MODELLING OF MEMS INERTIAL SENSORS AND ACCURACY IMPROVEMENT USING EXTENDED KALMAN FILTER

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Abstract *— Inertial navigation uses gyroscopes and accelerometers to maintain an estimate of the position, velocity, attitude and attitude rates of the aircraft. Gyroscopes are used in various applications to sense either the angle turned through by a vehicle or structure (displacement gyroscopes) or its angular rate of turn about some defined axis (rate gyroscopes). Micro Electro Mechanical System (MEMS) devices are one of the most exciting developments in inertial sensors*

Micro inertial sensors, such as MEMS gyroscopes, can provide small, inexpensive, low power devices; however, the accuracy of these devices is insufficient for many space applications. The aim of this project is to develop a mathematical model of the vibratory MEMS gyroscope and to derive the governing equations of motion of these systems using the Newton-Euler approach. The individual outputs of many nominally identical micro sensors can be combined to generate an arithmetic average of the sensor measurements and then processed this average in an appropriate Extended Kalman Filter (EKF). The estimates are generally much more accurate than the measurements taken directly from the sensors.

Keywords—*MEMS, EKF, Gyroscope, Accelerometers*

1. INTRODUCTION

Navigation, guidance, and control systems for small space vehicles, such as satellites, require compact inertial measurement sensors to provide accurate position, velocity, and angular information about the vehicle. Recent developments in Micro Electro Mechanical Systems technology promise sufficiently small inertial sensors, but these sensors are not sufficiently accurate for many space applications. On the other hand, the cost of these sensors promises to be quite inexpensive; therefore, one method of improving sensor accuracy is by using many sensors to make measurements of the same quantity and then combining these measurements to generate one accurate measurement. Note that the use of many inexpensive micro-sensors measuring the same quantity can also provide reliability, through redundancy, at a reasonable cost. The specific problem discussed here is that of combining the outputs of many micro-sensors, all measuring the same quantity, so that the accuracy of the combination greatly exceeds the accuracy of the individual micro-sensors.

2. MEMS Sensors

New applications that have demanded low-cost sensors for providing measurements of acceleration and angular motion have provided a major incentive for the development of micro-machined electromechanical system (MEMS) sensors. MEMS devices are one of the most exciting developments in inertial sensors in the last 30 years. These devices overcome many of the features that have impeded the adoption of inertial systems by many potential applications, especially where cost, size and power consumption have been governing parameters.

3. MEMS VIBRATORY GYROSCOPE

MEMS Gyroscopes are vibratory rate gyroscopes, which have no rotating parts that require bearings, and hence they can be easily miniaturized and batch fabricated using micromachining techniques. These structures fabricated on polysilicon or crystal silicon, and their main mechanical component is a two degree of freedom vibrating proof mass, which is capable of oscillating on two directions in a plane. Their operation is based on the Coriolis effect. When the gyroscope is subjected to an angular velocity along an axis (input axis) orthogonal to the axis of initial oscillation (driven axis), the Coriolis effect transfers energy from one vibrating mode to another. The response of the second vibrating mode, which is along a third axis (sense axis) orthogonal to the previous two, provides information about the applied angular velocity.

4. EXTENDED KALMAN FILTER

The Kalman filter addresses the general problem of trying to estimate the state of a discrete-time controlled process that is governed by a linear stochastic difference equation. A Kalman filter that linearizes about the current mean and covariance is referred to as an extended Kalman filter or EKF. The extended Kalman filter (EKF) has become a standard technique used in a number of nonlinear estimation and machine learning applications. These include estimating the state of a nonlinear dynamic system, estimating parameters for nonlinear system identification, where both states and parameters are estimated simultaneously.

5. CONFIGURATION OF MEMS GYROSCOPES

The simplest model for a vibratory rate Gyroscope is illustrated in Fig 1. The system consists of a proof mass suspended by a set of springs. The springs allow deflection in two orthogonal directions or modes. The mode along the X-axis is driven into oscillation and the second mode along the Y-axis is excited by Coriolis acceleration. The Coriolis acceleration results from the oscillatory motion in the driven mode and the rotation rate about the Z-axis.

