FEM Based Mathematical Model To Analyse The Various Thickness And Modular Ratio On The Free Vibration Analysis Of Laminated Composite Panel Using ANSYS

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ABSTRACT

In high performance engineering, laminated composites are used in a variety of applications, including as aeronautical, mechanical, chemical, civil, and spacecraft applications, and they are crucial for FEM-based mathematical models. Accurate design and later production depended on the analysis of these structures and its components utilizing mathematical, experimental, or simulation-based models. Over the course of their useful lives, these structures withstand extreme weather, vibration, inertia excitation, and intense aural stimulation. The initial vibration/fundamental frequency mode puts a structural component under a lot of stress and compression, which eventually wears it out since it is intrinsically linked to large amplitude. This emphasizes how important vibration analysis is for laminated constructions composed of composite and hybrid materials. We use code in the MATLAB environment to solve the equations and obtain the necessary results.Moreover, for each possible outcome, an ANSYS simulation model has been developed and validated to show that the model is applicable in all cases.

Keywords: Free vibration, Mathematical Model, FEM, ANSYS, Modular ratio.

1. Introduction

Nowadays, the structural components of a large number of contemporary cars, buildings, and historical and technical objects are made of laminated composite shells. Compared to conventional materials like concrete, metal, and wood, laminated composites are significantly lighter. Composite materials have exceptional chemical resistance, exceptional elastic characteristics, remarkable corrosion resistance, and a low coefficient of thermal expansion. They also show incredible strength, especially in relation to their weight or volume. The composites' adaptability for high-performance engineering applications is increased by their ability to modify their structural characteristics to satisfy particular needs. Since this has an immediate influence on the cost and availability of composite structures, mass production is required to overcome the existing economic problems. These components must be examined using a mathematical and/or simulation-based model before design and manufacture. Thin laminated composites with a panel-like form make up the outside skins of cars, spacecraft, and airplanes. As was previously indicated, aerodynamic heating from the operation of rockets, launch vehicles, and high-speed aircraft puts significant strain on the structural elements. These components' buckling, deformations, and natural frequencies are all significantly impacted by this pressure. Because the membrane stiffness of the shell panel is larger than its bending stiffness, it can absorb a large amount of strain energy in its membrane without deforming excessively.

Nowadays, designed structural components have mostly been replaced with laminated composite curved/flat panels because of their customized features. Panel structures are widely recognized for their outstanding capacity to absorb energy. Furthermore, it is accurate to say that the panel's fundamental shape is altered by the extra deformation brought on by the in-plane thermal/mechanical stress. As a result, the panel structure's stiffness characteristics are affected. Strong, high-amplitude vibrations increase stress levels and shorten life expectancy through exhaustion. It is commonly known that thin laminated structures are brittle, and that how well they function overall under combined loads is largely dependent on the structural geometry.

It is essential to have a practical grasp of how laminated composite curved and flat panels respond structurally to vibration. Because of these fascinating and difficult issues, the introduction of laminated composite structural elements has made modeling and analysis necessary. From a designer's perspective, the prediction of the fundamental frequency characteristics depends on the modeling of these structures. The restrictions of loading types, material properties, layered structure geometries, and their impacts are all explained via parametric analysis. In order to highlight the current issue, this part will discuss earlier research that has been done by different academics.

