss the applicability of Aboodh Transform [1, 2] method to the Newton's principle on cooling.

***IndexTerms* – Aboodh Transform, Differential Equation, Newton's Law**

1. INTRODUCTION

In the literature there are numerous integral transform techniques such as Fourier, Laplace, Henkel and Melin, Sumudu, Elzaki etc. are widely used for solving the differential equations, integral equations and also integro-differential equations. Aboodh transform [1,2] and Elzaki transform [3] are closely connected with the Laplace transform.

1.1 Definition: A new integral transform called the Aboodh transform defined for function of exponential order, we consider the set A defined by $A=\left\{f(t);\exists M,k_1,k_2>0,\left|f(t)\right|<M\,e^{-vt}\right\}$. For a given function f(t) in the set A, M must be a finite number, k_1,k_2 may be finite (or) infinite. The Aboodh transform of the given function f(t) is defined by the integral equation as $A\{f(t)\}=k(v)=\frac{1}{v}\int_0^\infty e^{-vt}f(t)dt, t\geq 0, k_1\leq v\leq k_2$

1.2. Aboodh transform of some standard functions

Function f(t)	1	e^{at}	e^{-at}	t	t^n	Sin(at)	Cos(at)	Sinh(at)	Cos(hat)
Aboodh Transform	$\frac{1}{v^2}$	$\frac{1}{v^2-av}$	$\frac{1}{v^2+av}$	$\frac{1}{v^3}$	$\frac{n!}{v^{n+2}}$	$\frac{a}{v(v^2+a^2)}$	$\frac{1}{(v^2+a^2)}$	$\frac{a}{v(v^2-a^2)}$	$\frac{1}{(v^2-a^2)}$

1.3 Aboodh Transform of Derivatives: Let A{f(t)}= k(v) then $A\{f'(t)\}=vk(v)-\frac{f(0)}{v}$

Proof: Given $A\{f(t)\}=k(v)=\frac{1}{v}\int_0^\infty e^{-vt}f(t)dt$. Now consider $A\{f'(t)\}=\frac{1}{v}\int_0^\infty \frac{e^{-vt}}{u}\frac{f'(t)}{v}dt$

$$=\frac{1}{v}\left\{\left\{e^{-vt}f(t)\right\}_0^\infty-\int_0^\infty e^{-vt}(-v)f(t)dt\right\}=\frac{1}{v}\left\{-f(0)+v\int_0^\infty e^{-vt}f(t)dt\right\}=-\frac{f(0)}{v}+\int_0^\infty e^{-vt}f(t)dt=\frac{f(0)}{v}+vk(v)$$

$$A\{f'(t)\}=vk(v)-\frac{f(0)}{v}$$

2. Application of Aboodh Transforms Technique on Newton's Law

Newton's principle on cooling states that the rate of change of the temperature of a body is proportional to the difference of the temperature of the body and of its surrounding medium. Let θ be the temperature of the body at any time t.θ_0 be the temperature of its surrounding medium then by Newton's law, we have $\frac{d\theta}{dt}\propto(\theta-\theta_0)\Rightarrow\frac{d\theta}{dt}=-k(\theta-\theta_0)$ where k is proportionality constant (k>0).

2.1.1 If the air maintained at 30^0c and the temperature of the body cools from 80^0c to 60^0c in 20 minutes. Find the temperature of the body after 40 minutes.

Solution: Given that air temperature θ=30^0c, at time t=0, θ=80^0c after t=20 min. θ=60^0c ,after time t=40min, need to find θ value.

From Newton's law, $\frac{d\theta}{dt}\alpha(\theta-\theta_0) \Rightarrow \frac{d\theta}{dt}=-l(\theta-\theta_0) \Rightarrow \frac{d\theta}{dt}=-l\theta+k\theta_0 \Rightarrow \theta'(t)+l\theta(t)=30l$ ________(1)

Now taking on both sides Aboodh Transform $A\{\theta'(t)+l\,\theta(t)\}=A\{30l\} \Rightarrow A\{\theta'(t)\}+l\,A\{\theta(t)\}=30l\,A\{1\}$

