RESTRAINED GIRTH DOMINATION NUMBER OF GRAPHS

G.Victor Emmanuel[1], A.Eskkimuthu[2], A.Nellai Murugan[3], G.Princeton Lazarus[4]
Asst. Prof. of Mathematics, St. Mother Theresa Engineering College, , Tamilnadu, India[1,4]
Asst. Prof. of Mathematics,Thiyagi Dharmakkan Amirtham College of Arts and Science ,Kannirajapuram, Ramanathapuram, Tamilnadu, India[2] ,
Assoc. Prof. of Mathematics,V.O.Chidambaram College,Thoothukudi-628 008, Tamilnadu, India[3]

Abstract: The concept of complete graphs with real life application was introduced in [18] and the Forbidden pairs and the existence of a dominating cycle was introduced in[19] . In this paper, we introduce a new domination parameter called girth 2-domination number , that is, if all the edges of the girth(cycle) graph are the edges of any other cycles in a graph G and let G is a connected graph then C_{ni} is the girth graph of G if $C_{ni} \leq C_{nj}$, i≠j. A subset S of V of a non trivial graph G is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating set in G. A subset S of V of a nontrivial graph G is said to be restrained girth dominating set, if every vertex in V-S is adjacent to at least one vertex of girth(cycle) graph as well as another vertex in V-S is called the restrained girth dominating set. The minimum cardinality taken over all restrained girth dominating set is called the restrained girth domination number and is denoted by $\gamma_{rg}(G)$.We determine this number for some standard graphs and obtain bounds for general graphs. Its relationship with other graph theoretical parameters are also investigated.

Mathematics Subject Classification: 05C69

Keywords: *Domination, Domination Number, Girth Domination, Girth Domination Number, and Restrained Girth domination number*

I Introduction:

The concept of domination in graphs evolved from a chess board problem known as the Queen problem- to find the minimum number of queens needed on an 8x8 chess board such that each square is either occupied or attacked by a queen. C.Berge[3] in 1958 and 1962 and O.Ore[8] in 1962 started the formal study on the theory of dominating sets. Thereafter several studies have been dedicated in obtaining variations of the concept. The authors in [7] listed over 1200 papers related to domination in graphs in over 75 variation.

Throughout this paper , G(V , E) a finite, simple , connected and undirected graph where V denotes its vertex set and E its edge set. Unless otherwise stated the graph G has n vertices and m edges. Degree of a vertex v is denoted by d(v), the *maximum degree* of a graph G is denoted by Δ(G). Let C_n a cycle on n vertices, P_n a path on n vertices by and a complete graph on n vertices by K_n. A graph is *connected* if any two vertices are connected by a path . A maximal connected subgraph of a graph G is called a *component* of G. The *number of components* of G is denoted by ω(G). The *complement* $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G. A tree is a connected acyclic graph. A *bipartite graph* is a graph whose vertex set can be divided into two disjoint sets V_1 and another in V_2. A *complete bipartite graph* is a bipartite graph with partitions of order $|V_1|$ =m and $|V_2|$=n, is denoted by $K_{m,n}$. A star denoted by $K_{1,n-1}$ is a *tree* with one root vertex and n-1 pendant vertices. A bistar , denoted by B(m,n) is the graph obtained by joining the root vertices of the stars denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A *wheel* graph denoted by W_n is a graph with n vertices formed by connecting a single vertex to all vertices of C_{n-1} . A *Helm* graph denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex to each vertex in the outer cycle of W_n.

The *chromatic number* of a graph G denoted by χ(G) is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colours . For any real number x , $\lceil x \rceil$ denotes the largest integer greater than or equal to x and $\lfloor x \rfloor$ the smallest integer less than or equal to x. A Nordhaus- Gaddum – type result is a lower or upper bound on the sum or product of a parameter of a graph and its complement. Throughout this paper, we only consider undirected graphs with no loops. The basic definitions and concepts used in this study are adopted from[11].

Given a graph G = (V(G) , E(G)), the cardinality $|V(G)|$ of the vertex set V(G) is the order of G is n. The distance $d_G(u,v)$ between two vertices u and v of G is the length of the *shortest path* joining u and v. If $d_G(u,v) = 1$, u and v are said to be adjacent.

For a given vertex v of a graph G , The open neighbourhood of v in G is the set $N_G(v)$ of all vertices of G that are adjacent to v.

The degree $deg_G(v)$ of v refers to $|N_G(v)|$, and $\Delta(G) = \max\{deg_G(v):\ v \in V(G)\}$. The closed neighbourhood of v is the set $N_G[v] = N_G(v) \cup v$ for $S \subseteq V(G)$, $N_G(S) = \cup_{v \in S} N_G(v)$ and $N_G[v] = N_G(S) \cup S$. If $N_G[v]$==V(G), then S is a dominating set in G. The minimum cardinality among dominating sets in G is called the *domination number* of G and is denoted by $\gamma(G)$.

Definition [17]:In a connected graph G, a chord of a spanning tree T is a line of G which is not in T.Clearly the subgraph of G consists of T and any chord of T has exactly one cycle.

Definition [17]:If T is a regular of degree 2,every component is a cycle and regular graphs of degree 3 are called cubic.

Definition [17]:If all the edges of the girth are the edges of any other cycles in a graph G.