2. Literature Review

Using the body of existing knowledge, this part investigates laminated constructions' vibration properties. When forecasting and developing structures with new and innovative concepts, experts are particularly concerned about the vibration behavior of laminated structures. An analytically computed mathematical model based on higher order refined theory was created by Kant and Swaminathan [1] to investigate the free vibration behavior of sandwich and laminated composite plates. Matsunaga [2–3] uses power series expansion to solve the issues of stability and free vibration in laminated (angle- and cross-ply) composite plates. Using an improved plate theory, Putcha and Reddy [4] investigated the stability and vibration properties of laminated plates. The static and vibration properties of laminated composite shells are analyzed and solved using Navier's-type exact solution in the HSDT kinematic model created by Reddy and Liu [5]. Using the FSDT, Ferreira et al. [6] investigated the buckling and vibration properties of laminated and isotropic plates. In the framework of the first-order shear deformation theory (FSDT) and the higher-order shear deformation theory (HSDT) kinematics, Bhar et al. [7] employed the finite element method (FEM) to ascertain the structural responses of laminated composite stiffened plates. Mantari et al. [8] investigate the static and dynamic properties of laminated composite plates using a novel higher order shear deformation theory. Using the CLPT, Xiang and Kang [9] examined the free vibration properties of moving laminated composite plates. Xiang et al. [10] investigate the natural vibration behavior of laminated composite shells using a meshless global collocation approach inside the First-order Shear Deformation Theory (FSDT) framework. Cui et al. [11] use the discrete shear gap approach, which is based on the midplane kinematics of the first-order shear deformation theory (FSDT), to address the bending and vibration behaviors of laminated composite plates. Hatami et al.'s study [12] used a meshless local collocation method based on thin plate spline radial basis functions to investigate the vibration properties of laminated composite plates. The application of the Generalized Differential Quadrature (GDQ) approach in the HSDT kinematics for the analysis of doubly-curved laminated shell panels' free vibration was investigated by Viola et al. [13]. To get the free vibration responses of doubly-curved laminated composite shell panels, Tornabene et al. [14] employed HSDT kinematics analysis. By developing a thorough and precise solution approach using the FSDT, Jin et al. [15–16] investigated the vibration responses of several composite laminated structures, such as annular plates, cylindrical, conical, and spherical shells.

3. Finite Element Method Based Mathematical Model

Since the finite element method (FEM) is more accurate than other analytical or numerical techniques, it is extensively utilized and considered the most reliable tool for planning any structure in today's world. Anticipating the reactions of different commodities, components, assemblies, and subassemblies is one of its main responsibilities. Due to its potential to expedite and enhance innovation with more precision, as well as drastically reduce the time and expense involved with physical testing, FEM is presently widely employed in all modern domains. Many industries and analysts utilize ANSYS, a respected and extensively used finite element analysis (FEA) technology in the market.

A large number of elements for laminated plates using HOT. Another approach is presented in this section. As this treatment of the unsymmetric laminated plate is more general, the theoretical formulation is based on this case only.

The displacement fields are expressed in Taylors series expansion is as follows

$$u(x,y,z) = u(x,y,0) + z\left(\frac{\partial u}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 u}{\partial z^2}\right) + \ldots.. + \infty$$

$$v(x,y,z) = v(x,y,0) + z\left(\frac{\partial v}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 u}{\partial z^2}\right) + \ldots.. + \infty$$

$$w(x,y,z) = w(x,y,0) + z\left(\frac{\partial w}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 w}{\partial z^2}\right) + \ldots.. + \infty \qquad (1)$$

After neglecting higher order strain terms, the appropriate displacement field can be expressed as

$$u(x,y,z) = u_0(x,y) + Z\theta_x(x,y) + Z^3\theta_x^*(x,y)$$

$$v(x,y,z) = v_0(x,y) + Z\theta_y(x,y) + Z^3\theta_y^*(x,y)$$

$$w(x,y,z) = w_0(x,y) \qquad (2)$$

Where

u_0, v_0 *are inplane displacements at a point in the middle plane*

w_0 *is the transverse displacement at a point in the middle plane*

θ_x, θ_y *are the rotions of the normal to the middle plane about y and x − axis respectively*

w_0, θ_x^* *and* θ_y^* Are higher order degree terms of Taylor's series expression.