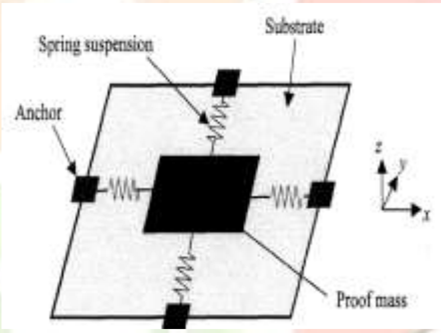

Fig. 1: Model of a Vibratory MEMS gyroscope

6. DYNAMICS OF MEMS GYROSCOPES

Here provided a vectorial derivation of the equations of motion characterizing the dynamic behavior of vibratory micro machined gyroscopes. The derivation presented here utilizes the Newton-Euler approach. The configuration of a vibratory rate Gyroscope is illustrated in figure 2.

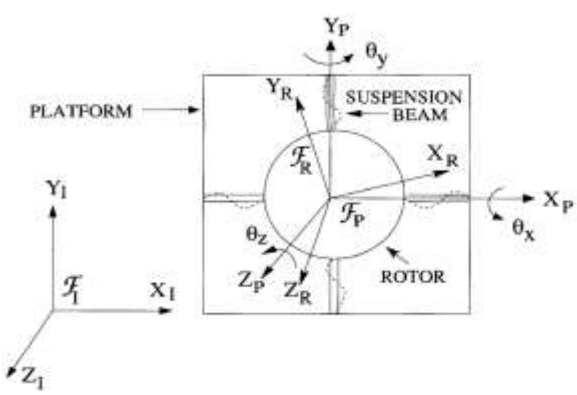

Fig.2: Angular Vibratory Rate Gyro

The rotor is angularly oscillated about the X_p If the spacecraft is rotating about the y,-axis (input-axis) with some Here provided a vectorial derivation of the equations of motion characterizing the dynamic behavior of vibratory micro machined gyroscopes. The derivation presented here utilizes the Newton-Euler approach.

The rotor is angularly oscillated about the X_p If the spacecraft is rotating about the y,-axis (input-axis) with some nonzero Z_p axis resulting in an oscillating angular momentum vector angular rate Ω_z, (i.e., the rate to be sensed), an oscillating Coriolis moment is developed across the transverse Y_p axis (sense axis).

7. VECTORIAL EQUATIONS OF MOTION

The following reference frames shown in Figure 2 will play an important role:

i. F_I - a suitable inertial frame of reference.

ii. F_P - The platform reference frame rigidly attached at the center of mass of the platform and rotating with the platform (the platform frame can be taken equal to the spacecraft's body fixed frame FB).

iii. F_R - The rotor frame rigidly attached at the mass center of the rotor and rotating with the same angular motion of the rotor.

Applying the balance of angular momentum about the center of mass of the system,

$$\sum M = \frac{dH}{dt}$$

$$H = J_R \cdot {}^I\omega^R$$

Where , H denotes the total angular momentum of the system about the system center-of-mass.

J_R denotes the inertia of the rotor about its center of mass,

${}^I\omega^R$ is the angular velocity of the rotor relative to F_I.

The time rate of change of a vector Q as seen by an observer rigidly fixed to F_A is related to the time rate of change of the same vector as seen by an observer rigidly fixed to F_B via the transport theorem.

$$\frac{{}^A dQ}{dt} = \frac{{}^B dQ}{dt} + {}^A\omega^B \times Q$$

Applying the transport theorem to the balance of angular momentum.

$$\frac{{}^I dH}{dt} = \frac{{}^P dH}{dt} + {}^I\omega^P \times H$$

Where $\frac{{}^P dH}{dt} = \frac{{}^R dH}{dt} + {}^P\omega^R \times H$ $\qquad \frac{{}^I dH}{dt} = \frac{{}^R dH}{dt} + ({}^I\omega^P + {}^P\omega^R) \times H$

$$= \frac{{}^R dH}{dt} + ({}^I\omega^P + {}^P\omega^R) \times H$$

This equation can be further simplified as follows

$$\frac{{}^R dH}{dt} = \frac{{}^R d}{dt}(J_R \cdot {}^I\omega^R)$$

$$= \left(\frac{{}^R dJ_R}{dt}\right) \cdot {}^I\omega^R + J_R \cdot \left(\frac{{}^R d\ {}^I\omega^R}{dt}\right)$$

