TWO-SIDED CIRCULAR NEIGHBOR BALANCED BLOCK DESIGN WITH UNEQUAL BLOCK SIZE OF COMETITION EFFECTS

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Abstract: The determination of this paper is to present a method of constructing equireplicate and unequal block size of one dimensional circular neighbour block design of competition effects. The designs are established to be variance balanced for the estimation of direct effect and neighbor effects of treatments.

Keywords Neighbour balanced design, Variance balanced design, Equireplicate and Unequal block size, Neighbour effects.

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1. Introduction

An equireplicate block design with parameters $v, b, r, k_1, ..., k_b$ is said to be balanced (variance balanced) if any elementary treatment contrasts are estimated with the same variance. In this case by use of the C-matrix of an incomplete block design it is known Kageyama(1988) . A block design is said to be balanced if every elementary contrast of treatment effects is estimated with the same variance. In this sense, the design is also called a variance balanced design.

It is well known that a block design is a VB if and only if it has

$$C = \eta\left[I_v - \frac{1}{v}1_v1'_v\right]$$

Where η is the unique non-zero eigenvalue of the C-matrix with multiplicity v-1, 1_v is the $v \times v$ identity matrix. Kageyama and Tsuji (1979) In the particular case, when the block design is equireplicated, then

$$\eta = \frac{vr - b}{v - 1}$$

Bailey (2003) has been deliberated design for one-sided neighbour effects and perfect Mendelsohn designs are presented table in different block sizes. Ahmed and Akhtar(2011) are given by several series of neighbour balanced designs are considered in circular blocks of six units. Azais et.al(1993) are explained in the construction of complete and incomplete block of initial design. Jaggi et.al (2017) are proposed in unequal block size of circular neighbour block design.

Let v be the number of treatments whose effects are to be studied. Under the block design set-up with b blocks of size $k_j\,(j=1,2,...,b)$, the following model is considered for analyzing a design with neighbour effects from immediate left and right neighbour plots:

$$Y = \mu 1 + E'\xi + E_1'\psi + E_2'\omega + H_1'\alpha + e \qquad (1.1)$$

Where Y is a $n\times 1$ vector of observations, 1 is a $n\times 1$ vector of ones, E' is a $n\times v$ incidence matrix of observations versus direct treatments, ξ is the $v\times 1$ vector of direct treatment effects, E_1' is a $n\times v$ matrix of observations versus left neighbor treatment, ψ is a $v\times 1$ vector of left neighbor effects, E_2' is a $n\times v$ matrix of observations versus right neighbor treatment , ω is $v\times 1$ vector of right neighbor effects, H' is a $n\times b$ incidence matrix of observations versus blocks, α is a $v\times 1$ vector of right neighbor effects and e is a $n\times 1$ vector of errors.

Further, let L_1 be a $v\times v$ incidence matrix of direct versus left neighbor treatments, L_2 be a $v\times v$ incidence matrix of direct versus right neighbor treatments, L_3 be a $v\times b$ incidence matrix of direct treatment versus blocks, L_4 be a $v\times v$ incidence matrix of left versus right neighbor treatments, L_5 be a $v\times b$ incidence matrix of left neighbor treatment versus blocks, L_6 be a $v\times b$ incidence matrix of right neighbor treatment versus blocks. Also $r'=(r_1,\ r_2,\ ...,\ r_v)$ is the $v\times 1$ replication vector of direct treatments with $r_i(l=1,\ 2,\ ...,\ v)$ being the number of times the l^{th} treatment treatments with r_{11} being the number of times the treatments in the design has l^{th} treatments as left neighbor, $r_2'=(r_{21},\ r_{22},\ ...,\ r_{2v})$ is the $v\times 1$ replication vector of right neighbor treatments with r_{21} being the number of times the treatments in the design has l^{th} treatment as right neighbour.

$R_\varepsilon = diag(r_1,\ r_2,\ ...,\ r_v)$; $R_\psi = diag(r_{11},\ r_{12},\ ...,\ r_{1v})$; $R_\omega = diag(r_{11},\ r_{12},\ ...,\ r_{1v})$; and $K = diag(k_1,\ k_2,\ ...,\ k_b)$.