$$\epsilon_{x=} \frac{\partial u}{\partial x}$$

$$= \frac{\partial_{u0}}{\partial_x} + Z\frac{\theta x}{\partial x} + Z^3\frac{\theta_x^*}{\partial_x}$$

$$\epsilon_{y=} \frac{\partial v}{\partial x}$$

$$= \frac{\partial_{v0}}{\partial_y} + Z\frac{\theta x}{\partial y} + Z^3\frac{\theta_y^*}{\partial_y}$$

$$\gamma_{xy} = \frac{\partial_{u0}}{\partial_y} + Z\frac{\theta x}{\partial y} + Z^3\frac{\theta_x^*}{\partial_y} + \frac{\partial_{v0}}{\partial_x} + Z\frac{\theta}{\partial x} + Z^3\frac{\theta_y^*}{\partial_x}$$

$$\varepsilon_{x=}\varepsilon_x^0 + zk_x + z^3k_x^*$$

$$\varepsilon_{y=}\varepsilon_y^0 + zk_y + z^3k_y^*$$

$$\gamma_{xy} = \gamma_{xy}^0 + Zk_{xy} + Z^3k_{xy}^* \qquad (3)$$

Where

$$\varepsilon_x^0 = \frac{\partial_{u0}}{\partial_x}$$

$$\varepsilon_y^0 = \frac{\partial_{v0}}{\partial_y}$$

Stress variation in laminates

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \qquad (4)$$

In which

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\left[\begin{Bmatrix}\epsilon_x^0\\ \epsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix} + z^3\begin{Bmatrix}k_x^*\\ k_y^*\\ k_{xy}^*\end{Bmatrix}\right]$$

$[\sigma]_k = \overline{[Q]}_k\{\epsilon\}$ and $\{\epsilon\} = \epsilon^0 + zk_+Z^3k^*$

Resultant force

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix} = \int_{\frac{-h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} dz$$

For lamina, Resultant force

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix} = \sum_{k=1}^{n}\left[\int_{Z_{k-1}}^{Z_k}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} \boldsymbol{dz}\right] \qquad (5)$$

Resultant moment

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z.\,dz$$

For lamina, Resultant moment

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{n} \left[\int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \mathbf{z.\,dz} \right]$$

$$\begin{bmatrix} M_x^* \\ M_y^* \\ M_{xy}^* \end{bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} Z^3.\,dz = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} Z^3.\,dz \tag{6}$$

Shear components

$$\{\theta_{xz}\} = \sum_{k=1}^{n} [Q^{-1}]\,(Z_k - Z_{k-1})\gamma_{xz} + \frac{1}{3}\sum_{k=1}^{n} [Q^{-1}]\,(Z_k^3 - Z_{k-1}^3).\,\gamma_{xz}^*$$

$$\{\theta_{yz}\} = \sum_{k=1}^{n} [Q^{-1}]\,(Z_k - Z_{k-1})\gamma_{yz} + \frac{1}{3}\sum_{k=1}^{n} [Q^{-1}]\,(Z_k^3 - Z_{k-1}^3).\,\gamma_{yz}^*$$

$$\{\theta_{xz}^*\} = \frac{1}{3}\sum_{k=1}^{n} [Q^{-1}]\,(Z_k^3 - Z_{k-1}^3)\gamma_{xz} + \frac{1}{5}\sum_{k=1}^{n} [Q^{-1}]\,(Z_k^5 - Z_{k-1}^5)\gamma_{xz}^*$$

$$\{\theta_{yz}^*\} = \frac{1}{3}\sum_{k=1}^{n} [Q^{-1}]\,(Z_k^3 - Z_{k-1}^3)\gamma_{yz} + \frac{1}{5}\sum_{k=1}^{n} [Q^{-1}]\,(Z_k^5 - Z_{k-1}^5)\gamma_{yz}^* \tag{7}$$