However, since the rotor is assumed rigid (i.e., does not suffer any deformation) and F_R is rigidly attached to the rotor structure,

$$\left(\frac{{}^{R}dJ_R}{dt}\right)=0$$

The resulting vectorial of motion are expressed concisely as,

$$\sum M=J_R.\left(\frac{{}^{R}d\ {}^{I}\omega^{R}}{dt}\right)+{}^{I}\omega^{R}\times J_R.{}^{I}\omega^{R} \ldots\ldots.(i)$$

It is also important to observe from applying the transport theorem to ${}^{I}\omega^{R}$ that

$$\frac{{}^{I}d\ {}^{I}\omega^{R}}{dt}=\frac{{}^{R}d\ {}^{I}\omega^{R}}{dt}+{}^{I}\omega^{R}\times{}^{I}\omega^{R} \ldots\ldots\ldots(ii)$$
$$=\frac{{}^{R}d\ {}^{I}\omega^{R}}{dt}$$

8. REPRESENTATION OF THE EQUATIONS OF MOTION IN ROTOR FRAME

The vectorial equation (i) along with the vectorial statement time rate of change (ii) provide a concise and exact characterization of the non-linear equations of motion of the micro gyroscope. As a first step in expressing the equations of motion in component form resolved in F_R we must describe the orientation of F_R relative to F_P. $C:F_p\rightarrow F_R$ describing the attitude of F_R relative to F_P is parameterized via Euler angles. Consider the following Euler sequence:

$$C=C_z(\theta_z)C_y(\theta_y)C_x(\theta_x)$$

where the principal rotation matrices are defined by

$$c_x(\theta_x)=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\theta_x) & \sin(\theta_x)\\ 0 & -\sin(\theta_x) & \cos(\theta_x)\end{pmatrix}$$
$$c_y(\theta_y)=\begin{pmatrix}\cos(\theta_y) & 0 & -\sin(\theta_y)\\ 0 & 1 & 0\\ \sin(\theta_y) & 0 & \cos(\theta_y)\end{pmatrix}$$
$$c_z(\theta_z)=\begin{pmatrix}\cos(\theta_z) & \sin(\theta_z) & 0\\ -\sin(\theta_z) & \cos(\theta_z) & 0\\ 0 & 0 & 1\end{pmatrix}$$

The Euler angles ($\theta_x,\theta_y,\theta_z$) correspond physically to small rotations about the drive axis ,sense axis and input-axis respectively. The principle rotation matrices yields,

$$c=\begin{pmatrix}c\theta_y c\theta_z & s\theta_x s\theta_y c\theta_z+c\theta_x s\theta_z & -c\theta_x s\theta_y c\theta_z+s\theta_x s\theta_z\\ -c\theta_y s\theta_z & -s\theta_x s\theta_y s\theta_z+c\theta_x c\theta_z & c\theta_x s\theta_y s\theta_z+s\theta_x c\theta_z\\ s\theta_y & -s\theta_x c\theta_y & c\theta_x c\theta_y\end{pmatrix}$$

Once the rotation matrix $C:F_p\rightarrow F_R$ is known, the angular velocity of F_R relative to F_P expressed in F_R is given by

$$\left[{}^{P}\omega^{R}\right]=-\dot{c}\,c^{T}$$

$$=-\dot{c}_z c_z^T - c_z(\dot{c}_y c_y^T)c_z^T - c_z c_y(\dot{c}_x c_x^T)c_y^T c_z^T$$

Where $[\omega] = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$

Denoting

$$[\omega_x] = -\dot{c}_x c^T{}_x$$

$$[\omega_y] = -\dot{c}_y c^T{}_y$$

$$[\omega_z] = -\dot{c}_z c^T{}_z$$

$$c[\omega\ ]c^T = [c\omega]$$

We get

$$^P\omega^R = \omega_z + c_z\,\omega_y + (c_z\,c_y)\omega_x$$

A straightforward calculation results in,

$$\omega_x = \left[\dot{\theta}_x, 0, 0\right]^T$$

$$\omega_y = \left[0, \dot{\theta}_y, 0\right]^T$$

$$\omega_z = \left[0, 0, \dot{\theta}_z\right]^T$$

Combining the above equations we get,

$$^P\omega^R = \begin{bmatrix} \dot{\theta}_x\, c\theta_y c\theta_z + \dot{\theta}_y\, s\theta_z \\ -\dot{\theta}_x\, c\theta_y s\theta_z + \dot{\theta}_y\, c\theta_z \\ \dot{\theta}_z + \dot{\theta}_x\, s\theta_y \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_y c\theta_z & s\theta_z & 0 \\ -c\theta_y s\theta_z & c\theta_z & 0 \\ s\theta_y & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{bmatrix}$$

