THEORITICAL ANALYSIS OF MATHEMATICAL MODELING IN NON-NEWTONIAN FLUID MECHANICS

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ABSTRACT

In this work, analysis the Thermodynamics and mechanics of non- Newtonian Fluids mechanics from Newtonian mechanics using different mathematical modeling, signified the natural science equations quantum mechanics, Newton's laws, Maxwell's equations and Schrödinger equation The non – Newtonian fluid classified under the operators and variables. Under the operator we use linear equations and non- linear equations. In mathematically Newton's method and Broyden's method are used for linear and non- linear. Continuum Mechanics Geometrically analysis scalar and vector quantity.

INTRODUCTION:

A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. Mathematical models are used in the natural sciences (such as physics, biology, Earth science, meteorology) and engineering disciplines (such as computer science, artificial intelligence), as well as in the social sciences (such as economics, psychology, sociology, political science). Physicists, engineers, statisticians, operations research analysts, and economists use mathematical models most extensively. A model may help to explain a system and to study the effects of different components, and to make predictions about behaviour.

SIGNIFICANCE IN THE NATURAL SCIENCES:

Mathematical models are of great importance in the natural sciences, particularly in physics. Physical theories are almost invariably expressed using mathematical models.

Throughout history, more and more accurate mathematical models have been developed. Newton's laws accurately describe many everyday phenomena, but at certain limits relativity theory and quantum mechanics must be used; even these do not apply to all situations and need further refinemen. It is possible to obtain the less accurate models in appropriate limits, for example relativistic mechanics reduces to Newtonian mechanics at speeds much less than the speed of light. Quantum mechanics reduces to classical physics when the quantum numbers are high. For example, the de Broglie wavelength of a tennis ball is insignificantly small, so classical physics is a good approximation to use in this case.

It is common to use idealized models in physics to simplify things. Massless ropes, point particles, ideal gases and the particle in a box are among the many simplified models used in physics. The laws of physics are represented with simple equations such as Newton's laws, Maxwell's equations and the Schrödinger equation. These laws are such as a basis for making mathematical models of real situations. Many real situations are very complex and thus modeled approximate on a computer, a model that is computationally feasible to compute is made from the basic laws or from approximate models made from the basic laws. For example, molecules can be modeled by molecular orbital models that are approximate solutions to the Schrödinger equation. In engineering, physics models are often made by mathematical methods such as finite element analysis.

Different mathematical models use different geometries that are not necessarily accurate descriptions of the geometry of the universe. Euclidean geometry is much used in classical physics, while special relativity and general relativity are examples of theories that use geometries which are not Euclidean.

CLASSIFICATIONS:

Mathematical models are usually composed of relationships and *variables*. Relationships can be described by *operators*, such as algebraic operators, functions, differential operators, etc. Variables are abstractions of system parameters of interest, that can be quantified. Several classification criteria can be used for mathematical models according to their structure:

- **Linear vs. nonlinear:** If all the operators in a mathematical model exhibit linearity, the resulting mathematical model is defined as linear. A model is considered to be nonlinear otherwise. The definition of linearity and nonlinearity is dependent on context, and linear models may have nonlinear expressions in them. For example, in a statistical linear model, it is assumed that a relationship is linear in the parameters, but it may be nonlinear in the predictor variables. Similarly, a differential equation is said to be linear if it can be written with linear differential operators, but it can still have nonlinear expressions in it. In a mathematical programming model, if the objective functions and constraints are represented entirely by linear equations, then the model is regarded as a linear model. If one or more of the objective functions or constraints are represented with a nonlinear equation, then the model is known as a nonlinear model.

Nonlinearity, even in fairly simple systems, is often associated with phenomena such as chaos and irreversibility. Although there are exceptions, nonlinear systems and models tend to be more difficult to study than linear ones. A common approach to nonlinear problems is linearization, but this can be problematic if one is trying to study aspects such as irreversibility, which are strongly tied to nonlinearity.