Combining

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_x \\ M_{xy} \\ M_x^* \\ M_y^* \\ M_{xy}^* \\ \theta_{xz} \\ \theta_{yz} \\ \theta_{xz}^* \\ \theta_{yz}^* \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & C_{11} & C_{12} & C_{16} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{22} & C_{21} & C_{22} & C_{26} & 0 & 0 & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & C_{16} & C_{26} & C_{66} & 0 & 0 & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & E_{11} & E_{12} & E_{16} & 0 & 0 & 0 & 0 \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & E_{21} & E_{22} & E_{26} & 0 & 0 & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & E_{16} & E_{26} & E_{66} & 0 & 0 & 0 & 0 \\ C_{11} & C_{12} & C_{16} & E_{11} & E_{12} & E_{16} & F_{11} & F_{12} & F_{16} & 0 & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{26} & E_{21} & E_{22} & E_{26} & F_{21} & F_{22} & F_{26} & 0 & 0 & 0 & 0 \\ C_{16} & C_{26} & C_{66} & E_{16} & E_{26} & E_{66} & F_{16} & F_{26} & F_{66} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{11} & H_{12} & K_{11} & K_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{12} & H_{22} & K_{12} & K_{22} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{11} & K_{12} & M_{11} & M_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{12} & K_{22} & M_{12} & M_{22} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ k_x^* \\ k_y^* \\ k_{xy}^* \\ \theta_{xz} \\ \theta_{yz} \\ \theta_{xz}^* \\ \theta_{yz}^* \end{bmatrix}$$

In abbreviated form the above equation is written as

$$\{N\} = [D]\,\{\varepsilon\}$$

In which [D] is the stiffness matrix of the composite laminate.

Where

$$\{N\} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_x \\ M_{xy} \\ M_x^* \\ M_y^* \\ M_{xy}^* \\ \theta_{xz} \\ \theta_{yz} \\ \theta_{xz}^* \\ \theta_{yz}^* \end{bmatrix}; \quad [D]$$

$$= \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & C_{11} & C_{12} & C_{16} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{22} & C_{21} & C_{22} & C_{26} & 0 & 0 & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & C_{16} & C_{26} & C_{66} & 0 & 0 & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & E_{11} & E_{12} & E_{16} & 0 & 0 & 0 & 0 \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & E_{21} & E_{22} & E_{26} & 0 & 0 & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & E_{16} & E_{26} & E_{66} & 0 & 0 & 0 & 0 \\ C_{11} & C_{12} & C_{16} & E_{11} & E_{12} & E_{16} & F_{11} & F_{12} & F_{16} & 0 & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{26} & E_{21} & E_{22} & E_{26} & F_{21} & F_{22} & F_{26} & 0 & 0 & 0 & 0 \\ C_{16} & C_{26} & C_{66} & E_{16} & E_{26} & E_{66} & F_{16} & F_{26} & F_{66} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{11} & H_{12} & K_{11} & K_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{12} & H_{22} & K_{12} & K_{22} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{11} & K_{12} & M_{11} & M_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{12} & K_{22} & M_{12} & M_{22} \end{bmatrix}$$

Where

$$[A]_{ij} = \sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (Z_k - Z_{k-1}), \quad [B]_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (Z_k^2 - Z_{k-1}^2)$$

$$[C]_{ij} = \frac{1}{4}\sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (Z_k^4 - Z_{k-1}^4), \quad [D]_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (Z_k^3 - Z_{k-1}^3)$$

$$[E]_{ij} = \frac{1}{5}\sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (Z_k^5 - Z_{k-1}^5), \quad [F]_{ij} = \frac{1}{7}\sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (Z_k^7 - Z_{k-1}^7)$$

$$[H] = \sum_{k=1}^{n} [Q^{-1}](Z_k - Z_{k-1}), \qquad [K] = \frac{1}{3}\sum_{k=1}^{n} [Q^{-1}](Z_k^3 - Z_{k-1}^3)$$

$$[M] = \frac{1}{5}\sum_{k=1}^{n} [Q^{-1}](Z_k^5 - Z_{k-1}^5)$$

Stress-strain Relationship

Total potential energy is given by

$$T = U - W \qquad (8)$$

Or

$$T = \frac{1}{2}\int \{\varepsilon\}^T \{\sigma\}\, dv - \int \{\delta\}^T \{P\}\, dv \qquad (9)$$

Where

U is the strain energy stored in the laminate

V is the work done by the external load

{P} is the load vector

$\{\delta\}$ is the generated displacement vector at the middle plane of the plate

