A Unified Study of the Inverse Laplace Transform of Aleph- Function Involving General Class of Polynomial and Their Associated Properties

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ABSTRACT:

The main purpose of this paper is to establish some useful formulae by using the inverse Laplace transform of various product of algebraic power and the Aleph function with general class of polynomial. Some special cases involving generalized hypergeometric function, Mittag-Leffler function, Hermite polynomial and Laguerre polynomial are presented to enhance the utility and importance of our main results.

Keywords: Laplace transform, Aleph function, Fox H-function, I-function, generalized hypergeometric function, Mittag-Leffler function, Hermite polynomial and Laguerre polynomial.

Mathematics Subject Classification 2010 : 44A10, 33-XX, 33Cxx, 33E12, 33C45.

1. Introduction: The Aleph function [ℵ] introduced by Südland [12] .The notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integrals.

$$\aleph[z] = \aleph^{m,n}_{p_i,q_i,c_i;r}[z] = \aleph^{m,n}_{p_i,q_i,c_i;r}\left[z\left|\begin{matrix}(a_j,A_j)_{1,n},[c_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right]$$
$$= \frac{1}{2\pi\omega}\oint \Omega^{m,n}_{p_i,q_i,c_i;r}(s)z^{-s}ds, \tag{1.1}$$

for all $z \neq 0$, where $\Omega = \sqrt{-1}$ and

$$\Omega^{m,n}_{p_i,q_i,c_i;r}(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j+B_js).\prod_{j=1}^{n}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}c_i\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}s).\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}s)}\ . \tag{1.2}$$

The integration path $\Omega = \Omega_{\omega\gamma\infty,}\gamma\epsilon R$ extends from $\gamma-\omega\infty$ to $\gamma+\omega\infty$, and is such that the poles of the gamma functions $\Gamma(1-a_j-A_js), j=\overline{1,n}$ do not coincide with the poles of the gamma functions $\Gamma(b_j+B_js)$, $j=\overline{1,m}$. The perameter p_i, q_i are non negative integers satisfying $0 \leq n \leq p_{i,} 0 \leq m \leq q_{i,} c_i > 0$ for $i=\overline{1,r}$. The parameters A_j, B_j, A_{ji}, B_{ji} are positive numbers and a_j, b_j, a_{ji}, b_{ji} are complex. All poles of the integrand (1.2) are assumed to be simple, and the empty product is interpreted as unity. The existence conditions for the defining integral (1.1) are given below:

$$\varphi_l > 0, |arg(z)| < \frac{\pi}{2}\varphi_l\ ,\ \ l=\overline{1,r}; \tag{1.3}$$

$$\varphi_l \geq 0, |arg(z)| < \frac{\pi}{2}\varphi_l \text{ and } \Re\{\zeta_l\}+1<0\ . \tag{1.4}$$

where

$\varphi_l = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - c_l \left(\sum_{j=n+1}^{p_l} A_{jl} + \sum_{j=m+1}^{q_l} B_{jl}\right)$,

$\zeta_l = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + c_l \left(\sum_{j=m+1}^{q_l} b_{lj} - \sum_{j=n+1}^{p_l} a_{jl}\right) + \frac{1}{2}(p_l - q_l)\ l = \overline{1,r}.$

Remarks 1. If we set $c_1 = c_2 = \cdots = c_r = 1$, then (1.1) reduces to the I-function[14]:

$$I[z] = \aleph_{p_i,q_i,1;r}^{m,n}[z] = \aleph_{p_i,q_i,1;r}^{m,n}\left[z \middle| \begin{matrix} (a_j,A_j)_{1,n}, [1(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [1(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix}\right] = \frac{1}{2\pi\omega}\oint \Omega_{p_i,q_i,1;r}^{m,n}(s) z^{-s} ds, \tag{1.5}$$

the existence conditions for the integral in (1.5) are the same as given in (1.3) and (1.4) with $c_1 = c_2 = \cdots = c_r = 1$.