Theorem [17]:Let x be a line of a connected graph G, The following statements are equivalent(1)x is a bridge of G.(2) x is not on any cycle of G.(3)There exist points u and v of G s.t the line x is on every path joining u and v.(4)There exists a partition of v into subsets U and W s.t for any points u∈Uand w∈W the line x is on every path joining u and w.

Theorem[17]:Let G be a connected graph with atleast three points. The following statements are equivalent. (1)G is a block (2)Every two points of G lie on a common cycle (3)Every point and line of G lie on a common cycle (4) Every two lines of G lie on a common cycle (5)Given two points and one line of G, there is a path joining the points which contains the line (6)For every three distinct points of G, There is a path joining any two of them which contains the third.

A set $S \subset V(G)$ is called a *girth dominating set* of G if every vertex in V-S is adjacent to at least one vertex in the girth cycle of G. The minimum cardinality of a girth dominating set of G is called its *girth domination number* of G denoted by $\gamma_g(G)$ [19].

II.MAIN RESULT

2.0.Definition: A subset S of V of a nontrivial graph G is said to be restrained girth dominating set, if every vertex in V-S is adjacent to at least one vertex of girth(cycle) graph as well as another vertex in V-S is called the restrained girth dominating set. The minimum cardinality taken over all restrained girth dominating set is called the restrained girth domination number and is denoted by $\gamma_{rg}(G)$.

2.1.Example: For any graph $|G| = C_4 + v = 5$ (or) $|G| = C_3 + e = 5$ .If $N(v_i) = (u_i, v_i)$ and $\cup N(v_i) = (C_3, v_i)$ where $u_i \in S$ and $v_i \in V - S$ has a restrained girth dominating set of G with $\gamma_{rg}(G)$=n-2=3 for n = 5 if Max{d(u_i, u_j)} ≥n-3,$i \neq j$ where $u_i \in C_3$ by fig:1

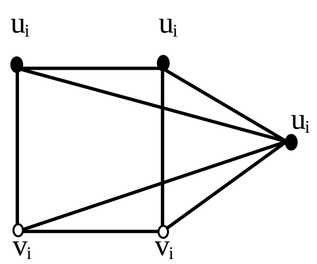

Fig1: $|G| = C_4 + v = 5$, $\gamma_{rg}(G)$**=n-2=3**

2.2.Result : For any graph |G| $=K_n = C_{n-2} + e = n$, $C_{n-2} = n - e$ and $\cup N(v_i) = (C_{n-2}, v_i)$ has a girth dominating set of G with $\gamma_{rg}(G)$ ≥n-2 for n ≥ 5 for every $v_i \in V - S$. Since if Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_g(G)$=4, Similarly we can have the restrained girth dominating set with $\gamma_{rg}(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k + 2$ for every $v_i \in V - S$.

2.3.Example: For any graph $|G|$=$K_n \odot K_{1,n-1}$=$[C_{n-1} + v] \odot K_{1,n-1} = 2n - 1$, $[C_{n-1}] \odot K_{1,n-1} + v = 2n - 1$ and $\cup N(v_i) = C_{n-1}$ has a girth dominating set of G but it is not a restrained girth dominating set of G since $\cup N(v_i) \neq (C_{n-1}, v_i)$ for n≥ 4 by fig:2.

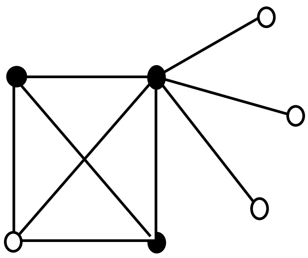

Fig2: $|G|$=$\boldsymbol{K_n \odot K_{1,n-1}}$**, Not a restrained girth dominating set**

2.4.Result: For any graph $|G|$=$K_n - 4e = (C_{n-3} + 3v) - 4e$=$K_6 - 4e = n$, $(C_{n-3} + 3v) = n$ and $C_{n-3} = n - 3$ with $\cup N(v_i) = (C_{n-3}, v_i)$ has a restrained girth dominating set of G with $\gamma_{rg}(G)$ ≥n-3 for n≥ 6 for every $v_i \in V - S$ by fig:3. Since if Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_{rg}(G)$=4,Similarly we can have the restrained girth dominating set with $\gamma_{rg}(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k + 3$, for every $v_i \in V - S$. By fig.3.

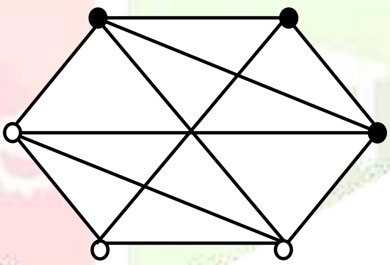

Fig 3: G=$\boldsymbol{K_n - 4e = (C_{n-3} + 3v) - 4e, \gamma_{rg}(G)}$**=3, for n≥ 6**

2.5.Result: For any graph $|G|$=$K_n - 6e = |(C_{n-3} + 3v) - 6e|$=$K_6 - 6e = n$, $(C_{n-3} + 3v) - 6e = n$ and $C_{n-3} = n - 3$ with $\cup N(v_i) = (C_{n-3}, v_i)$ has a restrained girth dominating set of G with $\gamma_{rg}(G)$ ≥n-3 for n≥ 6 for every $v_i \in V - S$ by fig:4. Since if Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_{rg}(G)$=4,Similarly we can have the restrained girth dominating set with $\gamma_{rg}(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k + 3$ for every $v_i \in V - S$ by fig.4.