$\{\sigma\}^T=\{\sigma_x\ \sigma_y\ \sigma_z\ \tau_{xy}\ \tau_{yz}\ \tau_{zx}\}$

$\{\varepsilon\}^T=\{\varepsilon_x\ \varepsilon_y\ \gamma_{xy}\ \gamma_{yz}\ \gamma_{zx}\}$

$\{\delta\}^T=\{u\ v\ w\}$

$\{P\}^T=\{P_x\ P_y\ P_z\}$ (10)

The expression of energy assumes the following form after the strain components are substituting in above

$$T=\frac{1}{2}\int\{\varepsilon\}^T\{\sigma\}\,dA-\int\{\delta\}^T\{P\}\,dA \quad (11)$$

Where

$\{\sigma\}^T=\{N_x\ N_y\ N_{xy}\ M_x\ M_y\ M_{xy}\ M_x^*\ M_y^*\ M_{xy}^*\ Q_{xz}\ Q_{yz}\ Q_{xz}^*\ Q_{yz}^*\}$

$\{\varepsilon\}^T=\{\varepsilon_{x0}\ \varepsilon_{y0}\ \gamma_{xy}\ \gamma_{yz}\ \gamma_{zx}\ k_x\ k_y\ k_{xy}\ k_x^*\ k_y^*\ k_{xy}^*\ \emptyset_{xz}\ \emptyset_{yz}\ \emptyset_{xz}^*\ \emptyset_{yz}^*\}$

$\{P\}^T=\{P_{x0}\ P_{y0}\ P_{z0}\}$

$\{\delta\}^T=\{u_0\ v_0\ w_0\ \theta_{xz}\ \theta_{yz}\ \theta_{xz}^*\ \theta_{yz}^*\}$ (12)

Then the resultant defined above are given by

$$\begin{bmatrix}N_x\\N_y\\N_{xy}\end{bmatrix}=\sum_{k=1}^{n}\left[\int_{Z_{k-1}}^{Z_k}\begin{Bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\end{Bmatrix}\boldsymbol{dz}\right] \quad (13)$$

$$\begin{bmatrix}M_x & M_x^*\\M_y & M_y^*\\M_{xy} & M_{xy}^*\end{bmatrix}=\sum_{k=1}^{n}\int_{Z_{k-1}}^{Z_k}\begin{Bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\end{Bmatrix}(Z,Z^3)dz \quad (14)$$

$$\begin{bmatrix}Q_{xz} & Q_{xz}^*\\Q_{yz} & Q_{yz}^*\end{bmatrix}=\sum_{k=1}^{n}\int_{Zk-1}^{Zk}\begin{Bmatrix}\gamma_{xz}\\\gamma_{yz}\end{Bmatrix}(1,Z^2)dz \quad (15)$$

Where ‘n’ is the number of layers of the laminate.

After integrating through the thickness, (13) to (15) can be combined and written as

$\{N\}=[D]\{\varepsilon\}$ (16)

Stiffness matrix

The displacement vector within an element is expresses as

$\{\delta\}=[N]\{X\}_e$ (17)

At each node, the unknown displacement are

$\{X\}_e^i=\{u_{0i}\ v_{0i}\ w_{0i}\ \theta_{xzi}\ \theta_{yzi}\ \theta_{xzi}^*\ \theta_{yzi}^*\}^T$ (18)

The strain vector corresponding to the membrane part is given by

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_x \\ k_{xy} \\ k_x^* \\ k_y^* \\ k_{xy}^* \\ \emptyset_{xz} \\ \emptyset_{yz} \\ \emptyset_{xz}^* \\ \emptyset_{yz}^* \end{bmatrix} = \sum_{i=1}^{NN} \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \\ 0 & 0 & \frac{\partial N_i}{\partial x} & N_i & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 3N_i & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & 3N_i \end{bmatrix} \begin{Bmatrix} u_{0i} \\ v_{0i} \\ w_{0i} \\ \theta_{xzi} \\ \theta_{yzi} \\ \theta_{xzi}^* \\ \theta_{yzi}^* \end{Bmatrix}_{i=1\ to\ 8}$$