The joint information matrix (C) for estimating direct left and right neighbour effects is obtained as

$$C = \begin{bmatrix} R_\varepsilon - L_3K^{-1}L_3' & L_1 - L_3K^{-1}L_5' & L_2 - L_3K^{-1}L_6' \\ L_1 - L_5K^{-1}L_3' & R_\psi - L_5K^{-1}L_5' & L_4 - L_5K^{-1}L_6' \\ L_2' - L_6K^{-1}L_3' & L_4' - L_6K^{-1}L_5' & R_\omega - L_6K^{-1}L_6' \end{bmatrix} \qquad (1.2)$$

The $3v\times 3v$ matrix C is symmetric, non-negative definite with zero row and column sums and $Rank(C)=v-1$. The information matrix for estimating the contrast pertaining to direct effect of treatments is obtained from (1.2) as given below:

$$C_\varepsilon = C_{11} - C_{12}C_{22}^{-}C_{21} \qquad (1.3)$$

Where $C_{11} = R_\varepsilon - L_3K^{-1}L_3'$, $C_{12} = \left[L_1 - L_3K^{-1}L_5' \quad L_2 - L_3K^{-1}L_6'\right]$,

$$C_{22} = \begin{bmatrix} R_{\psi} - L_5K^{-1}L_5' & L_4 - L_5K^{-1}L_6' \\ L_4' - L_6K^{-1}L_5' & R_{\omega} - L_6K^{-1}L_6' \end{bmatrix}$$

Similarly, the information matrices for estimation the contrast pertaining to right neighbour (C_{ψ}) and left neighbour (C_{ω}) effects can also be obtained.

2. Methods of Construction

2. 1 Circular Neighbor Balanced Block Design with Unequal Block Sizes

There may be subsequent to neighbor design where block designs with unequal block sizes are required. For example, the mountainous field for making blocks of equal sizes is not always possible. The need for blocks of different sizes in biological experiments was noted by Pearce (1957). A method for difference involving construction of interference effect in block designs with unequal block sizes for studying direct and neighbor effects is now illustration.

2.2 Method I

Let v=w is the number of treatments (w be the prime number). The following initial blocks are developed modulo (w)

$$I = \{Z^{w-2}, \quad Z^{w-3}, \quad ..., \quad Z^{1}, \quad Z^{w-1}\}$$

Z is a primitive element of GF(w). Making the block circular, the resulting design is a circular neighbor balanced block design with two different block sizes and with parameters $v = w, b = ws_2, k_1 = s_1, k_2 = s_2, r = s_1 + 2s_2$. In this area of circular block designs with unequal block sizes, every treatment has every other treatments as neighbor on both sides twice i.e.,

$L_1 = L_2 = L_4 = 2(J - I)$. Further,

$$L_3K^{-1}L_3' = L_3K^{-1}L_5' = L_3K^{-1}L_6' = L_5K^{-1}L_5' = L_6K^{-1}L_6' = \left[\frac{v-s_1}{s_1} + \frac{v-s_2}{s_2}\right]I + \left[\frac{s_1-1}{s_1} + \frac{s_2-1}{s_2}\right]J$$

The C-Matrix as given in expression (1.2) reduces here to,

$$C = \begin{bmatrix} \left[\frac{v(s_1-1)}{s_1} + \frac{v(s_2-1)}{s_2}\right]\left[I - \frac{J}{v}\right] & -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] & -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] \\ -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] & \left[\frac{v(s_1-1)}{s_1} + \frac{v(s_2-1)}{s_2}\right]\left[I - \frac{J}{v}\right] & -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] \\ -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] & -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] & \left[\frac{v(s_1-1)}{s_1} + \frac{v(s_2-1)}{s_2}\right]\left[I - \frac{J}{v}\right] \end{bmatrix}$$

Therefore the information matrix for estimating the contrast pertaining to direct effects, left and right neighbor effects of treatments is as given below:

$$C_{\varepsilon} = C_{\psi} = C_{\omega} = \frac{v(2s_1s_2 - 3s_1 - 3s_2)}{(s_1s_2 - s_1 - s_2)}\left[I - \frac{J}{v}\right]$$

Example 2.2.1 Let $v = 7, s_1 = 6, s_2 = 3.$ Developing in the initial blocks are $(5 \quad 4 \quad 6 \quad 2 \quad 3 \quad 1)$, $(5 \quad 6 \quad 3)$ and $(4 \quad 2 \quad 1)$. Developing these blocks mod (v) results in a circular neighbour balanced block design with parameters $v = 7, b = 21, k_1 = 6, k_2 = 4, r = 12$.

5	4	6	2	3	1
6	5	0	3	4	2
0	6	1	4	5	3
1	0	2	5	6	4
2	1	3	6	0	5
3	2	4	0	1	6
4	3	5	1	2	0

5	6	3
6	0	4
0	1	5
1	2	6
2	3	0
3	4	1
4	5	2
4	2	1
5	3	2
6	4	3
0	5	4
1	6	5
2	0	6
3	1	0

The various incidence matrices for this design are as follows

$$L_1 = L_2 = L_4 = 2(J - I)$$

$$L_3K^{-1}L_3' = L_3K^{-1}L_5' = L_3K^{-1}L_6' = L_5K^{-1}L_5' = L_6K^{-1}L_6' = 1.5I + 1.5J$$

The information matrix for estimating direct and neighbor effects are obtained as follows

$$C_{\varepsilon} = C_{\psi} = C_{\omega} = \begin{pmatrix} 6 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 6 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 6 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 6 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 6 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 6 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 6 \end{pmatrix}$$