Remarks 2: If we set $r = 1$, then (1.5) reduces to the familiar Fox H-Function introduced by Fox [5]:

$$H_{p,q}^{m,n}[z] = \aleph_{p_i,q_i,1;1}^{m,n}[z] = \aleph_{p_i,q_i,1;1}^{m,n}\left[z \middle| \begin{matrix} (a_p,A_p) \\ (b_q,B_q) \end{matrix}\right] = \frac{1}{2\pi\omega}\oint \Omega_{p_i,q_i,1;1}^{m,n}(s) z^{-s} ds. \tag{1.6}$$

Definition 1. The Wright generalized hypergeometric function $p\Psi_q$ called also the Fox-Wright function is defined as

$$p\Psi_q\left[\begin{matrix} (a_1,A_1),\ldots,(a_p,A_p) \\ (b_1,B_1),\ldots,(b_q,B_q) \end{matrix} \middle| z\right] = \sum_{k=0}^{\infty} \frac{\Gamma(a_1+kA_1)\ldots\Gamma(a_p+kA_p)}{\Gamma(b_1+kB_1)\ldots\Gamma(b_q+kB_q)} \frac{z^k}{k!}$$
$$= H_{p,q+1}^{1,p}\left[-z \middle| \begin{matrix} (1-a_1,A_1),\ldots,(1-a_p,A_p) \\ (0,1),(1-b_1,B_1),\ldots,(1-b_q,B_q) \end{matrix}\right] \tag{1.7}$$

Definition 2.The General class of polynomials introduced by Srivastava [13] is defined as

$$S_N^M(x) = \sum_{R=0}^{[N/M]} \frac{(-N)_{MR}}{R!} A_{N,R} x^R,\ N = 0,1,2,\ldots, \tag{1.8}$$

where M is a arbitrary positive integer and the coefficients $A_{N,R}(N,R \geq 0)$ are arbitrary constants, real or complex. By suitably specializing the coefficients $A_{N,R}$, the general class of polynomials can be reduced to a number of polynomials and $(\lambda)_n$ denotes the Pochhammer symbol defined by

$$(\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \begin{cases} 1, if\ n=0, \\ \lambda(\lambda+1)\ldots(\lambda+n-1),\ \forall n \in \{1,2,3,\ldots\}. \end{cases} \tag{1.9}$$

2. Main Results

Now we will obtain the Inverse Laplace transform of various products of algebraic powers and Aleph function with general class of polynomial.

$$(A)\ L^{-1}\left\{\begin{matrix} (p^2+b^2)^{-\left(\frac{2\mu+1}{2}\right)}.S_N^M\left[x(p^2+b^2)^{-\xi_1}\right] \\ \times \aleph_{p_i,q_i,c_i;r}^{m,n}\left[z(p^2+b^2)^{-\xi_2} \middle| \begin{matrix} (a_j,A_j)_{1,n}, [c_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [c_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix}\right]; t \end{matrix}\right\}$$

$$= \sum_{R=0}^{[N/M]} \sum_{r'=0}^{\infty} \frac{(-N)_{MR}}{R!} A_{N,R} x^R \sqrt{\pi} \frac{(-1)^{r}}{r'!} \frac{t^{2(\mu+\xi_1 R+r')}}{2^{2(\mu+\xi_1 R+r')}} b^{2r'}$$

$$\times \aleph_{p_i,q_i+2,c_i;r}^{m,n}\left[\frac{zt^{2\xi_2}}{2^{2\xi_2}} \middle| \begin{matrix} (a_j,A_j)_{1,n}, [c_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}, h \end{matrix}\right], \tag{2.1}$$

where $h = (\frac{1}{2} - \mu - \xi_1 R, \xi_2), (-\xi_1 R - r' - \mu, \xi_2)$,
provided that $\xi_1 \geq 0$, $\xi_2 \geq 0$, $\mathrm{Re}(p) > 0$ and $\mathrm{Re}(2\mu + 1) > 0$.

