ON A NEW SUBCLASS OF MEROMORPHIC FUNCTIONS WITH POSITIVE COEFFICIENTS DEFINED BY CONVOLUTION

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Abstract: The purpose of the present paper is to introduce a new subclass of meromorphic functions with positive coefficients defined by convolution. Coefficient inequalities, growth and distortion inequalities as well as closure result are obtained. Properties of an integral operator and its inversed defined on the new class is also discussed.

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I. INTRODUCTION

let $\sum$ denote the class of normalized meromorphic functions *f* of the form

$$f(z)=\frac{1}{z}+\sum_{n=1}^{\infty} f_n z^n \qquad (1.1)$$

defined on the punctured unit disk $\Delta^* := \{z \in C : 0 <| z |< 1\}$. A function $f \in \sum$ is meromorphic starlike of order $\alpha(0 \le \alpha < 1)$ if

$$-\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha, \qquad (z \in \Delta := \Delta^* \cup \{0\}).$$

The class of all such functions is denoted by $\sum^*(\alpha)$. Similarly the class of convex functions of order α is defined. Let Σ_p be the class of functions $f \in \sum$ with $f_n \ge 0$. The subclass of Σ_p consisting of starlike functions of order α is denoted by $\sum_p^*(\alpha)$.

The following class $MR_p(\alpha)$ is related to the class of functions with positive real part :

$MR_p(\alpha) := \{f : \Re\{-z^2 f'(z)\} > \alpha, \quad (0 \le \alpha < 1)\}$.

In Definition 1.1 below, we unify these classes by using convolution.

The Hadamard product or convolution of two functions *f(z)* given by (1.1) and

$$g(z)=\frac{1}{z}+\sum_{n=1}^{\infty} g_n z^n \qquad (1.2)$$

is defined by

$$(f * g)(z)=\frac{1}{z}+\sum_{n=1}^{\infty} f_n g_n z^n.$$

Definition 1.1. Let $0 \le \alpha < 1$ and $f \in \Sigma_p$ be given by (1.1) and $g(z) \in \Sigma_p$ be given by (1.2) and

$$h(z)=\frac{1}{z}+\sum_{n=1}^{\infty} h_n z^n.$$

Let h_n, g_n be real and

$[g_n + (1-2\alpha)\{\lambda g_n + (1-\lambda)h_n\}] \le 0 \le [\alpha\{\lambda g_n + (1-\lambda)h_n - g_n]$.

The class $Mp(g,h,\alpha,\lambda)$ is defined by

$$Mp(g,h,\alpha,\lambda)=\left\{f\in\sum_p Re\left(\frac{(f*g)(z)}{\lambda(f*g)(z)+(1-\lambda)(f*h)(z)}\right)>\alpha\right\}.$$

Of course, one can consider a more general class of functions satisfying the subordination:

$$\left\{\frac{(f*g)(z)}{\lambda(f*g)(z)+(1-\lambda)(f*h)(z)}\right\}\prec h(z) \qquad (z\in\Delta).$$

By specialzing the parameters in the class $Mp(g,h,\alpha,\lambda)$, we obtain the following known subclassses studied earlier by various researchers.

1. $Mp(g,h,\alpha,0)\equiv Mp(g,h,\alpha)$ studied by Kumar et al. [1].
2. $Mp\left(\frac{1}{z}-\frac{z}{(1-z)^2},\frac{1}{z(1-z)},\alpha,0\right)\equiv\sum_p^*(\alpha)$ studied by Ravichardran [4].
3. $Mp\left(\frac{1}{z}-\frac{z}{(1-z)^2},\frac{1}{z(1-z)},\alpha,0\right)\equiv\sum_p^*(\alpha)$ studied by Ravichardran [4].

In the present paper, motivating with the above mentioned work and work of ([2], [3], [5])coefficient inequalities, growth and distortion inequalities, convolution, convex combination, integral operator for functions of the class $Mp(g,h,\alpha,\lambda)$.

2. **Coefficients Inequalities**

First, we give a necessary and sufficient condition for the function f to be the class $Mp(g,h,\alpha,\lambda)$.