2.3 Method II

Let the number of treatments $v = f(f > 4)$ is a prime or a prime power. Then the neighbour design with parameters $v = f, b = f(f-2),\ k_1 = s_1, k_2 = s_2, r = s_1 + 2s_2$.

can be constructed from the initial blocks

$$I_g = \{Z^0,\quad Z^1,\quad Z^2,\quad ...,\quad Z^{f-2}\};\quad g = 0, 1, 2, ..., f-2.$$

Where Z is a primitive root of the Galois field $GF(f)$, making the block circular, the resulting design is a circular neighbor balanced block design with two different block sizes and with parameers $v = f, b = f(f-2),\ k_1 = s_1, k_2 = s_2, r = s_1 + 2s_2$, the method of difference involving neighour design has been used in the series Meitei (2010).

The structures of incidence matrices are as follows

$$L_3K^{-1}L_3' = L_3K^{-1}L_5' = L_3K^{-1}L_6' = L_5K^{-1}L_5' = L_6K^{-1}L_6'$$

$$= \left[\frac{v-s_1}{s_1} + \frac{v-s_2}{s_2}\right]I + \left[\frac{s_1-1}{s_1} + \frac{s_2-1}{s_2}\right]J$$

The C-Matrix as given in expression (1.2) reduces here to,

$$C = \begin{bmatrix} \left[\frac{v(s_1-1)}{s_1} + \frac{v(s_2-1)}{s_2}\right]\left[I - \frac{J}{v}\right] & -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] & -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] \\ -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] & \left[\frac{v(s_1-1)}{s_1} + \frac{v(s_2-1)}{s_2}\right]\left[I - \frac{J}{v}\right] & -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] \\ -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] & -\left[\frac{v}{s_1} + \frac{v}{s_2}\right]\left[I - \frac{J}{v}\right] & \left[\frac{v(s_1-1)}{s_1} + \frac{v(s_2-1)}{s_2}\right]\left[I - \frac{J}{v}\right] \end{bmatrix}$$

Therefore the information matrix for estimating the contrast pertaining to direct effects of treatments is as given below:

$$C_\varepsilon = \frac{v(2s_1s_2 - 3s_1 - 3s_2)}{(s_1s_2 - s_1 - s_2)}\left[I - \frac{J}{v}\right]$$

The information for estimating the left and right neighbor effects of treatments are given below

$$C_\psi = C_\omega = \frac{v(3s_1s_2 - 3s_1 - 3s_2)}{s_1(s_1s_2 - s_1 - s_2)}\left[I - \frac{J}{v}\right]$$

Example 2.3.1

Developing in the initial blocks are $(1\quad 2\quad 4\quad 3), (1\quad 4)$ and $(2\quad 3)$. Developing these blocks mod (v) results in a circular neighbour balanced block design with parameters $v = 5, b = 15, k_1 = 4, k_2 = 2, r = 8$.

1	2	4	3
2	3	0	4
3	4	1	0
4	0	2	1
0	1	3	2

1	4
2	0
3	1
4	2
0	3
2	3
3	4
4	0
0	1
1	2

The various incidence matrices for this design are as follows

$L_1 = L_2 = 2(J - I)$

$L_3 K^{-1} L_3' = L_3 K^{-1} L_5' = L_3 K^{-1} L_6' = L_5 K^{-1} L_5' = L_6 K^{-1} L_6' = 1.75I + 1.25J$

Therefore the information matrix for estimating the contrast pertaining to direct effects of treatments is as given below

$$C_\varepsilon = \begin{pmatrix} 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 2 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 2 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 2 \end{pmatrix}$$

The information matrix for estimating the contrast pertaining to left and right neighbor effect of treatments is as given below

$$C_{\psi} = C_{\omega} = \begin{pmatrix} 3 & -\frac{3}{4} & -\frac{3}{4} & -\frac{3}{4} & -\frac{3}{4} \\ -\frac{3}{4} & 3 & -\frac{3}{4} & -\frac{3}{4} & -\frac{3}{4} \\ -\frac{3}{4} & -\frac{3}{4} & 3 & -\frac{3}{4} & -\frac{3}{4} \\ -\frac{3}{4} & -\frac{3}{4} & -\frac{3}{4} & 3 & -\frac{3}{4} \\ -\frac{3}{4} & -\frac{3}{4} & -\frac{3}{4} & -\frac{3}{4} & 3 \end{pmatrix}$$

Conclusions

The target of this paper is to prepare the construction of neighbour balanced design with more flexible competition effects. In this sense, a class of block design with unequal block sizes for studying direct and left and right neighbour effect has also been deliberated which may be useful for situations where block designs with unequal block sizes are needed in the existence of interference effects.

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