Study Of Bianchi Type Model In Lyra Geometry Considering Barber's Second Self-Creation Theory Taking Constant Deceleration Parameter

Dr. Priya Advani ,
Department of Mathematics,
Government Mahila Engineering College
Ajmer, India

Abstract : The Bianchi type-I cosmological model in Lyra geometry and Barber's second self-creation theory with disordered radiation are studied. Special law of variation for Hubble's parameter which results in constant value of deceleration parameter , is taken into consideration. Some physical properties of the model have also been discussed.

Keywords: Bianchi type-I , Hubble's parameter

Introduction

Barber [12] proposed two self-creation cosmologies by modifying the scalar tensor theories of gravity which were first proposed by Jordan and then by Brans and Dicke and Dicke as an alternative to Einstein's general theory of gravitation. Brans and Dicke have formulated scalar tensor theory of gravitation which develops Mach's principle in a relativistic framework, assuming that inertial masses of fundamental particles are not constant, but are dependent upon the particles' interaction with some cosmic scalar field coupled to the large scale distribution of matter in motion.

Out of two theories given by Barber, the first self-creation theory is a modified Brans and Dicke theory that is rejected on the grounds of violation of the equivalence principle. The second one is an adoption of general relativity to include continuous creation and is within limits of the observations.

A number of scholars have investigated the Barber's second theory in different contexts. Pradhan and Vishwakarma [2] have studied LRS Bianchi type-I cosmological models in self-creation theory. Pradhan and Pandey [4] have obtained a class of LRS Bianchi type-I models in Barber's self- creation theory in the presence of bulk viscous fluid for constant deceleration parameter. Recently Katore et al.[8] have obtained plane symmetric cosmological models with negative constant deceleration parameter in self-creation theory. Lyra [13] suggested a modification of Riemannian geometry by introducing a gauge function. Subsequent investigations were done by Sen [7] within the framework of Lyra geometry. The close connection between these models and general relativistic models have often been noted. Pradhan et al. [1] described isotropic homogeneous universe with a bulk viscous fluid in Lyra geometry. Reddy [9] studied plane symmetric cosmic strings. Reddy and SubbaRao [10] studied axially symmetric cosmic strings and domain walls in this theory. Rahman and Mal [11] studied local cosmic strings in Lyra geometry.

Recently, Singh and Kumar [6] have studied LRS Bianchi type II models with perfect fluid in Barber's second self-creation theory of gravitation by using a special law of variation for Hubble parameter. Very recently Pradhan and Singh [3] explored anisotropic Bianchi type-I string cosmological models in normal gauge for Lyra manifold with constant deceleration parameter.

Model and Field Equations

We consider the Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 , \tag{1}$$

where A, B, C are metric potentials depending on time alone.

The energy momentum tensor has the form

$$T_i^j = (\rho + p) v_i v^j + p g_i^j . \tag{2}$$

In equation (2) ρ is energy density, p is the pressure and v^i is the four velocity vector satisfying the relation

$$g_{ij} v^i v^j = -1 . \tag{3}$$

Einstein's field equations in normal gauge for Lyra Manifold in Barber's theory of self-creation are given by

$$R_i^j - \frac{1}{2} R g_i^j + \frac{3}{2} \alpha_i \alpha^j - \frac{3}{4} \alpha_k \alpha^k g_i^j = -8\pi \phi^{-1} T_i^j , \tag{4}$$

$$\text{and } \phi^k_{;k} = \frac{8\pi}{3} \eta T , \tag{5}$$

where $v_i = (0,0,0,-1)$, $\alpha_i = (0,0,0,\beta(t))$, $v_4 = -1$ and β is the gauge function.