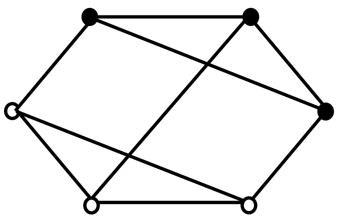

Fig 4: G=$\boldsymbol{K_n - 6e = (C_{n-3} + 3v) - 6e, \gamma_{rg}(G)}$**=3, for n≥ 6**

2.6.Result: For any graph $|G|=K_n - 5e = |(C_{n-3} + 3v) - 5e|=K_6 - 5e = n$, $(C_{n-3} + 3v) - 5e = n$ and $C_{n-3} = n - 3$ with $\cup N(v_i) = (C_{n-3}, v_i)$ has a restrained girth dominating set of G with $\gamma_{rg}(G) \geq$n-3 for n$\geq$ 6 for every $v_i \in V - S$. Since if Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_{rg}(G)$=4,Similarly we can have the restrained girth dominating set with $\gamma_{rg}(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k + 3$ for every $v_i \in V - S$.

2.7.Example: For any wheel graph G=W_n , n = 4 is a restrained girth dominating set with $\gamma_{rg}(G)$ =3 with $\cup N(v_i) = (C_{n-1}, v_i)$ since Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=3 and $|\cup N(v_i)| = 4$, for every $v_i \in V - S$ by fig:5.

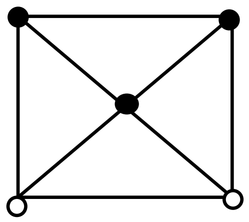

Fig 5: G=W_n , n=4, $\gamma_{rg}(G)$=3,

2.8.Example : For any complete bipartite graph G=$K_{m,n}$ is a restrained girth dominating set and its restrained girth domination number $\gamma_{rg}(G)$=4 for m,n $\geq$ 3 since Max{d(u_i, u_j)} = 3 and Min{d(u_i, u_j)} = 1 and $|\cup N(v_i)| = 6$, for every $v_i \in V - S$ by fig:6.

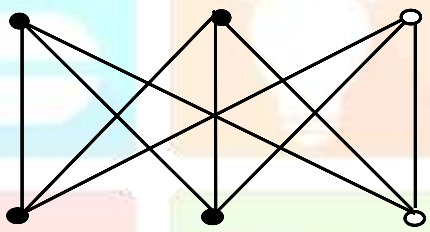

Fig 6: G=$K_{m,n}$; m,n $\geq$ 3, $\gamma_{rg}(G)$=4

2.9.Example: For any Helm graph G=H_n is not a girth dominating set for n$\geq$ 4 Since $|N(u_i) \cap (V - S)| \neq 1$, i≠1 where V-S =$(H_n - C_3)$.Hence it is not a restrained girth dominating set of G.

2.10.Example: For any graph G= $K_4 - e$ is a girth dominating set and its girth domination number $\gamma_{rg}(G)$=3 with $|M|$=1 then we have $|N(u_i) \cap (V - S)| = 1$,i≠1 but $\cup N(v_i) \neq (C_{n-1}, v_i) \neq (S, V - S)$ by fig:7.Hence it is not a restrained girth dominating set of G.By fig.7.

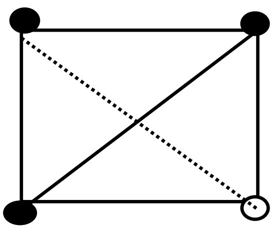

Fig 7: G= $K_4 - e, \gamma_{rg}(G)$=3

2.11.Example : For any graph G= $K_5 - ie$; i=1,2,3 is a restrained girth dominating set and its restrained girth domination number $\gamma_{rg}(G)$=3 by fig:8.

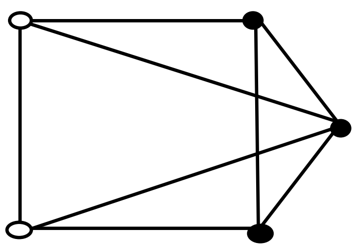

Fig 8: G= $K_5 - 2e, \gamma_{rg}(G)$=3

2.12.Example: For any graph G= $K_5 - 4e$ is a restrained girth dominating set and its restrained girth domination number $\gamma_{rg}(G)$=3 with $|M|$=1 then we have $|N(u_i) \cap (V-S)| = 1$,i≠1 by fig:9.

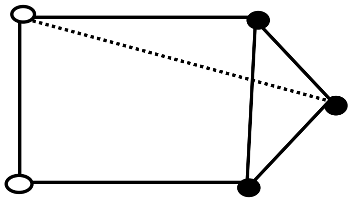

Fig 9: $\boldsymbol{K_5 - 4e}$ **,** $\boldsymbol{\gamma_{rg}(G)}$**=3**

2.13.Lemma : Let G is a connected graph then C_{ni} is the girth graph of G if $C_{ni} \leq C_{nj}$, i≠j.