$$(B)\ L^{-1}\left\{\begin{matrix} \{p + (p^2+b^2)^{1/2}\}^{-\upsilon} S_N^M\left[x\{p + (p^2+b^2)^{1/2}\}^{-\xi}\right] \\ \times \aleph_{p_i,q_i,c_i;r}^{m,n}\left[z\{p + (p^2+b^2)^{1/2}\}^{-\rho}\left|\begin{matrix}(\mathrm{a_j},\mathrm{A_j})_{1,n}, [\mathrm{c_i}(\mathrm{a_{ji}},\mathrm{A_{ji}})]_{n+1,p_i;r} \\ (\mathrm{b_j},\mathrm{B_j})_{1,m}, [\mathrm{c_i}(\mathrm{b_{ji}},\mathrm{B_{ji}})]_{m+1,q_i;r}\end{matrix}\right.\right]; t\end{matrix}\right\}$$

$$= \sum_{R=0}^{[N/M]} \sum_{r'=0}^{\infty} \frac{(-N)_{MR}}{R!} A_{N,R} x^R \frac{(-1)^{r'}}{r'!}\left(\frac{t}{2}\right)^{\nu+2r'-\xi R-1} . b^{2r'}$$
$$\times \aleph_{p_i+1,q_i+2,c_i;r}^{m,n+1}\left[z\left(\frac{t}{2}\right)^{\rho}\left|\begin{matrix}h_1,(a_j,A_j)_{1,n}, [c_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}, h_2\end{matrix}\right.\right], \tag{2.2}$$

where $h_1 = (-\nu - \xi R, \rho)$ and $h_2 = (-\nu - r' - \xi R, \rho), (1 - \upsilon - \xi R, \rho)$,
provided that $\xi \geq 0, \rho \geq 0, \mathrm{Re}(p) > 0$ and $\mathrm{Re}(\nu + \rho s) > 0$.

$$(C)\ L^{-1}\left\{\begin{matrix} p^{-2\lambda}(p^2+b^2)^{-\upsilon} S_N^M\left[xp^{-2\delta}(p^2+b^2)^{-\mu}\right] \\ \times \aleph_{p_i,q_i,c_i;r}^{m,n}\left[zp^{-2\rho}(p^2+b^2)^{-\sigma}\left|\begin{matrix}(a_j,A_j)_{1,n}, [c_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right]; t\end{matrix}\right\}$$

$$= \sum_{R=0}^{[N/M]} \sum_{r'=0}^{\infty} \frac{(-N)_{MR}}{R!} A_{N,R} x^R \frac{(-1)^{r'}}{r'!}\left(\frac{b}{2}\right)^{2r'} . t^{2(\lambda+\upsilon+r'+\delta R+\mu R)-1}$$
$$\times \aleph_{p_i+3,q_i+4,c_i;r}^{m,n+3}\left[zt^{2(\rho+\sigma)}\left|\begin{matrix}h_3,(a_j,A_j)_{1,n}, [c_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}, h_4\end{matrix}\right.\right], \tag{2.3}$$

where $h_3 = (1 - \upsilon - r' - \mu R, \sigma), (1 - \upsilon - \lambda - \delta R - \mu R, \rho + \sigma)$,
$\left(\frac{1}{2} - \lambda - \upsilon - \delta R - \mu R, \rho + \sigma\right)$ and $h_4 = \left(\frac{1}{2} - \lambda - \upsilon - r' - \delta R - \mu R, \rho + \sigma\right)$,
$(1 - 2\lambda - 2\upsilon - 2\delta R - 2\mu R, 2\rho + 2\sigma), (1 - \upsilon - \mu R, \sigma), (1 - \lambda - \upsilon - \delta R - \mu R, \rho + \sigma)$,
provided that $\mathrm{Re}\,(p) > 0, \mathrm{Re}(\lambda) > 0, \mathrm{Re}(\upsilon) \geq 0$.