Thus the stiffness matrix is

$$[K]_e = \int [B]^T [D][B]\, dz \qquad (19)$$

4. Results and Discussions

It was suggested and described in the prior discussion that ANSYS is used in order to develop a finite element code that is dependent on the mathematical panel model that was produced. A free vibration analysis of laminated composite shell panels has been generated for the five degrees of freedom (DOFs) model. This study was carried out. As part of the investigation into the validity and accuracy of the algorithm that is presently being researched, a comparison of the results with those that are available in the literature is carried out. In addition to this, a simulation model is also built in ANSYS by using code written in ANSYS parametric design language (APDL) in order to cross validate the mathematical model that is already in place. In order to validate the model that has been created, a comparison is done between the answers that are generated via the use of the MATLAB code and ANSYS (by means of the Block-Lanczos technique) and those that are available in the published literature. It is possible to observe, on the basis of the results of the validation and convergence study, that the present findings are displaying a high degree of concordance with the literature that is already available. Within the scope of this paper, we investigate the influence that a number of different combinations of parameters, such as the thickness ratio (a/h), the modular ratio (E_1/E_2), the lay-up scheme, and the support condition, have on the vibration responses of composite shell panels. Material properties uesd are shown in the table 1.

Table 1 Material properties of the laminated composite structures

M1:	E_1/E_2=25 $v_{12}=v_{13}$=0.25	$G_{12}=G_{13}=0.5E_2$ ρ=1	$G_{23}=0.2E_2$
M2:	E_1/E_2=open $v_{12}=v_{13}$=0.25	$G_{12}=G_{13}=0.6E_2$ ρ=1	$G_{23}=0.5E_2$

4.1 Free vibration analysis using ANSYS model

Figure 1 presents the free vibration responses of cross-ply $(0^0/90^0)_2$ laminated composite flat panel for SSSS and CCCC supports. The responses are obtained by varying the thickness ratio (a/h=10, 20, 50 and 100) and the modular ratio (E_1/E_2=10, 20, 30, 40 and 50).

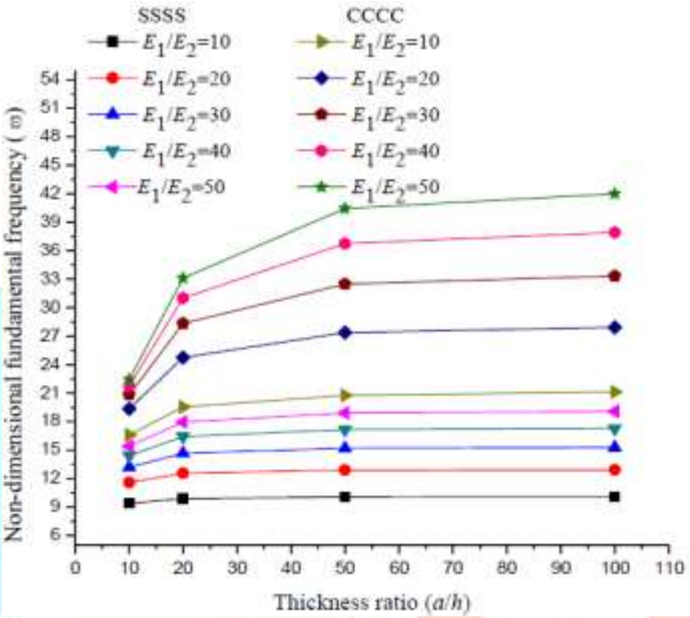

Figure 1 Non-dimensional fundamental frequency of cross-ply $(0^0/90^0)_2$ laminated composite flat panel

In this current examples, the effect of number of layers and support conditions (CCCC and CSCS) on the vibration behaviour is examined and presented in Figure 2 and Figure 2. In this numerical examples, six thickness ratios (a/h = 5, 10, 15, 20, 50, and 100), five modular ratios (E_1/E_2=10, 20, 30, 40 and 50) and different lay-up scheme [$(0^0/90^0)_2$, $(0_0/90^0)_3$, $(0_0/90_0)_5$] is used for the computation using the material property M2. It is clear from the figure that, the thickness ratio and the modular ratio increases the nondimensional fundamental frequency increases and this behavior is expected for any laminated structure. It is noted that the numbers of layers of the flat panel increases the non-dimensional fundamental frequency of the flat panel also increases.