Theorem 2.1. Let $f\in\Sigma_p$ be given by (1.1). Then $f\in Mp(g,h,\alpha,\lambda)$ if and only if

$$\sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]f_n\le 1-\alpha.$$

Proof. If $f\in Mp(g,h,\alpha,\lambda)$, then

$$\Re\left\{\frac{(f*g)(z)}{\lambda(f*g)(z)+(1-\lambda)(f*h)(z)}\right\}>\alpha$$

$$\Re\left\{\frac{1+\sum_{n=1}^{\infty}f_ng_nz^{n+1}}{\lambda\left(1+\sum_{n=1}^{\infty}f_ng_nz^{n+1}\right)+(1-\lambda)\left(1+\sum_{n=1}^{\infty}f_nh_nz^{n+1}\right)}\right\}>\alpha.$$

By letting $z\to1^-$, we have

$$\Re\left\{\frac{1+\sum_{n=1}^{\infty}f_ng_n}{\lambda\left(1+\sum_{n=1}^{\infty}f_ng_n\right)+(1-\lambda)\left(1+\sum_{n=1}^{\infty}f_nh_n\right)}\right\}>\alpha.$$

This show that(2.1)

Conversely, assume that (2.1) holds. Since $\Re\{w\}>\alpha$,

if and only if $|w-1|<|w+1-2\alpha|$, it is sufficient to show that

$$\left|\frac{(f*g)(z)-[\lambda(f*g)(z)+(1-\lambda)(f*h)(z)]}{(f*g)(z)+(1-2\alpha)[\lambda(f*g)(z)+(1-\lambda)(f*h)(z)]}\right|$$

$$=\left|\frac{\sum_{n=1}^{\infty}f_n[h_n(1-\lambda)-g_n(1-\lambda)]z^{n+1}}{2(1-\alpha)+\sum_{n=1}^{\infty}[g_n+(1-2\alpha)\{\lambda g_n-(\lambda-1)h_n\}]f_nz^{n+1}}\right|$$

$$\le\frac{\sum_{n=1}^{\infty}f_n[h_n(1-\lambda)-g_n(1-\lambda)]}{2(1-\alpha)+\sum_{n=1}^{\infty}[g_n+(1-2\alpha)\{\lambda g_n-(\lambda-1)h_n\}]f_n}$$

$$\le 1.$$

Thus we have $f\in Mp(g,h,\alpha,\lambda)$.

If we put $\lambda=0$ in Theorem 2.1 then we obtain the following result of Kumar et al. [1].

Corollary 2.2. Let $f\in\Sigma_p$ be given by (1.1). The $f\in Mp(g,h,\alpha)$, if and only if $\sum_{n=1}^{\infty}(\alpha h_n-g_n)f_n\le1-\alpha.$

Theorem 2.3. If $f\in Mp(g,h,\alpha,\lambda)$. Then

$$f_n \le \frac{(1-\alpha)}{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]}, \quad n=1,2,3,\ldots$$

The result is sharp for the function $F_n(z)$ given by

$$F_n(z)=\frac{1}{z}+\frac{(1-\alpha)z^n}{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]}, \quad n=1,2,3,\ldots$$

Proof. If $f\in Mp(g,h,\alpha,\lambda)$, then we have for each n,

$$[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]f_n \le \sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]f_n \le 1-\alpha.$$

Therefore we have

$$f_n \le \frac{1-\alpha}{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]f_n}.$$

Since

$F_n(z)=\frac{1}{z}+\frac{1-\alpha}{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]}z^n$ satisfies the condition of Theorem 2.1, $F_n(z)\in Mp(g,h,\alpha,\lambda)$ and the inequality is attained for this function.

Theorem 2.4. Let $\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1 \le [\alpha\{\lambda g_n+(1-\lambda)h_n\}$, if $f\in Mp(g,h,\alpha,\lambda)$. Then

$$\frac{1}{r}-\frac{(1-\alpha)r}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]} \le |f(z)| \le \frac{1}{r}+\frac{(1-\alpha)r}{r[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]}, \quad (|z|=r).$$

The result is sharp for

$$f(z)=\frac{1}{z}+\frac{(1-\alpha)(z)}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]}. \tag{2.2}$$

Proof. Since $f(z)=\frac{1}{z}+\sum_{n=1}^{\infty} f_n z^n$ we have

$$|f(z)| \le \frac{1}{r}+\sum_{n=1}^{\infty} f_n r^n \le \frac{1}{r}+r\sum_{n=1}^{\infty} f_n.$$