$$(\boldsymbol{D})\ L^{-1}\left\{\begin{matrix} (p^2+b^2)^{-1/2}\{p + (p^2+b^2)^{1/2}\}^{-\lambda} S_N^M\left[x\{p + (p^2+b^2)^{1/2}\}^{-\gamma}\right] \\ \times \aleph_{p_i,q_i,c_i;r}^{m,n}\left[z\{p + (p^2+b^2)^{1/2}\}^{-\rho}\left|\begin{matrix}(a_j,A_j)_{1,n}, [c_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right]; t\end{matrix}\right\}$$

$$= \sum_{R=0}^{[N/M]} \sum_{r'=0}^{\infty} \frac{(-N)_{MR}}{R!} A_{N,R} x^R \frac{(-1)^{r'}}{r'!}\left(\frac{t}{2}\right)^{\lambda+2r'+\gamma R} b^{2r'}$$
$$\times \aleph_{p_i,q_i+1,c_i;r}^{m,n}\left[z\left(\frac{t}{2}\right)^{\rho}\left|\begin{matrix}(a_j,A_j)_{1,n}, [c_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}, (-\lambda-r'-\gamma R,\rho)\end{matrix}\right.\right], \tag{2.4}$$

provided *that* $Re(p) > 0, Re(\lambda) > -1$ *and*, $Re(\rho s + \lambda) > -1$.

Proof: By using the definition of the Aleph-function {Eq.(1.1) and (1.2)} and the general class of polynomial {Eq.(1.7)} in the L.H.S of (2.1), we have

$$= \frac{1}{2\pi\omega}\oint \Omega^{m,n}_{p_i,q_i,c_i;r}(s) z^{-s} \sum_{R=0}^{[N/M]} \frac{(-N)_{MR}}{R!} A_{N,R} x^R L^{-1}\left\{(p^2+b^2)^{-\left(\frac{2\mu+1}{2}-+\xi_1 R-\xi_2 s\right)}; t\right\} ds,$$

Now, by using the known Result [4,p.239,Equ.(18)].

$$= \sum_{R=0}^{N/M} \frac{(-N)_{MR}}{R!} A_{N,R} x^R \sqrt{\pi}.\left(\frac{t}{2b}\right)^{\mu}$$

$$\times \frac{1}{2\pi\omega}\oint \Omega^{m,n}_{p_i,q_i,c_i;r}(s) \frac{1}{\Gamma\left(\mu+\xi_1 R-\xi_2 s+\frac{1}{2}\right)} z^{-s}\left(\frac{t}{2b}\right)^{-\xi_2 s} J_{\mu+\xi_1 R-\xi_2 s}(bt) ds,$$

Now, expanding $J_{\mu+\xi_1 R-\xi_2 s}(bt)$ in summation form and after little simplification we get

$$= \sum_{R=0}^{[N/M]} \sum_{r'=0}^{\infty} \frac{(-N)_{MR}}{R!} A_{N,R} x^R \sqrt{\pi} \frac{(-1)^{r'}}{r'!} \frac{t^{2(\mu+\xi_1 R+r')}}{2^{2(\mu+\xi_1 R+r')}} b^{2r'}$$

$$\times \aleph^{m,n}_{p_i,q_i+2,c_i;r}\left[\frac{zt^{2\xi_2}}{2^{2\xi_2}}\left|\begin{matrix}(a_j,A_j)_{1,n},[c_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r},h\end{matrix}\right.\right], \qquad (2.5)$$

where $h=(\frac{1}{2}-\mu-\xi_1 R,\xi_2),(-\xi_1 R-r'-\mu,\xi_2)$,

similarly ,The proofs of (2.2) through (2.4) can be established by using the definition of the Aleph- function and known result [4,p.240,Eq.(21)], [4,p.238,Eq.(10)] & [4,p240,Eq.23].

3. Particular Cases

(a) As mention above, when $c_i = 1, \forall\, i = 1,\ldots,r$, the Aleph-function reduces to the I-function and as well as If we take r = 1 in L.H.S. of (2.1), then I-function reduces to familiar Fox's H-function, we arrive at the following result

$$L^{-1}\left\{\begin{matrix}(p^2+b^2)^{-\left(\frac{2\mu+1}{2}\right)}.S_N^M\left[x(p^2+b^2)^{-\xi_1}\right]\\ .\aleph^{m,n}_{p_i,q_i,1;1}\left[z(p^2+b^2)^{-\xi_2}\left|\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right.\right];t\end{matrix}\right\}$$

$$= \sum_{R=0}^{[N/M]}\sum_{r'=0}^{\infty}\frac{(-N)_{MR}}{R!}A_{N,R}x^R\sqrt{\pi}\frac{(-1)^{r'}}{r'!}\frac{t^{2(\mu+\xi_1 R+r')}}{2^{2(\mu+\xi_1 R+r')}}b^{2r'}.H^{m,n}_{p,q+2}\left[\frac{zt^{2\xi_2}}{2^{2\xi_2}}\left|\begin{matrix}(a_j,A_j)_{1,n},\ (a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j)_{m+1,q},h\end{matrix}\right.\right], \qquad (3.1)$$

where $h=\left(\frac{1}{2}-\mu-\xi_1 R,\xi_2\right),(-\xi_1 R-r'-\mu,\xi_2)$.