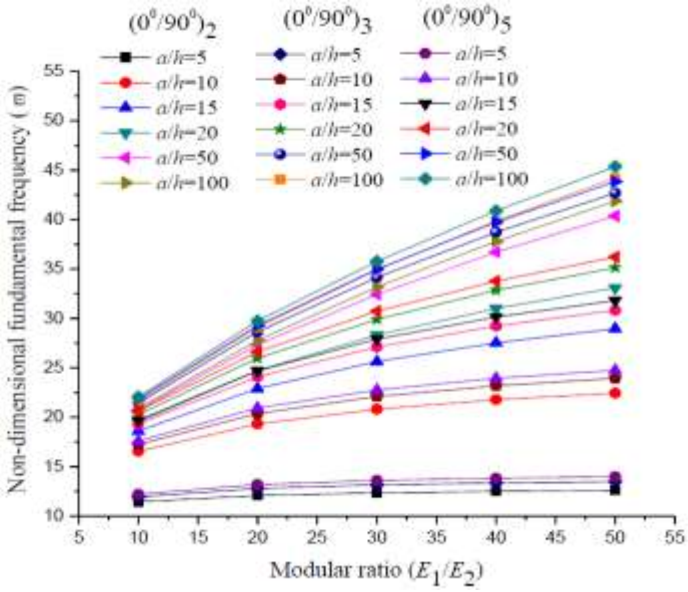

Figure 2 Non-dimensional fundamental frequency of clamped laminated composite flat panel

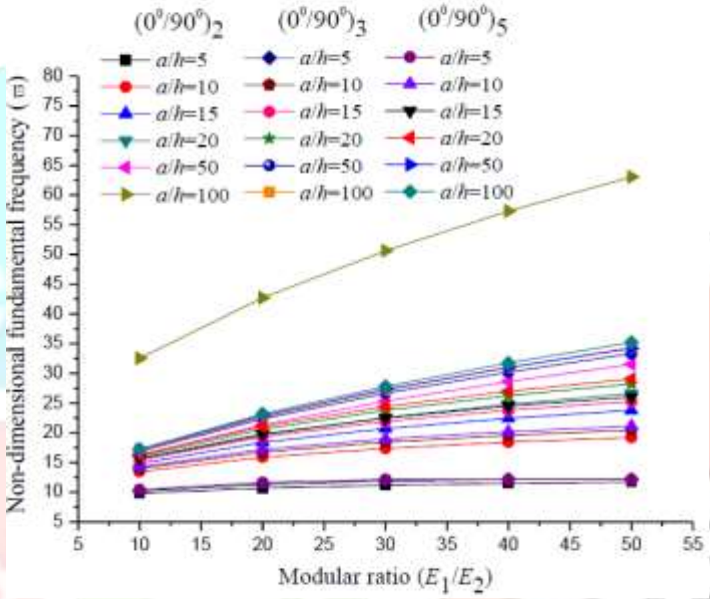

Figure 3 Non-dimensional fundamental frequency of CSCS laminated composite flat panel

5. Conclusions

This section makes use of the generic panel model that was built to look at laminated composite panels' free vibration properties. The finite element method (FEM) code implemented in MATLAB and the APDL code in ANSYS are used to solve the issue and calculate the free vibration of the panel using the eigenvalue formulation. The study looks at how different support conditions, stacking sequence, thickness ratio and modular ratio affect the fundamental frequency of different geometries. It is possible to make the following inferences from the numerical data. The non-dimensional fundamental frequency responses are enhanced by increasing the modular ratio and the aspect ratio, and they are decreased by increasing the curvature ratio. The modular ratio, and support conditions have a major impact on the non-dimensional fundamental frequency. There is a significant effect of the lay-up configuration on the dimensionless fundamental frequency.