- **Static vs. dynamic:** A *dynamic* model accounts for time-dependent changes in the state of the system, while a *static* (or steady-state) model calculates the system in equilibrium, and thus is time-invariant. Dynamic models typically are represented by differential equations or difference equations.
- **Explicit vs. implicit:** If all of the input parameters of the overall model are known, and the output parameters can be calculated by a finite series of computations, the model is said to be *explicit*. But sometimes it is the *output* parameters which are known, and the corresponding inputs must be solved for by an iterative procedure, such as Newton's method (if the model is linear) or Broyden's method (if non-linear). In such a case the model is said to be *implicit*. For example, a jet engine's physical properties such as turbine and nozzle throat areas can be explicitly calculated given a design thermodynamic cycle (air and fuel flow rates, pressures, and temperatures) at a specific flight condition and power setting, but the engine's operating cycles at other flight conditions and power settings cannot be explicitly calculated from the constant physical properties.
- **Discrete vs. continuous:** A discrete model treats objects as discrete, such as the particles in a molecular model or the states in a statistical model; while a continuous model represents the objects in a continuous manner, such as the velocity field of fluid in pipe flows, temperatures and stresses in a solid, and electric field that applies continuously over the entire model due to a point charge.
- **Deterministic vs. probabilistic (stochastic):** A deterministic model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables; therefore, a deterministic model always performs the same way for a given set of initial conditions. Conversely, in a stochastic model—usually called a "statistical model"—randomness is present, and variable states are not described by unique values, but rather by probability distributions.
- **Deductive, inductive, or floating:** A deductive model is a logical structure based on a theory. An inductive model arises from empirical findings and generalization from them. The floating model rests on neither theory nor observation, but is merely the invocation of expected structure. Application of mathematics in social sciences outside of economics has been criticized for unfounded models.] Application of catastrophe theory in science has been characterized as a floating model.

APPLICATIONS:

Prehistorical times simple models such as maps and diagrams have been used.

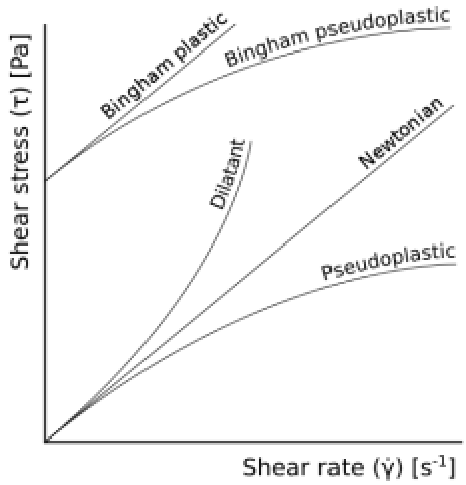
Often when engineers analyze a system to be controlled or optimized, they use a mathematical model. In analysis, engineers can build a descriptive model of the system as a hypothesis of how the system could work, or try to estimate how an unforeseeable event could affect the system. Similarly, in control of a system, engineers can try out different control approaches in simulations.

A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. Variables may be of many types; real or integer numbers, boolean values or strings, for example. The variables represent some properties of the system, for example, measured system outputs often in the form of signals, timing data, counters, and event occurrence (yes/no). The actual model is the set of functions that describe the relations between the different variables.

A **non-Newtonian fluid** is a fluid that does not follow Newton's Law of Viscosity. Most commonly, the viscosity (the measure of a fluid's ability to resist gradual deformation by shear or tensile stresses) of non-Newtonian fluids is dependent on shear rate or shear rate history. Some non-Newtonian fluids with shear-independent viscosity, however, still exhibit normal stress-differences or other non-Newtonian behavior. Many salt solutions and molten polymers are non-Newtonian fluids, as are many commonly found substances such as ketchup, custard, toothpaste, starch suspensions, maizena, paint, blood, and shampoo.

In a Newtonian fluid, the relation between the shear stress and the shear rate is linear, passing through the origin, the constant of proportionality being the coefficient of viscosity. In a non-Newtonian fluid, the relation between the shear stress and the shear rate is different. The fluid can even exhibit time-dependent viscosity. Therefore, a constant coefficient of viscosity cannot be defined.

Although the concept of viscosity is commonly used in fluid mechanics to characterize the shear properties of a fluid, it can be inadequate to describe non-Newtonian fluids. They are best studied through several other rheological properties that relate stress and strain rate tensors under many different flow conditions such as oscillatory shear or extensional flow—which are measured using different devices or rheometers. The properties are better studied using tensor valued constitutive equations.



Non Newtonian Viscosity are Stress depends on normal and shear strain rates and also the pressure applied on it like Blood plasma, custard, water.

TYPES OF NON-NEWTONIAN BEHAVIOUR:

SHEAR THICKENING FLUID:

The viscosity of a shear thickening fluid, or dilatant fluid, appears to increase when the shear rate increases. Corn starch dissolved in water is a common example: when stirred slowly it looks milky, when stirred vigorously it feels like a very viscous liquid.

SHEAR THINNING FLUID:

A familiar example of the opposite, a shear thinning fluid, or pseudoplastic fluid, is wall paint: The paint should flow readily off the brush when it is being applied to a surface but not drip excessively. Note that all thixotropic fluids are extremely shear thinning, but they are significantly time dependent, whereas the colloidal "shear thinning" fluids respond instantaneously to changes in shear rate. Thus, to avoid confusion, the latter classification is more clearly termed pseudoplastic.

Another example of a shear thinning fluid is blood. This application is highly favoured within the body, as it allows the viscosity of blood to decrease with increased shear strain rate.