Here $\phi^k_{;k}$ is invariant d'Alembertian and the contracted tensor T is trace of energy momentum tensor describing all non-gravitational and non-scalar field matter and energy. Here η is a coupling constant to be determined from experiments. The measurements of the deflection of light restrict the value of coupling to $|\eta| < 10^{-1}$. In the limit $\eta \to 0$ the Barber's second theory approaches the standard general relativity in every respect. Because of the homogeneity condition imposed by the metric, the scalar field ϕ will be a function of t only. For the line element (1) field equations (4) and (5) lead to the following system of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{3}{4}\beta^2 = -8\pi G \phi^{-1} p , \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{3}{4}\beta^2 = -8\pi G \phi^{-1} p , \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{3}{4}\beta^2 = -8\pi G \phi^{-1} p , \tag{8}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{3}{4}\beta^2 = 8\pi G \phi^{-1} \rho , \tag{9}$$

$$\phi_{44} + \phi_4 \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = \frac{8\pi}{3} \eta (\rho - 3p) , \tag{10}$$

where the suffix '4' denotes differentiation with respect to time t.

Using correspondence to general relativity we define equivalent densities and pressure (Soleng (1987a, 1987b)) as

$$\rho_{eq} = \frac{\rho}{\phi} \ , \tag{11}$$

$$p_{eq} = \frac{p}{\phi}, \tag{12}$$

If we use the equivalent energy conservation equation of general relativity on these quantities, we find that

$$\left(\frac{\rho}{\phi}\right)_4 + \left(\frac{\rho + p}{\phi}\right)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \ , \tag{13}$$

The conservation of left hand side of equation (4) leads to

$$\left(R_i^j - \frac{1}{2} R g_i^j\right)_{;j} + \frac{3}{2}(\alpha_i \alpha^j)_{;j} - \frac{3}{4}(\alpha_k \alpha^k g_i^j)_{;j} = 0, \tag{14}$$

which leads to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \ . \tag{15}$$

The scalar expansion θ and components of shear tensor σ_{ij} are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}, \tag{16}$$

$$\sigma_1^1 = \frac{1}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right], \tag{17}$$

$$\sigma_2^2 = \frac{1}{3}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right], \tag{18}$$

$$\sigma_3^3 = \frac{1}{3}\left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right], \tag{19}$$

$$\sigma_4^4 = 0 \ , \tag{20}$$

Therefore

$$\sigma^2 = \frac{1}{3}\left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{C_4 A_4}{AC}\right]. \tag{21}$$

The spatial volume (V) is given by

$$V = R^3 \quad . \tag{22}$$

The average Hubble's parameter H may be generalized in anisotropic cosmological models as

$$H = \frac{1}{3}(\log V)_4 = (\log R)_4 = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \tag{23}$$

We also have Hubble parameter as

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{24}$$

Where $H_1 = \frac{A_4}{A}, H_2 = \frac{B_4}{B}, H_3 = \frac{C_4}{C}$ are Hubble's factors in the direction of x, y and z respectively.

Solution of Field Equations

There are six independent field equations havingseven unknowns $A, B, C, \rho, \mathrm{p}, \phi$ and β. To find the deterministic solutions of the field equations (6)-(10), we assume the variation of the Hubble parameter given by Berman (1982);

$$H = k_1 R^{-m} = k_1 (ABC)^{-\frac{m}{3}} \tag{25}$$

Where $k_1 > 0$ and $m \geq 0$ are constants.

From (23) and (25) we get

$$R = (k_1 mt + c_1)^{\frac{1}{m}} \text{ when } m \neq 0 \tag{26}$$

$$\text{and } R = c_2 e^{k_1 t} \quad \text{when} \quad m = 0 \tag{27}$$

where c_1 and c_2 are constants of integration.