2.14.Lemma : Let G be any graph and C_{ni} is the cycle then $\cap\, C_{ni}$ ≥e if $C_{ni} \leq C_{nj}$, i≠j.

Proof: Given G be a connected graph , By the definition of a cycle , $\exists C_{ni} \leq C_{nj}$ and i≠j

We have atleast one edge is common to $C_{ni} \leq C_{nj}$.Hence we have $\cap\, C_{ni}$ ≥e.

2.15.Lemma: Let G be any connected graph and C_{ni} is the cycle then $\cap\, C_{ni}$ ≥2v if $C_{ni} \leq C_{nj}$, i≠j.

Proof: Given G be a connected graph , By the definition of a cycle , $\exists C_{ni} \leq C_{nj}$ and i≠j

We have at least two vertices are in common to $C_{ni} \leq C_{nj}$.Hence we have $\cap\, C_{ni}$ ≥2v.

2.16.Lemma: Let G be a connected graph and $\exists u_i \in C_{ni}$ is the restrained girth dominated if $\bigcup_{i=1}^{n} N(v_i)$=$(u_i,v_i) = (S, V-S) = V$ Where v_i ∈V-S

Proof: Given G be a connected graph , By lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is a restrained girth dominated which gives $|N(u_i) \cap (V-S)| = 1$ and $N(v_i)$ = u_i Where $u_i \in C_{ni}$ and $v_i \in$ V-S, Hence $\bigcup_{i=1}^{n} N(v_i)$=$(u_i,v_i) = (S, V-S) = V$.

2.17.Lemma: For any graph $|G| = C_{n-2} + e = n$ then S=C_{n-2}=n-e where C_{n-2} is the restrained girth dominating set of a graph G and $v \in V-S$ if then $|N(u_i) \cap (V-S)| = 1$ and $\bigcup_{i=1}^{n} N(v_i)$=$(u_i,v_i) = (S, V-S) = V$

Proof: For any graph $|G|$=n and if there exists a cycle $C_{n-2} \leq C_{n-1}$ that is $C_{n-2} + e$=n we have S=$C_{n-2} = n - 2v$ means that the graph is G-2v and the vertex v is non adjacent with any vertex of G. If v_i is adjacent with atleast one vertex. Hence $G = C_{n-2} + e$ then $|N(u_i) \cap (V-S)| = 1$ and $\bigcup_{i=1}^{n} N(v_i) = (u_i,v_i) = (S, V-S) = V$ where v_i ∈V-S.

2.18.Lemma : For any graph $|G| = C_{n-2} + e = n$ then S=C_{n-2}=n-2 where C_{n-2} is the restrained girth dominating set of a graph G and $v_i \in V-S$ if $|N(u_i) \cap (V-S)| = 2$ with $\bigcup_{i=1}^{n} N(v_i)$=$(u_i,v_i) = (S, V-S) = V$ where v_i ∈V-S.

Proof: For any graph $|G|$=n and if there exists a cycle $C_{n-2} \leq C_{n-1}$ that is $C_{n-2} + e$=n where $v \in V-S$ we have S=$C_{n-2} = n - 2$ means that the graph is G-e, There fore the vertices v_i i≥ 2 is non adjacent with any two vertices of G. If v_i is adjacent with atleast 2 vertices the $|N(u_i) \cap (V-S)| = 2$ and $\bigcup_{i=1}^{n} N(v_i)$=$(u_i,v_i) = (S, V-S) = V$ where v_i ∈V-S.

2.19.Lemma : Let G be any complete graph and $\exists u_i \in C_{ni}$ ⊆S is the girth dominating set of G then $|N(u_i) \cap (V-S)| \geq 1$ where v_i ∈V-S with $\bigcup_{i=1}^{n} N(v_i)$=$(u_i,v_i) = (S, V-S) = V$

Proof: Given G be a connected graph by lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is a restrained girth dominated ,which gives $\bigcup_{i=1}^{n} N(v_i)$=(u_i,v_i) where $u_i \in C_{ni}$, i= 1,2,..n and then for any graph $|G|$=n and if there exists a cycle $C_{n-2} \leq C_{n-1}$ that is $C_{n-2} + e$=n we have S=$C_{n-2} = n - 2v$ means that the graph is G-e and the vertex v is non adjacent with any vertex of G. If v_i is adjacent with atleast one vertex then $|N(u_i) \cap (V-S)| \geq 1$ where v_i ∈V-S and $\bigcup_{i=1}^{n} N(v_i)$=$(u_i,v_i) = (S, V-S) = V$.

2.20.Theorem : For any graph G .Let S be a restrained girth dominating set of G if S ={u_i} for i=1,2….n. is $\gamma_{rg}(G-v) \leq \gamma_{rg}(G)$

Proof: For any graph G .Let S be a restrained girth dominating set of G and V-S ={v_i} for i=1,2,…n. we have given G be a connected graph by lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is restrained girth dominated ,which gives $\bigcup_{i=1}^{n} N(v_i)$=(u_i, v_i) where $u_i \in C_{ni}$, i= 1,2,..n and then $|N(u_i) \cap (V-S)| \geq 1$ where v_i ∈V-S.