Similarly, we can find (2.2), (2.3) and (2.4) .

(**b**) Replacing $\aleph^{m,n}_{p_i,q_i,c_i;r}\left[z\{p+(p^2+b^2)^{1/2}\}^{-\rho}\left|\begin{matrix}(a_j,A_j)_{1,n},\ [c_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},\ [c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right]$

by $\aleph^{m,n}_{p_i,q_i,1;1}\left[z\{p+(p^2+b^2)^{1/2}\}^{-\rho}\left|\begin{matrix}(a_j,A_j)_{1,n},\ (a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j)_{m+1,q}\end{matrix}\right.\right]$,

in L.H.S of (2.2), we obtained the following results

$$L^{-1}\left\{\begin{array}{c}\{p+(p^2+b^2)^{1/2}\}^{-\upsilon}S_N^M[x\{p+(p^2+b^2)^{1/2}\}^{-\xi}]\\ \times\aleph_{p_i,q_i,1;1}^{m,n}\left[z\{p+(p^2+b^2)^{1/2}\}^{-\rho}\left|\begin{array}{l}(a_j,A_j)_{1,n},(a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j)_{m+1,q}\end{array}\right.\right];t\end{array}\right\}$$

$$=A^*.\sum_{r'=0}^{\infty}\frac{(-1)^{r'}}{r'!}\left(\frac{t}{2}\right)^{\nu+2r'}.t^{-1}b^{2r'}\times H_{p+1,q+2}^{m,n+1}\left[z\left(\frac{t}{2}\right)^{\rho}\left|\begin{array}{l}h_1,(a_j,A_j)_{1,n},(a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j)_{m+1,q},h_2\end{array}\right.\right], \tag{3.2}$$

where $h_1=(-\nu-\xi R,\rho)$ and $h_2=(-\nu-r'-\xi R,\rho),(1-\upsilon-\xi R,\rho)$,

Further, if we use the identity (1.7) in eq.(3.2), then after a little simplification, we arrive at the following result

$$=A^*.\sum_{r'=0}^{\infty}\frac{(-1)^{r'}}{r'!}\left(\frac{t}{2}\right)^{\nu+2r'}.t^{-1}b^{2r'}p+1^{\Psi}q+2\left[{}_{(b_q,B_q),(-\nu-r'-\xi R,\rho),(1-\upsilon-\xi R,\rho)}^{(-\nu-\xi R,\rho),(a_p,A_p)};-z\left(\frac{x}{2}\right)^{\rho}\right], \tag{3.3}$$

where $A^*=\sum_{R=0}^{[N/M]}\frac{(-N)_{MR}}{R!}A_{N,R}x^R$ and $p+1^{\Psi}q+2$ generalized hypergeometric function [7].
Similarly, we can find (2.1), (2.3) and (2.4).

(**c**) Reducing the General class of polynomial S_N^M to $H_V(x)$ Hermite polynomial ([1] and [2]) and also if we take $c_i=1,\ \ i=1\ldots r$ in L.H.S of (2.3), then Aleph-function reduces to the *I*- function [14], we obtain the following result after a little simplification.