Since

$$\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]\sum_{n=1}^{\infty} f_n \le \sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n],$$

we have

$[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]\sum_{n=1}^{\infty} f_n \le \sum_{n=1}^{\infty}[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]f_n \le 1-\alpha$ and therefore

$$\sum_{n=1}^{\infty} f_n \le \frac{1-\alpha}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]}.$$

Using this, we have

$$|f(z)| \le \frac{1}{r}+\frac{(1-\alpha)r}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]}.$$

Similarly

$$|f(z)| \ge \frac{1}{r}-\frac{(1-\alpha)r}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]}.$$

The result is sharp for

$$f(z)=\frac{1}{z}+\frac{(1-\alpha)z}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]}.$$

Similarly we have the following:

Theorem 2.5. Let

$$\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1] \le \frac{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]}{n}.$$

If $f\in Mp(g,h,\alpha,\lambda)$, then

$$\frac{1}{r^2}-\frac{1-\alpha}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]} \le |f'(z)| \le \frac{1}{r^2}-\frac{1-\alpha}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]} \quad \{|z|=r\}.$$

The result is sharp for the function given by (2.2).

3. Closure Theorems

Let the functions $F_k(z)$ be given by

$$F_k(z)=\frac{1}{z}+\sum_{n=1}^{\infty} f_{n,k}(z)z^n, \quad k=1,2,3,.,m\ . \tag{3.1}$$

We shall prove the following closure theorems for the class $Mp(g,h,\alpha,\lambda)$.

Theorem 3.1. Let the function $F_k(z)$ defined by (3.1) be in the class $Mp(g,h,\alpha,\lambda)$ for every *k=1,2,3,...m*. Then the function *f(z)* defined by

$$f(z)=\frac{1}{z}+\sum_{n=1}^{\infty}a_n z^n \qquad (a_n\geq 0)$$

belongs to the class $Mp(g,h,\alpha,\lambda)$, where $a_n=\frac{1}{m}\sum_{k=1}^{\infty}f_{n,k} \quad (n=1,2,3,....)$.

Proof. Since $F_n(z)\in Mp(g,h,\alpha,\lambda)$, it follows from Theorem 2.1 that

$\sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]f_{n,k}\leq 1-\alpha$ for every *k=1,2,...,m*. Hence

$$\sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]a_n=\sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]\left(\frac{1}{m}\sum_{k=1}^{\infty}f_{n,k}\right)$$

$$=\frac{1}{m}\sum_{k=1}^{m}\left(\sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]f_{n,k}\right)$$

$$\leq 1-\alpha.$$

By Theorem 2.1, it follows that $f(z)\in Mp(g,h,\alpha,\lambda)$.

Theorem 3.2. The class $Mp(g,h,\alpha,\lambda)$ is closed under convex linear combination.

Proof. Let the function $F_k(z)$ given by (3.1) be in the class $Mp(g,h,\alpha,\lambda)$, then it is enough to show that the function

$H(z)=\lambda F_1(z)+(1-\lambda)F_2(z), \quad (0\leq\lambda\leq 1)$ is also the class $Mp(g,h,\alpha,\lambda)$. Since for $0\leq\lambda\leq 1$ $H(z)=\frac{1}{z}+\sum_{n=1}^{\infty}[\lambda f_{n,1}+(1-\lambda)f_{n,2}]z^n$

we observe that

$$\sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n][\lambda f_{n,1}+(1-\lambda)f_{n,2}]$$

$$=\lambda\sum_{n=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]f_{n,1}+(1-\lambda)\sum_{k=1}^{\infty}[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]f_{n,2}$$

$$\leq 1-\alpha.$$

By Theorem 2.1, we have $H(z)\in Mp(g,h,\alpha,\lambda)$.

Theorem 3.3. Let $F_0(z)=\frac{1}{z}$ and $F_n(z)=\frac{1}{z}+\frac{1-\alpha}{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]z^n}$, for n=1,2,3,....