References

1. T. Kant and K. Swaminathan, “Analytical solutions for free vibration of laminated composite and sandwich plates based on a higher order refined theory,” Compos. Struct., vol. 53, pp. 73–85, 2001.
2. H. Matsunaga, “Free vibration and stability of angle-ply laminated composite and sandwich plates under thermal loading,” Compos. Struct., vol. 77, no. 2, pp. 249–262, Jan. 2007.
3. H. Matsunaga, “Vibration of cross-ply laminated composite plates subjected to initial in-plane stresses,” Thin-Walled Struct., vol. 40, no. 7–8, pp. 557–571, Jan. 2002.
4. N. S. Putcha and J. N. Reddy, “Stability and natural vibration analysis of laminated plates by using a mixed element based on a refined plate theory,” Journal of Sound and Vibration, pp. 285-300, 1986.
5. J. N. Reddy and C. F. Liu, “A higher order shear deformation theory of laminated elastic shells,” Int. J. of Eng. Sci., vol. 23, pp. 319–30, 1985.
6. A. J. M. Ferreira, C. M. C. Roque, A. M. A. Neves, R. M. N. Jorge, C. M. M. Soares, and K. M. Liew, “Buckling and vibration analysis of isotropic and laminated plates by radial basis functions,” Compos. Part B Eng., vol. 42, no. 3, pp. 592–606, Apr. 2011.
7. A. Bhar, S. S. Phoenix, and S. K. Satsangi, “Finite element analysis of laminated composite stiffened plates using FSDT and HSDT: A comparative perspective,” Compos. Struct., vol. 92, no. 2, pp. 312–321, Jan. 2010.
8. J. L. Mantari, A. S. Oktem, and C. Guedes Soares, “Static and dynamic analysis of laminated composite and sandwich plates and shells by using a new higher order shear deformation theory,” Compos. Struct., vol. 94, no. 1, pp. 37–49, Dec. 2011.
9. S. Xiang and G. W. Kang, “Local thin plate spline collocation for free vibration analysis of laminated composite plates,” European J. of Mecha. A/Solids, vol. 33, pp. 24-30, 2012.
10. S. Xiang, Z. Y. Bi, S. X. Jiang, Y. X. Jin and M. S. Yang, “Thin plate spline radial basis function for the free vibration analysis of laminated composite shells,” Compo. Struct., vol. 93, pp. 611–615, 2011.
11. X. Y. Cui, G. R. Liu and G. Y. Li, “Bending and vibration responses of laminated composite plates using an edge-based smoothing technique,” Engineering Analysis with Boundary Elements, vol. 35, pp. 818–826, 2011.
12. S. Hatami, M. Azahari and M. M. Sadatpour, “Free vibration of moving laminated composite plates,” Compo. Struct., vol. 80, pp. 609-620, 2007.
13. E. Viola, F. Tornabene, and N. Fantuzzi, “General higher order shear deformation theories for the free vibration analysis of completely doubly-curved laminated shells and panels,” Compos. Struct., vol. 95, pp. 639–666, Jan. 2013.
14. F. Tornabene, E. Viola, and N. Fantuzzi, “General higher order equivalent single layer theory for free vibrations of doubly-curved laminated composite shells and panels,” Compos. Struct., vol. 104, pp. 94–117, Oct. 2013.
15. G. Jin, T. Ye, X. Jia, and S. Gao, “A general Fourier solution for the vibration analysis of composite laminated structure elements of revolution with general elastic restraints,” Compos. Struct., vol. 109, pp. 150–168, Mar. 2014.
16. G. Jin, T. Ye, Y. Chen, Z. Su, and Y. Yan, “An exact solution for the free vibration analysis of laminated composite cylindrical shells with general elastic boundary conditions,” Compos. Struct., vol. 106, pp. 114–127, Dec. 2013.