If any one of v_i is removed then the edges incident on v_i is removed then it may not be the restrained girth dominating set of G, Since $\cup N(v_i) = (S, V-S)$ which gives N(v_i) ≠ {u_i,v_i} . There fore $\cup N(v_i) \neq (S, V-S) = V$. Hence $\gamma_{rg}(G-v) \leq \gamma_{rg}(G)$.

2.21.Theorem : For any graph G with girth cycle is girth dominated if it has atleast one matching.

Proof : For any graph with girth cycle will be the restrained girth dominating set S of G. If every vertex of V-S is adjacent to atleast one vertex of S also another vertex of V-S . If not then there exist one v ∈ $V-S$ is not adjacent to one vertex of S then we made a matching to this vertex with S then it gives the restrained girth dominating set of G with $|M|$=1 if $|N(u_i) \cap (V-S)| \geq 1$ and $\bigcup_{i=1}^{n} N(v_i)$=$(u_i, v_i) = (S, V-S) = V$. Hence we can find atleast one matching to find the restrained girth dominating set of G, that is $|M|$ ≥1

2.22.Theorem: Every restrained girth dominating set and it is of chromatic number $\chi(G)$ ≥3

Proof: Given the graph G is a restrained girth dominating set since it is restrained girth dominated the graph G must have at least a girth cycle of C_3.If every cycle C_n ,Since if Max{d(u_i,u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=3 must have 3 colourable and every vertex of V-S is adjacent to at least one vertex of S and also another vertex of V-S have 4[Th] vertex may have the 3[rd] colour other than the colours which had in the restrained girth cycle and which is adjacent to the vertex. Hence there must have at least 3 colours needed to colour the restrained girth dominated graph ,that is $\chi(G)$ ≥3.

2.23.Lemma : If $|N(u_i) \cap (V-S)| \geq 1$, Then $(u_1, u_2 \ldots, u_{n-i})$ are the restrained girth dominating set of any graph G.

Proof: For any graph $|G|$=n and if there exists a cycle $C_{n-i} \leq C_n$, $\left|C_{n-i} + \bigcup_{2}^{n-3} v_i\right|$ =n where $v_i \in V-S$ and we have S=C_{n-i} =n-v_i;S=n-(n-3), $i < n-2$ means that the graph G-v_i. There fore any v_i is non adjacent with any vertex of G. If v_i is adjacent with C_{n-i} by at least one vertex then $|N(u_i) \cap (V-S)| \geq 1$ then $\cup N(v_i) = (u_1, u_2 \ldots, u_{n-i}, v_i)$ =V that is $\cup\, N(v_i) = V$.

2.24.Theorem: Let S be any restrained girth dominating set in a graph G then their domination numbers are

(i)$\gamma_{rg}(G)$=n-2=3 if $|G|$ =$C_4 + v = 5$ (or) $|G| = C_3 + e = 5$ **(or)** |G| =K_n =$C_{n-2} + e = n$

(ii) $\gamma_{rg}(G)$ ≥n-3 if $|G|$=$K_n - 2e = (C_{n-3} + 3v) - 2e$=$K_6 - 2e = n\ (or) |G|$ =$K_n - 4e = |(C_{n-3} + 3v) - 4e|$ for n≥ 6 .

(iii) $\gamma_{rg}(G)$=(n-1)=3 if G≅ W_n , $n = 4$

(iv) $\gamma_{rg}(G)$=4 if G=$K_{m,n}$ for m,n ≥ 3 if G≅ K_n-2e =$[(C_{n-3} + 3v) - 2e]$ for n≥ 6.

(v) $\gamma_{rg}(G)$=3 if G≅ ($K_5 - ie$) for i=1,2,3 (or) G≅ $(K_5 - 4e)$ with $|M|$=1.

Proof: Given S be a restrained girth dominating set of any graph G. By lemma 2.13 ,For any graph $|G|$=n and if there exists a cycle $C_{n-i} \leq C_n$, $\left|C_{n-i} + \bigcup_{1}^{n-3} v_i\right|$ =n where $v_i \in V-S$ and we have S=C_{n-i} =n-v_i; S=n-(n-3), $i \leq n-3$ means that the graph G-v_i. There fore any v_i is non adjacent with any vertex of G. If v_i is adjacent with C_{n-i} by atleast one vertex then $|N(u_i) \cap (V-S)| \geq 1$ then $\cup N(v_i) = (u_1, u_2 \ldots, u_{n-i})$ that is, $\cup N(v_i) = V$ and By lemma 2.16 Given G be a connected graph , By lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is a restrained girth dominated which gives $|N(u_i) \cap (V-S)| = 1$ and $N(v_i)$ = u_i Where $u_i \in C_{ni}$ and $v_i \in$ V-S, Hence $\bigcup_{i=1}^{n} N(v_i)$=$(u_i, v_i) = (S, V-S) = V$.

Case(i) For any graph $|G| = C_4 + v = 5$ (or) $|G| = C_3 + e = 5$.If $\mathrm{N}(v_i) = (u_i, v_i)$ and $\cup N(v_i) = (C_3, v_i)$ where $u_i \in S$and $v_i \in V - S$ has a restrained girth dominating set of G with $\gamma_{rg}(G)$=n-2=3 for n= 5 if Max{d(u_i, u_j)} ≥n-3,$i \neq j$ where $u_i \in C_3$.