Then $f(z)\in Mp(g,h,\alpha,\lambda)$, if and only if *f(z)* can be expressed in the form

$$f(z)=\sum_{n=0}^{\infty}\gamma_n F_n(z), \text{where } \lambda_n\geq 0 \text{ and } \sum_{n=0}^{\infty}\lambda_n=1.$$

Proof. Let $f(z)=\sum_{n=0}^{\infty}\lambda_n F_n(z)=\frac{1}{z}+\sum_{n=1}^{\infty}\frac{\lambda_n(1-\alpha)}{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]}z^n.$

Then

$$\sum_{n=1}^{\infty}\frac{\lambda_n g_n+(1-\alpha)[\alpha\{\lambda_n g_n+(1-\lambda_n)h_n\}-g_n]}{[\alpha\{\lambda_n g_n+(1-\lambda_n)h_n\}-g_n](1-\alpha)}$$

$$=\sum_{n=1}^{\infty}\lambda_n$$

$$=1-\lambda_0\leq 1.$$

By Theorem 2.1, we have $f(z)\in Mp(g,h,\alpha,\lambda)$.

Conversely, let $f(z)\in Mp(g,h,\alpha,\lambda)$. From Theorem 2.3 we have

$f_n\leq\frac{1-\alpha}{[\alpha\{\lambda g_n+(1-\lambda)h_n-g_n]}$, for n=1,2,3,. we may take

$\gamma_n=\frac{[\alpha\{\lambda g_n+(1-\lambda)h_n-g_n]}{1-\alpha}$ for n=1,2,3,. and $\lambda_0=1-\sum_{n=1}^{\infty}\lambda_n$.

Then $f(z)=\sum_{n=1}^{\infty}\lambda_n F_n(z)$.

4. Integral Operators

In this section, we consider integral transform of functions in the class $Mp(g,h,\alpha,\lambda)$.

Theorem 4.1. Let the function *f(z)* given by (1.1) be in $Mp(g,h,\alpha,\lambda)$. Then the integral operator

$$F(z)=c\int_0^1 u^c f(uz)du \qquad (0<u\leq 1, 0<c<\infty)$$

is in $Mp(g,h,\alpha,\lambda)$, where

$$\delta=\frac{(c+2)[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]+(1-\alpha)cg_1}{c[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]+(1-\alpha)(c+2)g_1}.$$

The result is sharp for the function $f(z)=\frac{1}{z}+\frac{(1-\alpha)z}{[\alpha\{\lambda g_1+(1-\lambda)h_1\}-g_1]}$.

Proof. Let $f(z)\in Mp(g,h,\alpha,\lambda)$. Then

$$F(z)=c\int_0^1 u^c f(uz)du$$

$$F(z)=c\int_0^1 u^c[\frac{1}{uz}+\sum_{n=1}^{\infty}u^n z^n f_n]du$$

$$F(z)=\frac{1}{z}+\sum_{n=1}^{\infty}\frac{c}{(c+n+1)}f_n z^n.$$

It is sufficient to show that

$$\sum_{n=1}^{\infty}\frac{c[\delta\{g_n+(1-\lambda)h_n\}-g_n]f_n}{(c+n+1)(1-\delta)}\le 1. \qquad (4.1)$$

Since $f(z)\in Mp(g,h,\alpha,\lambda)$, we have

$$\sum_{n=1}^{\infty}\frac{[\alpha\{-\lambda g_n+(1-\lambda)h_n\}-g_n]}{1-\alpha}\le 1.$$

Note that (4.1) satisfied if

$$\frac{c[\delta\{\lambda g_n+(1-\lambda)h_n\}-g_n]}{(c+n+1)(1-\delta)}\le\frac{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]}{1-\alpha}.$$

Re writing the inequality, we have

$$c[\delta\{\lambda g_n+(1-\lambda)h_n\}-g_n](1-\alpha)$$
$$\le(c+n+1)(1-\delta)[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n]$$
$$[\delta c\{\lambda g_n+(1-\lambda)h_n\}-cg_n](1-\alpha)$$
$$\le(c+n+1)(1-\delta)\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n.$$

Solving for δ we have

$$\delta\le\frac{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n](c+n+1)+cg_n(1-\alpha)}{ch_n(1-\alpha)+[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n](c+n+1)}$$

=F(n)