BINGHAM PLASTIC:

Fluids that have a linear shear stress/shear strain relationship require a finite yield stress before they begin to flow (the plot of shear stress against shear strain does not pass through the origin). These fluids are called Bingham plastics. Several examples are clay suspensions, drilling mud, toothpaste, mayonnaise, chocolate, and mustard. The surface of a Bingham plastic can hold peaks when it is still. By contrast Newtonian fluids have flat featureless surfaces when still.

RHEOPECTIC OR ANTI-THIXOTROPIC:

There are also fluids whose strain rate is a function of time. Fluids that require a gradually increasing shear stress to maintain a constant strain rate are referred to as rheopectic. An opposite case of this is a fluid that thins out with time and requires a decreasing stress to maintain a constant strain rate (thixotropic).

EXAMPLES:

- Many common substances exhibit non-Newtonian flows. These includes Soap solutions, cosmetics and toothpaste
- Food such as butter, cheese, jam, ketchup, mayonnaise, soup, taffy, and yogurt
- Natural substances such as magma, lava, gums, and extracts such as vanilla extract
- Biological fluids such as blood, saliva, semen, mucus and synovial fluid
- Slurries such as cement slurry and paper pulp, emulsions such as mayonnaise, and some kinds of dispersions.

OOBLECK:

Oobleck on a subwoofer. Applying force to oobleck, by sound waves in this case, makes the non-Newtonian fluid thicken.An inexpensive, non-toxic example of a non-Newtonian fluid is a suspension of starch (e.g. cornstarch) in water, sometimes called "Oobleck", "ooze", or "magic mud" (1 part of water to 1.5–2 parts of corn starch). Uncooked cornflour has the same properties. The name "oobleck" is derived from the Dr. Seuss book *Bartholomew and the Oobleck*. Because of its properties, oobleck is often used in demonstrations that exhibit its unusual behavior. A person may walk on a large tub of oobleck without sinking due to its shear thickening properties, given the individual moves quickly enough to provide enough force with each step to cause the thickening. Also, if oobleck is placed on a large subwoofer driven at a sufficiently high volume, it will thicken and form standing waves in response to low frequency sound waves from the speaker.

FLUBBER:

Flubber is a non-Newtonian fluid, easily made from polyvinyl alcohol–based glues (such as white "school" glue) and borax. It flows under low stresses but breaks under higher stresses and pressures. This combination of fluid-like and solid-like properties makes it a Maxwell fluid. Its behaviour can also be described as being viscoplastic or gelatinous.

CHILLED CARAMEL TOPPING:

Another example of this is chilled caramel ice cream topping (so long as it incorporates hydrocolloids such as carrageenan and gellan gum). The sudden application of force—by stabbing the surface with a finger, for example, or rapidly inverting the container holding it—causes the fluid to behave like a solid rather than a liquid. This is the "shear thickening" property of this non-Newtonian fluid. More gentle treatment, such as slowly inserting a spoon, will leave it in its liquid state. Trying to jerk the spoon back out again, however, will trigger the return of the temporary solid state.

SILLY PUTTY:

Silly Putty is a silicone polymer based suspension which will flow, bounce, or break depending on strain rate.

PLANT RESIN:

Plant resin is a viscoelastic solid polymer. When left in a container, it will flow slowly as a liquid to conform to the contours of its container. If struck with greater force, however, it will shatter as a solid.

KETCHUP:

Ketchup is a shear thinning fluid. Shear thinning means that the fluid viscosity decreases with increasing shear stress. In other words, fluid motion is initially difficult at slow rates of deformation, but will flow more freely at high rates.

DRY GRANULAR FLOWS:

Under certain circumstances, flows of granular materials can be modelled as a continuum, for example using the $\mu(I)$ rheology. Such continuum models tend to be non-Newtonian, since the apparent viscosity of granular flows increases with pressure and decreases with shear rate.

GEOMETRY OF DEFORMATION OF CONTINUUM MECHANICS:

- Body, congurations, and motion,
- Description of motion, Lagrangian and Eulerian coordinates,
- Lagrangian and Eulerian variables ,
- Deformation gradient
- Polar decomposition of the deformation gradient,
- Measures of deformation,
- Length and angle changes,

- Surface and volume changes,
- Strain invariants, principal strains
- Displacement vector, Geometrical linearization,
- Linearized analysis of deformation,
- Length and angle changes,
- Surface and volume changes

CONCLUSION :

About the analysis the Thermodynamics and mechanics of non- Newtonian Fluids mechanics from Newtonian mechanics using different mathematical modeling, signified the natural science equations quantum mechanics, Newton's laws, Maxwell's equations and Schrödinger equation. The non -Newtonian fluid classified under the operators and variables. Under the operator we use linear equations and non- linear equations. In mathematically Newton's method and Broyden's method are used for linear and non- linear. Continuum Mechanics Geometrically analysis scalar and vector quantity are size, shape,

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