Subcase(a) Suppose for any graph |G| $= K_n = C_{n-2} + e = n$, $C_{n-2} = n - e$ and $\cup N(v_i) = (C_{n-2}, v_i)$ has girth dominating set of G with $\gamma_{rg}(G)$ ≥n-2 for n ≥ 5 for every $v_i \in V - S$. Since if Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_g(G)$=4,Similarly we can have the restrained girth dominating set with $\gamma_{rg}(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k + 2$, for every $v_i \in V - S$.

Case(ii) For any graph $|G|$=$K_n - 2e = (C_{n-3} + 3v) - 2e$=$K_6 - 2e = n$, $(C_{n-3} + 3v) = n$ and $C_{n-3} = n - 3$ with $\cup N(v_i) = (C_{n-3}, v_i)$ has restrained girth dominating set of G with $\gamma_{rg}(G)$ ≥n-3 for n≥ 6 for every $v_i \in V - S$. Since if Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$ =3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_{rg}(G)$=4,

Similarly we can have the restrained girth dominating set with $\gamma_{rg}(G)$=k if Max{d(u_i, u_j)} = k-1 and $|\cup N(v_i)| \geq k + 3$, for every $v_i \in V - S$.

Subcase(a) Suppose for any graph $|G|$=$K_n - 4e = |(C_{n-3} + 3v) - 4e|$=$K_6 - 4e = n$, $(C_{n-3} + 3v) - 4e = n$ and $C_{n-3} = n - 3$ with $\cup N(v_i) = (C_{n-3}, v_i)$ has a restrained girth dominating set of G with $\gamma_{rg}(G)$ ≥n-3 for n≥ 6 for every $v_i \in V - S$.

Since if Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_{rg}(G)$=4,

Similarly we can have the restrained girth dominating set with $\gamma_{rg}(G)$=k if Max{d(u_i, u_j)} = k-1 and $|\cup N(v_i)| \geq k + 3$ for every $v_i \in V - S$.

Case (iii) For any wheel graph G= W_n , n = 4 is a restrained girth dominating set with $\gamma_{rg}(G)$ =3 with $\cup N(v_i) = (C_{n-1}, v_i)$ since Max{d(u_i, u_j)} = 2 then we can have the restrained girth dominating set and its $\gamma_{rg}(G)$=(n-1)=3 and $|\cup N(v_i)| = 4$ for every $v_i \in V - S$

Case(iv) For any complete bipartite graph G=$K_{m,n}$ is a restrained girth dominating set and its restrained girth domination number $\gamma_{rg}(G)$=4 for m,n ≥ 3 since Max{d(u_i, u_j)} = 3 and Min{d(u_i, u_j)} = 1 and $|\cup N(v_i)| \geq 6$ for every $v_i \in V - S$.

Case(v) For any graph G=$K_5 - ie$; i=1,2,3 is a restrained girth dominating set and its restrained girth domination number $\gamma_{rg}(G)$=3with $\cup N(v_i) = (C_3, v_i)$

Subcase(a) For any graph G= $K_5 - 4e$ is a restrained girth dominating set and its restrained girth domination number $\gamma_{rg}(G)$=3 with $|M|$=1 then we have $|N(u_i) \cap (V - S)| = 1$,i≠1 and $\cup N(v_i) = (C_3, v_i)$.

2.25.Theorem: If $\cup N(v_i) = (u_i, v_i)$ and $|\cup N(v_i)| \geq 3$ then $\cup u_i$ are the restrained girth dominating set of any graph G.

Proof: Given G be any graph and we have in $|G| = \left|C_{n-k} + \cup_{i=2}^{k} v_i\right|$=n then S=$C_{n-k} = n - \cup_{i=1}^{k} v_i$;n≥ $k + 3$ and S=n-k, S ≥ $k + 3 - k$, S ≥ 3 and if $\cup_{i=1}^{k} v_i$ is non adjacent with any vertices of C_{n-k} then we have $|N(u_i) \cap (V - S)| \neq n - k$ .Hence N(v_i) must have adjacent with C_{n-k} by atleast one vertex also another vertex of v_i which gives $|N(u_i) \cap (V - S)| = 1$ and N(v_i)=(u_i, v_i) and already we have S≥ 3, There fore we have $\cup N(v_i) = (u_i, v_i)$ and $|\cup N(v_i)| \geq 3$ and in general we have $\cup N(v_i) = (u_1, u_2, \ldots . u_{n-i}, v_i)$, $|\cup N(v)| = n - i$ where $u_i \in C_{n-i}$, i=1,2,.... and $v_i \in V - S$. Hence $\gamma_{rg}(G)$ ≥n-i and Max{d(u_i, u_j)} ≥n-(i+1),$i \neq j$.

2.26.Theorem : If $|\cup N(v_i)| \geq k$ then $\cup_{i=1}^{k} u_i$ are the restrained girth dominating set of G, Then any two vertices of C_n and Max{d(u_i, u_j)} = $k - 1$, k ≥ 3 that is i,j are non adjacent and d(u_i, u_j) ≠ 1.