$$= S(\Theta)\dot{\Theta}$$

Where

$$\Theta = \left[\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z\right]^T$$

The angular velocity of the platform F_P relative to F_I expressed in F_P given by

$$^{I}\omega^{P}=\left[\Omega_x,\Omega_y,\Omega_z\right]^T$$

Assuming that F_R is aligned with the principle axes of the rotor. As a result the inertia matrix of the rotor J is diagonal, $J=Diag\left[J_{xx},J_{yy},J_{zz}\right]$

Euler's equations take the form,

$$M_x=J_{xx}\dot{\omega}_x+(J_{zz}-J_{yy})\omega_y\omega_z$$

$$M_y=J_{yy}\dot{\omega}_y+(J_{xx}-J_{zz})\omega_x\omega_z$$

$$M_z=J_{xx}\dot{\omega}_z+(J_{yy}-J_{xx})\omega_x\omega_z$$

Assume that the rotor is symmetric about the x and y axes so that $J_{xx}=J_{yy}$, then the resultant moment about the center of mass of the system can be expressed in F_P as

$$\sum M=\begin{bmatrix}-k_{xx}\theta_x-c_{xx}\dot{\theta}_x+M^e{}_x\\ -k_{yy}\theta_y-c_{yy}\dot{\theta}_y+M^e{}_y\\ -k_{zz}\theta_z-c_{zz}\dot{\theta}_z+M^e{}_z\end{bmatrix}$$

The restoring torques due to stiffness and damping along axis $i=(x,y,z)$ of F_P are denoted as $-k_{ii}\theta_i$ and $-c_{ii}\dot{\theta}_i$ respectively, and the components of the external moment acting about axis $i=(x,y,z)$ are denoted $M^e{}_i$.

Simplifying the above equations using equations neglecting all cross-axis coupling and angular acceleration effects the equations of motion are approximated by the following linear model.

$$J_{xx}\ddot{\theta}_x+c_{xx}\dot{\theta}_x+k_{xx}\theta_x=-J_{zz}\dot{\theta}_z\Omega_y+M^e{}_x$$

$$J_{yy}\ddot{\theta}_y+c_{yy}\dot{\theta}_y+k_{yy}\theta_y=J_{zz}\dot{\theta}_z\Omega_x+M^e{}_y$$

$$J_{zz}\ddot{\theta}_z+c_{zz}\dot{\theta}_z+k_{zz}\theta_z=M^e{}_z$$

Dividing each equation (2.29) by the inertia about each respective axis transforms the dynamics to standard second order form,

$$\ddot{\theta}_x+\frac{\omega_{xn}}{Q_x}\dot{\theta}_x+\omega_{xn}^2\theta_x=-\frac{J_{zz}}{J_{xx}}\dot{\theta}_z\Omega_y+\frac{M^e{}_z}{J_{xx}}$$

$$\ddot{\theta}_y+\frac{\omega_{yn}}{Q_y}\dot{\theta}_y+\omega_{yn}^2\theta_y=\frac{J_{zz}}{J_{yy}}\dot{\theta}_z\Omega_x+\frac{M^e{}_y}{J_{yy}}$$

$$\ddot{\theta}_z+\frac{\omega_{zn}}{Q_z}\dot{\theta}_z+\omega_{zn}^2\theta_z=\frac{M^e_z}{J_{zz}}$$

Where $Q_i=\frac{1}{2\zeta_i}$ are the x,y and z-axes quality factors and $\omega_{in}=\sqrt{\frac{k_{ii}}{J_{ii}}}$, ζ_i denotes the damping ratio associated with axis $i=(x,y,z)$ .and k denotes spring constant.