$\Rightarrow\left[vk(v)-\frac{\theta(0)}{v}\right]+lk(v)=\frac{30l}{v^2}\Rightarrow\left[vk(v)-\frac{80}{v}\right]+lk(v)=\frac{30l}{v^2}\Rightarrow(v+l)k(v)=\frac{30l}{v^2}+\frac{80}{v}\Rightarrow k(v)=\frac{30l}{v^2(v+l)}+\frac{80}{v(v+l)}$

$=30\left[\frac{1}{v^2}-\frac{1}{v^2+vl}\right]+\frac{80}{(v^2+l)} \Rightarrow k(v)=\left[\frac{30}{v^2}+\frac{50}{v^2+vl}\right]$ Taking inverse Aboodh Transform on both sides we get $\theta(t)=30+50e^{-lt}$ __(2). Given at t=20m, θ=60^0c substituting in (2), $60=30+50e^{-20l}\Rightarrow 30=50e^{-20l}\Rightarrow e^{-20l}=\frac{3}{5}$

$\Rightarrow e^{-l}=\left(\frac{3}{5}\right)^{1/20}$ ___(3) Now the temperature of body at t=40m. $\theta(t)=30+50e^{-40l}=30+50\left(e^{-l}\right)^{40}=30+50\left(\frac{3}{5}\right)^2$

$=30+50\left(\frac{9}{25}\right)\Rightarrow\theta(t)=48^0c.$

2.1.2 <u>Estimation of murder:</u> A body of murder victim was discovered at 11.00Am, the doctor took the temperature of body at 11.30Am which was 93.4^{0}F .He took again the temperature after half an hour when it is showed 92^{0}F and noticed that temperature of room was 70^{0}F . Estimate the time of death.(Normal temperature of human body is 98.6^{0}F)

Solution: Given that at the initial time (11.30AM) t=0, $\theta(t)=93.4^0F$. At t=30m (12.00PM) $\theta(t)=92^0F$ Room temperature $\theta_0=70^0F$ From Newton's law of cooling we have $\frac{d\theta}{dt}\alpha(\theta-\theta_0)\Rightarrow\frac{d\theta}{dt}=-l(\theta-\theta_0) \Rightarrow\frac{d\theta}{dt}+l\theta=l\theta_0 \Rightarrow\theta'(t)+l\theta(t)=70l.$

Now taking on both sides Aboodh Transform $A\{\theta'(t)+l\theta(t)\}=A\{70l\}\Rightarrow A\{\theta'(t)\}+lA\{\theta(t)\}=70lA\{1\}$

$\Rightarrow\left[vk(v)-\frac{\theta(0)}{v}\right]+lk(v)=\frac{70l}{v^2} \Rightarrow(v+l)k(v)=\frac{70l}{v^2}+\frac{93.4}{v}\Rightarrow k(v)=\frac{70l}{v^2(v+l)}+\frac{93.4}{v(v+l)}k(v)=\left[\frac{70}{v^2}+\frac{23.4}{v^2+vl}\right]$

Taking Inverse Aboodh Transform on both sides we get $\theta(t)=70+23.4e^{-lt}$ __(6). Given at t=30min, θ(t)=92^{0}c. From the equation (6), we get $92=70+23.4e^{-30l} \Rightarrow 22=23.4e^{-30l} \Rightarrow e^{-30l}=\frac{22}{23.4} \Rightarrow e^{-l}=\left(\frac{22}{23.4}\right)^{1/30}$ _____(7)

But the human body temperature before the death is 98.6^0F, From (6), we get $98.6=70+23.4e^{-lt}\Rightarrow 28.6=23.4e^{-lt}$

$\Rightarrow 28.6=23.4(e^{-l})^t\Rightarrow(e^{-l})^t=\frac{28.6}{23.4}$. From (7), $\left(\frac{22}{23.4}\right)^{t/30}=\frac{28.6}{23.4}$. Applying natural logarithms on both sides, we get

$\Rightarrow\frac{t}{30}=\frac{\log\left(\frac{28.6}{23.4}\right)}{\log\left(\frac{22}{23.4}\right)} \Rightarrow t=30\frac{\log\left(\frac{28.6}{23.4}\right)}{\log\left(\frac{22}{23.4}\right)} \approx 98\,\text{min}\,utes$. The Estimated time of death was11.30AM-1.38min=9.52AM

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