The deceleration parameter q is given as

$$q = -\frac{RR_{44}}{R_4^2} \tag{28}$$

From equations (25) and (28) we get

$$q = m - 1 \tag{29}$$

Which shows that the law (25) gives a constant deceleration parameter

For deterministic solution we use disordered radiating condition

$$\rho = 3p \tag{30}$$

Using equation (30) in (13) and integrating we get

$$\frac{\rho}{\phi} = \frac{k_2}{R^4} \tag{31}$$

where k_2 is constant of integration

Equations (8)-(10) on integration yield

$$\frac{A_4}{A} = \frac{R_4}{R} + \frac{(2a_2 + a_3)}{3R^3} \tag{32}$$

$$\frac{B_4}{B} = \frac{R_4}{R} + \frac{(a_3 - a_2)}{3R^3} \tag{33}$$

$$\frac{C_4}{C} = \frac{R_4}{R} - \frac{(a_2 + 2a_3)}{3R^3} \tag{34}$$

Where a_2 and a_3 are constants of integration. We solve (32)-(34) by using power law cosmology $(m \neq 0)$ and exponential cosmology $(m = 0)$ given by (26) and (27) respectively.

Power Law Cosmology $(m \neq 0)$

Using equation (26) in equations(32)-(34), we obtain the line element (1) in the form

$$\begin{aligned} ds^2 = -dt^2 &+ (k_1 mt + c_1)^{2/m} \exp\left[\frac{2(2a_2 + a_3)}{3k_1(\mathrm{m-3})}(k_1 mt + c_1)^{\frac{m-3}{m}}\right] dx^2 + \\ &(k_1 mt + c_1)^{2/m} \exp\left[\frac{2(a_3 - a_2)}{3k_1(\mathrm{m-3})}(k_1 mt + c_1)^{\frac{m-3}{m}}\right] dy^2 + \\ &(k_1 mt + c_1)^{2/m} \exp\left[-\frac{2(a_2 + 2a_3)}{3k_1(\mathrm{m-3})}(k_1 mt + c_1)^{\frac{m-3}{m}}\right] dz^2 \end{aligned} \tag{35}$$

where $m \neq 3$.

After taking suitable transforms it takes the form

$$\begin{aligned} dS^2 = -dt^2 + T^{2/m} \exp\left[\frac{2(2a_2 + a_3)}{3k_1(\mathrm{m-3})} T^{\frac{m-3}{m}}\right] dX^2 + T^{2/m} \exp\left[\frac{2(a_3 - a_2)}{3k_1(\mathrm{m-3})} T^{\frac{m-3}{m}}\right] dY^2 + \\ T^{2/m} \exp\left[-\frac{2(a_2 + 2a_3)}{3k_1(\mathrm{m-3})} T^{\frac{m-3}{m}}\right] dZ^2 \end{aligned} \tag{36}$$

Some Physical Properties:

In equation (10) using equation (30) and integrating twice we get

$$\phi = \frac{c_3 m}{k_1 (m-3)} T^{m-3/m} + c_4 \tag{37}$$

Where c_3 and c_4 are constants of integration.

Equation (15) gives either $\beta = 0$ or

$$\frac{\beta_4}{\beta} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \tag{38}$$

Which leads to

$$\beta = \frac{a_1}{R^3} = \frac{a_1}{T^{3/m}} \tag{39}$$

Using (37) in (31) we get

$$\rho = \frac{k_2}{T^{4/m}} \left[\frac{c_3 m}{k_1 (m-3)} T^{m-3/m} + c_4 \right] \tag{40}$$

From (30) and (40) we get

$$p = \frac{k_2}{3T^{4/m}} \left[\frac{c_3 m}{k_1 (m-3)} T^{m-3/m} + c_4 \right] \tag{41}$$

Scalar expansion (θ) is given as

$$\theta = \frac{3k_1}{T} \tag{42}$$

Components of shear tensor (σ_i^j) are given as

$$\sigma_1^1 = \frac{(2a_2 + a_3)}{3T^{3/m}} \tag{43}$$

$$\sigma_2^2 = \frac{(a_3 - a_2)}{3T^{3/m}} \tag{44}$$

$$\sigma_3^3 = -\frac{(a_2 + 2a_3)}{3T^{3/m}} \tag{45}$$

$$\sigma_4^4 = 0 \tag{46}$$

Hence shear tensor (σ) is given as

$$\sigma^2 = \frac{\left(a_2^2 + a_3^2 + a_2 a_3\right)}{3T^{6/m}} \tag{47}$$

The ratio of shear tensor (σ) and scalar expansion (θ) is calculated as

$$\frac{\sigma}{\theta} = \frac{\left(a_2^2 + a_3^2 + a_2 a_3\right)}{3\sqrt{3}k_1 T^{3/m - 1}} \tag{48}$$