This gives $\omega_{xn}=\sqrt{k_{xx}/m\omega_0^2}$ and $\omega_{yn}=\sqrt{k_{yy}/m\omega_0^2}$

Where m is the proof mass and ω_0 is the natural resonant frequency.

Generally, sense axis is under force to balance control so the control signal can be used to measure the desired rotation rate. therefore, we can assume that $\theta_y=\dot{\theta}_y=0$ and simplifying (2.30) yielding system transfer function for the driven axis.

$$G(s)=\frac{1/m}{S^2+2\zeta\omega_n S+\omega_n^2}$$

Where $\zeta=\frac{1}{2Q_x}$ and $\omega_n=\omega_x$

9. DEVELOPMENT OF STATE EQUATIONS

A sinusoidal torque, T(t), is applied to the plate about the *x*-axis producing a periodic motion about the *x*-axis. The other torques about the *x*-axis are a damping torque and a spring torque, both due to the mechanical properties of the supporting wires along the *x*-axis. The applied torque about the *y*-axis is a damping torque and a spring torque due to the supporting wires. The damping coefficients and spring constants are assumed to be equal about the *x*-axis and the *y*-axis because of the symmetry of the plate in the *x* − *y* plane. When the housing is rotated at an angular rate of Ω_z rad/sec about the *z*-axis, a torque is transmitted to the plate by the support wires and consists of a damping torque and a spring torque given by

$$C_Z(\Omega_Z-\dot{\theta}_Z)$$

$$K_Z(\theta\int_0^t \Omega_Z(t')dt'-\theta_Z)$$

where C_Z and K_Z are the damping coefficient and the spring constant about the *z*-axis. The input rate Ω_Z induces a motion in the *y*-axis and Ω_Z is determined from the measurement of the angular motion about the *y*-axis.

10. EULER'S EQUATIONS

Based on the above conditions, simplified Euler's equations can be written as,

$$\ddot{\theta}_x+a_1\dot{\theta}_x+a_0\theta_x+A\dot{\theta}_y\dot{\theta}_z=T_0\sin\omega_0 t$$

$$\ddot{\theta}_y+a_1\dot{\theta}_y+a_0\theta_y-A\dot{\theta}_x\dot{\theta}_z=0$$

$$\ddot{\theta}_z+b_1\dot{\theta}_z+b_0\theta_z=b_1\Omega_z+b_0\int_0^t \Omega_Z(t')dt'$$

The desired output of the gyroscope is the input angular rate $\Omega_Z(t)$ This is obtained by measuring and processing the angular rate $\dot{\theta}_y(t)$.

Assuming that the rate of change of the angular rate input $\Omega_Z(t)$ is small relative to the rates of change induced by the applied torque $T_0\sin\omega_0 t$ and that the nonlinear effects in the *x* and *y* equations are relatively small, the steady-state value of the $\dot{\theta}_y(t)$ can be written as,

$$\dot{\theta}_y(t)=K\Omega_Z(t)\sin\omega_0 t$$

Thus $\dot{\theta}_y(t)$ is the sine wave sine wave amplitude-modulated by the applied angular rate $\Omega_Z(t)$. $\Omega_Z(t)$ can therefore be obtained by a simple amplitude modulation (AM) demodulator.

11. STATE EQUATIONS

$$x_1=\theta_x \qquad x_2=\dot{\theta}_x \qquad x_3=\theta_y$$

$$x_4=\dot{\theta}_y \qquad x_5=\theta_z \qquad x_6=\dot{\theta}_z$$

Resulting in the state equations,

$$\dot{x}_1=x_2$$

$$\dot{x}_2=-a_1x_2-a_0x_1-Ax_4x_6+T_0\sin\omega_0 t$$

$$\dot{x}_3=x_4$$

$$\dot{x}_4=-a_1x_4-a_0x_3+Ax_2x_6$$

$$\dot{x}_5=x_6$$

$$\dot{x}_6=x_7-b_1x_6-b_0x_5+b_1\Omega_z$$

$$\dot{x}_7=b_0\Omega_z$$

AM demodulation of the variable $x_4(t)=\dot{\theta}_y(t)$ generates the desired output $\Omega_Z(t)$. The equation defining the demodulator is,