Proof: Given $|\cup N(v_i)| \geq k$ then $\cup_{i=1}^{k} u_i$ are the restrained girth dominating set of G and we have $|G| = |C_n + \cup_{i=1}^{r} v_i|$; n ≥ r+k and k ≥ 3 we have S=n-r which implies that S≥ $r + k - r$, that is S≥ k and if $\cup_{i=1}^{r} v_i$ is non adjacent with any vertices of C_n then we have $|N(u_i) \cap (V - S)| \neq n - r$.

Hence N(v_i) must have adjacent with C_{n-k} by atleast one vertex that is $|N(u_i) \cap (V - S)| = 1$ and $|\cup N(v_i)| \geq k$ and we have S≥ k, $\cup N(v_i)$= $(\cup_{i=1}^{k} u_i, v_i)$ and we must have d(u_i,u_j) = 1, i≠j and nonadjacent if d(u_i,u_j) = 2 then we can have the restrained girth dominating set and $\gamma_{rg}(G)$=3.

If Max{d(u_i,u_j)} =3 then we can have $\gamma_{rg}(G)$=4, Similarly we can have the restrained girth dominating set with $\gamma_{rg}(G)$=k if Max{d(u_i,u_j)} =k-1 and $|\cup N(v_i)| \geq k$, for every $v_i \in V - S$.

2.27.Theorem : Every Connected graph is of restrained girth dominating set $C_3$with $|M| \leq 3$.

Proof: Let C_n be the girth subgraph of G and S = C_n with V-S=G-S. If $|\cup N(v - S)| = 3$ and its $\cup N(v - S)$=(S,V-S)=V then C_n is the restrained girth dominating set of G with n≥ 3. If d(u_i,u_j)=1 where $u_i \in C_n$ then C_n is the girth subgraph of G . If d(u_i,u_j)≥2 then add a chord to the subgraph which gives d(u_i,u_j)=1 and $|\cup N(v - S)| = 3$ and $\cup N(v - S)$=(S,V-S)=V then it is the restrained girth dominating set of G. If N(V-S)= (C_3,$V - S$) and if $|\cup N(v - S)| < 2$ add one edge of matching with any one vertex of C_3 again if $|\cup N(v - S)| < 2$ add another vertex of C_3 ,Since the restrained girth graph is of cycle 3 we can add maximum 3 matchings then it becomes restrained girth dominating set of G , That is $|M| \leq 3$. Otherwise the graph has no restrained girth dominating set.

2.28.Corollary: Every connected graph is of restrained girth dominating set C_n with $|M| \leq n$; n≥ 3.

Proof is obvious from Theorem 2.26 and 2.27.

2.29.Theorem: Every Corona graph of a restrained girth graph G is not a restrained girth dominating set, that is $|\cup N(H_v)| \neq n - k$.

Proof: Let S be a restrained girth graph of any number of vertices of G and S be the set of n-k vertices of a restrained girth cycle then $|N(u_i) \cap N(H_v)| \neq n - k$ and V-S=G-S∪ H_v.Hence we have N(H_v) ≠ $(S, V - S)$ also ∪ N(H_v) ≠ V which implies that $|\cup N(H_v)| \neq (u_1, u_2, \ldots . u_{n-k}) = n - k$.

Hence for any graph $|G|$=n and if there exists a cycle $C_{n-2} \leq C_{n-1}$ that is $C_{n-i} + \cup_{i=k}^{n-3} iv$=n we have S=$C_{n-k} = n - iv$,i=2.....,k and k≥ 2 means that the graph is G-iv and the vertex v is non adjacent with any vertex of G then it gives $|N(u_i) \cap (V - S)| = 1$ and $|\cup N(v_i)| \geq k$ and we have S≥ k, $\cup N(v_i) \neq (\cup_{i=1}^{k} u_i, v_i)$.

Hence every vertex of V-S is non adjacent to at least one vertex of S that is , $|\cup N(H_v)| = (u_1, u_2, \ldots . . u_{n-k}, v_i) \neq n - k$ and $|N(u_i) \cap (N(H_v)| \neq n - k$.

Hence it is not a restrained girth dominating set of G by the definition restrained girth domination but if S=G then $|N(u_i) \cap (V - S)| = 1$ and $\cup N(v_i) \neq (\cup_{i=1}^{k} u_i, v_i)$ which gives the restrained girth dominating set G.

2.30.Lemma : Let G be any connected graph with a restrained girth dominating set then $\cup N(S) = (S, V - S)$

Proof: For any graph G .Let S be a restrained girth dominating set of G and V-S ={v_i} for i=1,2,...n. we have given G be a connected graph by lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is a restrained girth dominated ,which gives $\cup N(v_i) = (\cup_{i=1}^{k} u_i, v_i)$ where $u_i \in C_{ni}$, i= 1,2,..n and we have N(u_i)=(u_i, v_i) and ∪N(u_i)=($S, V - S$) and $|N(u_i) \cap (V - S)| \geq 1$ where v_i ∈V-S.

2.31.Theorem : Let S_1 and S_2 be any two restrained girth dominating set of same order in G_1 and G_2 respectively then $[\cup N(S_1) \cdot \cup N(S_2)] = \cup N(S) = [S, V - S]$

Proof: By lemma 2.30 , For any graph G .Let S be a restrained girth dominating set of G and V-S ={v_i} for i=1,2,...n. we have given G be a connected graph by lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is a restrained girth dominated which gives $\cup N(v_i) = (\cup_{i=1}^{k} u_i, v_i)$ where $u_i \in C_{ni}$, i= 1,2,..n and we have N(u_i)=(u_i, v_i) and ∪N(u_i)=($S, V - S$) and $|N(u_i) \cap (V - S)| \geq 1$ where v_i ∈V-S.