$$L^{-1}\left\{\begin{array}{l}p^{-2\lambda}(p^2+b^2)^{-\upsilon}[xp^{-2\delta}(p^2+b^2)^{-\mu}]^{\frac{N}{2}}H_V\left[\frac{1}{2\sqrt{xp^{-2\delta}(p^2+b^2)^{-\mu}}}\right]\\ \times\aleph_{p_i,q_i,1;r}^{m,n}\left[zp^{-2\rho}(p^2+b^2)^{-\sigma}\left|\begin{array}{l}(a_j,A_j)_{1,n},[1(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[1(b_{ji},B_{ji})]_{m+1,q_i;r}\end{array}\right.\right];t\end{array}\right\}$$

$$=\sum_{R=0}^{[N/2]}\sum_{r'=0}^{\infty}\frac{(-N)_{2R}}{R!}(-1)^Rx^R\frac{(-1)^{r'}}{r'!}\left(\frac{b}{2}\right)^{2r'}.t^{2(\lambda+\upsilon+r'+\delta R+\mu R)-1}$$

$$\times I_{p_i+3,q_i+4;r}^{m,n+3}\left[zt^{2(\rho+\sigma)}\left|\begin{array}{l}h_3,(a_j,A_j)_{1,n},[(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[(b_{ji},B_{ji})]_{m+1,q_i;r},h_4\end{array}\right.\right], \tag{3.4}$$

where $h_3=(1-\upsilon-r'-\mu R,\sigma),(1-\upsilon-\lambda-\delta R-\mu R,\rho+\sigma)$,
$\left(\frac{1}{2}-\lambda-\upsilon-\delta R-\mu R,\rho+\sigma\right)$ and $h_4=\left(\frac{1}{2}-\lambda-\upsilon-r'-\delta R-\mu R,\rho+\sigma\right)$,
$(1-2\lambda-2\upsilon-2\delta R-2\mu R,2\rho+2\sigma),(1-\upsilon-\mu R,\sigma),(1-\lambda-\upsilon-\delta R-\mu R,\rho+\sigma)$,
Similarly, we can find (2.1), (2.2) and (2.4).

(**d**) If we reduce the $S_N^M(x)$ polynomial to the Laguerre polynomial $L_N^{\alpha'}(x)$ ([1] and [2]) and the Aleph-function to the familiar Fox's H-function [6], we obtain the following result after a little simplification.

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$$L^{-1}\left\{\begin{array}{c}(p^2+b^2)^{-1/2}\{p+(p^2+b^2)^{1/2}\}^{-\lambda}L^{\alpha'}{}_N[x\{p+(p^2+b^2)^{1/2}\}^{-\gamma}]\\ \times\aleph_{p_i,q_i,1;1}^{m,n}\left[z\{p+(p^2+b^2)^{1/2}\}^{-\rho}\left|\begin{array}{l}(a_j,A_j)_{1,n},[c_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{array}\right.\right];t\end{array}\right\}$$

$$= \sum_{R=0}^{[N]} \sum_{r'=0}^{\infty} \frac{(-N)_R}{R!} \binom{N+\alpha'}{N} \frac{1}{(\alpha'+1)_R} x^R \frac{(-1)^{r'}}{r'!} \left(\frac{t}{2}\right)^{\lambda+2r'} b^{2r'}$$

$$\times H_{p,q+1}^{m,n}\left[z\left(\frac{t}{2}\right)^{\rho} \middle| \begin{matrix} (a_j,A_j)_{1,n},(a_j,A_j)_{n+1,p} \\ (b_j,B_j)_{1,m},(b_j,B_j)_{m+1,q},(-\lambda-r'-\gamma R,\rho) \end{matrix} \right], \quad (3.5)$$

If now we set $m = n = p = q - 1 = 1$ and use the identity [5, section 18.1] in (3.5), the H-function reduces to the Mittag-Leffler function and the following result is obtained

$$= \sum_{R=0}^{[N]} \sum_{r'=0}^{\infty} \frac{(-N)_R}{R!} \binom{N+\alpha'}{N} \frac{1}{(\alpha'+1)_R} x^R \frac{(-1)^{r'}}{r'!} \left(\frac{t}{2}\right)^{\lambda+2r'} b^{2r'} E_{-\rho,\lambda+r'+\gamma R+1}\left[-z\left(\frac{t}{2}\right)^{\rho} \right], \quad (3.6)$$

where $E_{-\rho,\lambda+r-\xi_1 R+1}$ is Mittag-Leffler function [3,p.65,Eq.(2.9.28)]. Similarly, we can find (2.1), (2.2) and (2.3).

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