$$\ddot{z}=-c_0z-c_1\dot{z}+c_0x_4\sin\omega_0 t$$

where the coefficients in Equation are chosen to provide low pass filtering, with a corner frequency at $\omega_0/10$ rad/sec, after the demodulation. The output measurement is the demodulator output $z(t)$. Ideally, $\Omega_Z(t)$ is given by

$$\Omega_z(t)=\frac{K}{2}z(t)$$

The design coefficients for the gyroscope simulated in this work are

$$a_1=1.8621\times10^4 \quad b_1=9.3750\times10$$

$$a_0=3.4483\times10^8 \quad b_0=3.3750\times10^8$$

$$c_1 = 1200\pi \qquad A = 4.5172$$
$$c_0 = (600\pi)^2 \quad \omega_0 = 2\pi \times 3000 rad / sec$$

ω_0 was chosen to be near the resonant frequency of the linear part of the *y*-axis equation and the magnitude T_0 of the forcing function was chosen to be

$$T_0 = 1.5 \times 10^8$$

The resulting equations of motion, using these coefficients, provide the 'truth model' of the gyroscope.

To generate a set of Kalman filter equations the unknown Ω_z cannot appear in the equations; therefore the following modification was made to the gyroscope state equations. Let:

$$x_6 = \Omega_z + n_6$$

where n_6 is the unknown difference between the input Ω_z and the state $x_6 = \Omega_z$. The revised state equations can now be rewritten:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -a_1 x_2 - a_0 x_1 - A x_4 x_6 + T_0 \sin\omega_0 t + n_2$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = -a_1 x_4 - a_0 x_3 + A x_2 x_6 + n_4$$
$$\dot{x}_5 = x_6$$
$$\dot{x}_6 = x_7 - b_0 x_5 + b_1 n_6$$
$$\dot{x}_7 = b_0 x_6 - b_0 n_6$$

The demodulator equation is rewritten in state form as

$$\dot{x}_8 = x_9$$
$$\dot{x}_9 = -c_0 x_8 - c_1 x_9 + c_0 x_4 \sin\omega_0 t + n_8$$

The measurement equation is given by

$$z = x_8 + v$$

The quantities n_2, n_4, n_6, n_8 and V are random quantities, including random noise and modeling errors, which in the development of the extended Kalman filter (EKF) are assumed to be white noises. For convenience, the gyroscope model is now rewritten in the vector-matrix form:

$$\dot{X} = FX + f_1(x) + Gn + g(X)\sin\omega_0 t$$

and

$$z = h^T X + v$$

Where

$$X=\begin{bmatrix} x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\\ x_7\\ x_8\\ x_9 \end{bmatrix} \quad g(x)=\begin{bmatrix} 0\\ T_0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ c_0 x_4 \end{bmatrix} \quad h=\begin{bmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 1\\ 0 \end{bmatrix} \quad X=\begin{bmatrix} 0\\ -Ax_4x_6\\ 0\\ Ax_2x_6\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix} \quad n=\begin{bmatrix} n_1\\ n_4\\ n_6\\ n_8 \end{bmatrix}$$

$$F=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ -a_0 & -a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -a_0 & -a_1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -b_0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & b_0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_0 & -c_1 \end{bmatrix} \quad G=\begin{bmatrix} 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -b_1 & 0\\ 0 & 0 & -b_0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12. KALMAN FILTER (EKF) EQUATIONS USED FOR GYROSCOPE MODEL

The EKF equations can be written for gyroscope model as as

$$X=f(x)+k(z-h^T X)+g(X)\sin\omega_0 t$$

where $f(x)=FX+f_1(x)$

The output of the Kalman filter, x , is the estimate of the combined state vectors of the gyroscope and the demodulator. The estimate of the applied torque $\Omega_Z(t)$ is obtained from the state estimate $x_8(t)$ as

$$\Omega_z(t)=\frac{K}{2}x_8(t)$$

The Kalman gain vector is given by k=Ph/R and the error covariance P is calculated by the equation

$$\dot{P}=\bar{F}P+P\bar{F}-\frac{Phh^T P}{R}+GQG^T$$

where $\bar{F}=F+F_1$.The matrix F is defined previously and

$$\mathbf{F}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -A\hat{x}_6 & 0 & -A\hat{x}_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A\hat{x}_6 & 0 & 0 & 0 & A\hat{x}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