The components of Hubble parameter are given as

$$H_1 = \frac{A_4}{A} = \frac{k_1}{T} + \frac{(2a_2 + a_3)}{3T^{\frac{3}{m}}} \tag{49}$$

$$H_2 = \frac{B_4}{B} = \frac{k_1}{T} + \frac{(a_3 - a_2)}{3T^{\frac{3}{m}}} \tag{50}$$

$$H_3 = \frac{C_4}{C} = \frac{k_1}{T} - \frac{(a_2 + 2a_3)}{3T^{\frac{3}{m}}} \tag{51}$$

Hence the Hubble parameter (H) is given as

$$H = \frac{k_1}{T} \tag{52}$$

The anisotropy parameter $\overline{A}$ is defined as

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{53}$$

In our model the anisotropy parameter $\left(\overline{A}\right)$ is evaluated as

$$\overline{A} = \frac{2\left(a_2^2 + a_3^2 + a_2 a_3\right)}{9k_1^2 T^{6-2m/m}} \tag{54}$$

The spatial volume (V) is given by

$$V = T^{\frac{1}{m}} \tag{55}$$

The energy conditions given by Hawking and Ellis (1973) $\rho + p > 0$, $\rho + 3p > 0$ are satisfied if c_2, c_3 and c_4 are all positive and $m \neq 3$.

It is possible to discuss entropy. To solve entropy we have $dS > 0$ necessarily.

The conservation equation $T^{j}_{i;j} = 0$ for the metric (1) is

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{56}$$

In our case $S^3 = ABC$

$$\text{Since } \rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) > 0 \tag{57}$$

It is evident that

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) < 0 \tag{58}$$

Which leads to $\beta < 0$.

Thus the displacement vector β affects entropy because for entropy $dS > 0$ which leads to $\beta < 0$.In this case the universe starts with an infinite rate of expansion from $t = t_0$ where $t_0 = -\frac{c_1}{k_1 m}$. The energy density (ρ) scalar field (ϕ) and pressure (p) are infinite at the initial singularity provided $m < 3$. The space time exhibits 'POINT TYPE' singularity at $t = t_0$. As t increases, the spatial volume V increases, but the rate of expansion slows down. All physical parameters decrease with time. Spatial volume V becomes infinitely large as $T \to \infty$. The energy density, pressure and directional Hubble factors tend to zero as $T \to \infty$. The scalar field (ϕ) is constant for $m < 3$, but anisotropic parameter $(\bar{A})$ vanishes at $T \to \infty$. The shear σ is infinite at T = 0 and tends to zeroas $T \to \infty$. Since $\lim_{T \to \infty} \frac{\sigma}{\theta} \to 0$, which shows that the model approaches isotropy for large values of T. Therefore the model represents non-rotating, shearing and expanding universe with a big-bang start.

Conclusion

In this paper we have obtained exact solutions of the field equations for Bianchi type –I space time in Barber's second self-creation theory of gravitation and Lyra geometry. Cosmological models with constant deceleration parameter have been presented for $m \neq 0$ cosmology. We have also discussed geometrical and kinematical properties of different parameters in detail in each phase. The nature of singularities of the models has been clarified and explicit forms of the scale factors have been obtained in each case. For $m \neq 0$ the spatial volume V grows linearly with cosmic time. It has been observed that the model represents shearing, non-rotating and expanding universe with a big bang start. If the deceleration parameter q is positive $(m > 3)$, the model decelerates and for q to be negative $(m < 3)$, the model inflates. The model has singular origin for $m \neq 0$.

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