Let G_1 and G_2 be any two graphs of n_1 and n_2 vertices and its restrained girth dominating sets are S_1 and S_2 respectively. If we have $\cup N(S_1) = (S_1, V - S_1)$ and $\cup N(S_2) = (S_2, V - S_2)$ and its $[\cup N(S_1) \cdot \cup N(S_2)] = [S_1 \cdot S_2, (V_1 - S_1) \cdot (V_2 - S_2)]$=[S, $V_1 \cdot V_2 - S$] where $S_1.S_2$ =S and $v_1 \cdot v_2$ =V which gives $\cup N(S) = [S, V - S]$.

III.Conclusion:

In this paper we found an upper bound for the Restrained girth domination number and relationship between Restrained girth domination numbers of graphs and characterized the corresponding extremal graphs. Similarly Restrained girth domination numbers with other graph theoretical parameters can be considered.

References

[1] R.B. Allan and R. Laskar, *On domination and independent domination numbers of a graph*, Discrete Mathematics, Vol. 23, No. 2, 73-76, 1978.
[2] I.S. Aniversario, F.P. Jamil and S.R. Canoy Jr., *The closed geodetic numbers of graphs*, Utilitas Mathematica, Vol. 74, pp. 3-18, 2007.
[3] C. Berge, *theory of Graphs and its Applications*, Methuen, London, 1962.
[4] F. Buckley, F. Harary. *Distance in graphs*. Redwood City. CA: Addition-Wesley. 1990.
[5] W. Duckworth and N. C. Wormald, *On the independent domination number of random regular graphs*, Combinatorics, Probabilty and Computing, Vol. 15, 4, 2006.
[6] T. Haynes, S. Hedetniemi and M. Henning , *Domination in graphs applied to electrical power networks*, J. Discrete Math. 15(4), 519-529, 2000.
[7] T.W. Hanes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs*. Marcel Dekker, Inc. New York (1998).
[8] O. Ore, *Theory of graphs* , Amer. Math. Soc. Colloq. Publ., Vol.38, Providence, 1962.
[9] L.Sun and J. Wang, *An upper bound for the independent domination number*, Journal of Combinatorial Theory, Vol.76, 2. 240-246, 1999.
[10] H. Walikar, B. Acharya and E. Sampathkumar, *Recent developments in the theory of domination in graphs*, Allahabad, 1, 1979.
[11] Teresa L. Tacbobo and Ferdinand P.Jamil, *Closed Domination in Graphs*, International Mathematical Forum, Vol. 7, 2012, No. 51, 2509-2518.
[12] T.R.Nirmala Vasantha, *A study on Restrained Domination number of a graph*, Thesis 2007, M.S.U, Tirunelveli.
[13] B.D.Acharya, H.B.Walikar, and E.Sampathkumar, *Recent developments in the theory of domination in graphs*. In MRI Lecture Notes in Math. Mehta Research Instit, Allahabad No.1, (1979).
[14] A.Nellai Murugan and G.Victor Emmanuel, *Degree Equitable Domination Number and Independent Domination Number of a Graph ,International Journal of Innovative Research in Science,Engineering and Technology(An ISO 3297:2007 Certified organization), Vol.2,Issue 11, November 2013*

[15] A.Nellai Murugan and G.Victor Emmanuel, *Complete Dominating Number Of Graphs, Indian Journal Of Applied Research*, *Volume : 4 , Issue : 1 , Jan 2014 , ISSN - 2249-555X.*

[16] J. A.Bondy and U. S. R. Murty, Graph Theory, Springer, 2008.
[17] F .Harary, Graph Theory, Addison-Wisley Publishing Company, Inc
[18] Shuya Chiba, Michitaka and Shoichi Tsuchi, *Forbidden Pairs and the Existence of a Dominating Cycle*, *Discrete Mathematics,Vol:338,Issue:12,2442-2452,Dec 2015.*

[19]Nellai Murugan. A and Victor Emmanuel.G, Generalised Girth Domination Number of Graphs, Inter national Journal of Mathematics and Computer research ,issue 06 (2016) 1404--1409.

[20] Nellai Murugan.A, Victor Emmanuel.G, Irredundant Complete Domination Number of Graphs, International Journal of Emerging Trends in Science and Technology}, 03 (2016) 3638--3646.

[21]Nellai Murugan.A and Victor Emmanuel.G, Generalised Girth two Domination Number of Graphs, paper presented in Inter national Conference on Discrete Computational Mathematics, (February 16-18,2017) and communicated.

[22] Nellai Murugan.A and G.Victor Emmanuel.G, Irredundant n-complete Domination Number of Graphs, paper presented in National Conference On Recent Advances In Fuzzy Mathematics And Applications 9th ,10th February 2017 and Communicated.

[23] Nellai Murugan.A and Victor Emmanuel.G, Restrained Irredundant complete domination number of graphs, Outreach Journal ,VOC College,Volume IX,(2017) .