A computation show that

F(n+1)-F(n)

$$\frac{[\alpha\{\lambda g_{n+1}+(1-\lambda)h_{n+1}\}-g_{n+1}](c+n+1)+cg_{n+1}(1-\alpha)}{ch_{n+1}(1-\alpha)+[\alpha\{\lambda g_{n+1}+(1-\lambda)h_{n+1}\}-g_{n+1}](c+n+2)}$$
$$-\frac{[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_{n+1}](c+n+1)+cg_n(1-\alpha)}{ch_n(1-\alpha)+[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n](c+n+1)}$$
$$=\frac{(1-\alpha)c[(1-\alpha)(n+1)g_n h_{n+1}+(h_n-g_n)(\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n)]}{[ch_n(1-\alpha)+[\alpha\{\lambda g_n+(1-\lambda)h_n\}-g_n](c+n+1)]\,[(ch_{n+1}(1-\alpha))+(\alpha\{\lambda g_{n+1}+(1-\lambda)h_{n+1}\}-g_{n+1})(c+n+2)]}$$

>0

for all *n*. This means that *F(n)* is increasing and $F(n)\ge F(1)$. Using this, the result follows.

If we put $\lambda=0$ in Theorem 4.1 then we obtain the following result of Kumar et al. [1].

Corollary 4.2. Let the function *f(z)* defined by (1.1) be in $\Sigma_p^*(\alpha)$. Then the integral operator

$$F(z)=c\int_0^1 u^c f(uz)du, \qquad (0,<u\le 1,=0<c<\infty)$$

is in $\Sigma_p^*(\alpha)$., where $\delta=\frac{1+\alpha+c\alpha}{1+\alpha+c}$. The result is sharp for the function

$$f(z)=\frac{1}{z}+\frac{(1-\alpha)}{1+\alpha)}z.$$

Also, we have the following.

Corollary 4.3. Let the function *f(z)* defined by (1.1) be in $MR_p(\alpha)$. Then the integral operator

$$F(z)=c\int_0^1 u^c f(uz)du, \qquad (0,<u\le 1,0<c<\infty)$$

is in $MRp\left(\frac{2+c\alpha}{c+2}\right)$. The result is sharp for the function $f(z)=\frac{1}{z}+(1-\alpha)z.$

Theorem 4.4. Let *f(z)* given by (1.1), be in $Mp(g,h,\alpha,\lambda)$.

$$F(z)=\frac{1}{c}[(c+1)f(z)+zf'(z)]=\frac{1}{z}+\sum_{n=1}^{\infty}\frac{c+n+1}{c}f_n z^n, c>0.$$

Then *F(z)* is in $Mp(g,h,\beta,\lambda)$ for $|z| \le r(\alpha,\beta)$, where

$$r(\alpha,\beta) = \inf_n \left(\frac{c(1-\beta)[\alpha\{\lambda g_n + (1-\lambda)h_n\} - g_n]}{(1-\alpha)(c+n+1)[\beta\{\lambda g_n + (1-\lambda)h_n\} - g_n]} \right)^{1/n+1}, n = 1,2,3...$$

The results is sharp the function $f_n(z) = \frac{1}{z} + \frac{(1-\alpha)z^n}{[\alpha\{\lambda g_n + (1-\lambda)h_n\} - g_n]}, n = 1,2,3....$

Proof. Let $w = \frac{(f*g)(z)}{\lambda(f*g)(z) + (1-\lambda)(f*h)(z)}$.

Then it is sufficient to show that

$$\left| \frac{w-1}{w+1-2\beta} \right| < 1.$$

A computation shows that this is satisfied if

$$\sum_{n=1}^{\infty} \frac{\beta\{\lambda g_n + (1-\lambda)h_n\} - g_n] f_n]}{(1-\beta)c} |z|^{n+1} f_n \le 1. \qquad (4.2)$$

Since $f \in Mp(g,h,\alpha,\lambda)$, by Theorem 2.1, we have

$$\sum_{n=1}^{\infty} [\alpha\{\lambda g_n + (1-\lambda)h_n\} - g_n] f_n \le 1-\alpha .$$

The equation (4.2) is satisfies if

$$\frac{[\beta\{\lambda g_n + (1-\lambda)h_n\} - g_n]}{(1-\beta)c} f_n |z|^{n+1} \le \frac{[\alpha\{\lambda g_n + (1-\lambda)h_n\} - g_n]}{1-\alpha}$$

Solving for $|z|$, we get the result.

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