13. SIMULATION RESULTS

Using values $m = 10^{-9} Kg$, $\omega_n = 1.57 \times 10^3 rad/\sec$, $\zeta = 0.5 \times 0^{-3}$ the system transfer function for the driven axis of gyroscope is obtained as

$$G(s) = \frac{10^9}{S^2 + 1.57S + 2.46 \times 10^6}$$

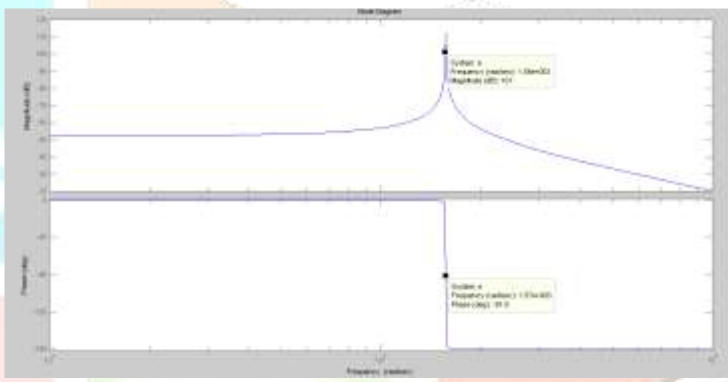

Fig :1. Bode diagram of MEMS Gyro driven axis

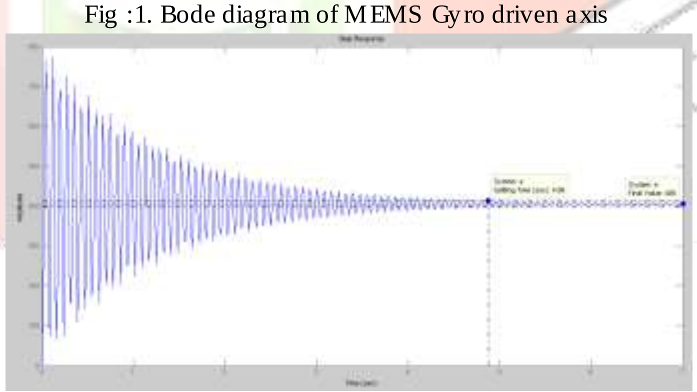

Fig:2. Step response of MEMS Gyro driven axis

Measurement noises, v_i with standard deviations of twenty per cent of the maximum value (.02) of the output signal of the truth model was used to corrupt the output of each micro gyroscope. Simulation results are shown below.

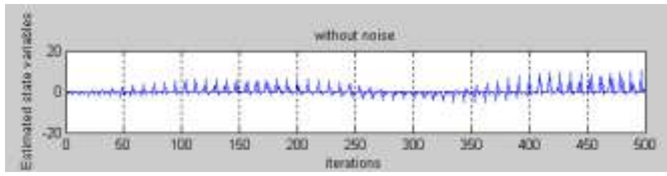

Fig 3 .Plot of estimated state variables without adding noise

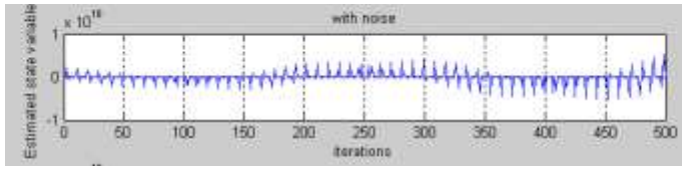

Fig 4 .Plot of estimated state variables with added noise

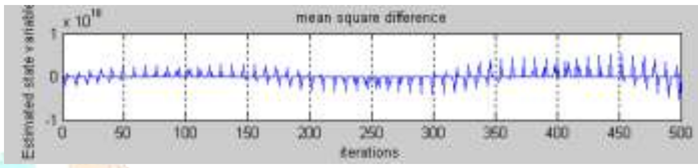

Fig 5. Plot of mean square error

14. CONCLUSION

In this paper, studied the configuration of MEMS vibratory Gyroscopes and derived the governing equations of motion of these systems using the Newton- Euler approach and modeled them based on a second order system. Because of the relatively low cost of micro-sensors, accuracy can be increased by using a large number of micro sensors to measure the same quantity and then using extended kalman filtering to combine the measurements.

15. REFERENCES

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