BIANCHI TYPE COSMOLOGICAL MODEL IN LYRA GEOMETRY AND SELF-CREATION THEORY WITH CONSTANT DECELERATION PARAMETER

Priya Advani ,
Assistant Professor
Department of Mathematics,
Government Mahila Engineering College , Ajmer, India

Abstract: The present study deals with the exact solutions of Bianchi type-I cosmological model in Lyra geometry and Barber's second self-creation theory with disordered radiation. We have assumed a special law of variation for Hubble's parameter that yield a constant value of deceleration parameter. Some physical consequences of the model have also been discussed.

***Keywords**:* Bianchi space-times, Deceleration parameter, Cosmological models, Hubble's parameter

1. INTRODUCTION

In general theory of relativity, Einstein described gravitation in terms of geometry and it motivated him to geometrize the other physical fields also. Weyl [14] made one of the best attempts in this direction. He introduced a generalization of Riemannian geometry in an attempt to unify gravitation and electromagnetism. Weyl's theory was not taken seriously because it was based on the non- integrability of length transfer. Later Lyra [13] suggested a modification of Riemannian geometry which has a close resemblance to Weyl's geometry, the connection is metric preserving as in Riemannian geometry and length transfer are integrable. Lyra introduced a gauge function which removed the non-integrability condition of the length of a vector under parallel transport. Thus Riemannian geometry was modified by Lyra and was given new name Lyra's geometry. Subsequent investigations in this field were done by Sen [7], Halford [26] and Bhamra [19]. The close connection between these models and general relativistic models has often been noted. Pradhan et al. [1] described isotropic homogeneous universe with a bulk viscous fluid in Lyra geometry. Reddy [9] studied plane symmetric cosmic strings. Reddy and Subba Rao [10] studied axially symmetric cosmic strings and domain walls in this theory. Rahman and Mal [11] studied local cosmic strings in Lyra geometry.

Barber [12] proposed two self-creation cosmologies by modifying the scalar tensor theories of gravity which were first proposed by Jordan and then by Brans and Dicke as an alternative to Einstein's general theory of gravitation. Brans and Dicke have formulated scalar tensor theory of gravitation which develops Mach's principle in a relativistic framework, assuming that inertial masses of fundamental particles are not constant, but are dependent upon the particles' interaction with some cosmic scalar field coupled to the large scale distribution of matter in motion.
Out of two theories given by Barber, the first self-creation theory is a modified Brans and Dicke theory that is rejected on the grounds of violation of the equivalence principle. The second one is an adoption of general relativity to include continuous creation and is within limits of the observations.
A number of scholars have investigated the Barber's second theory in different contexts. Various aspects of the self-creation theories have been investigated by Pimental [20], Soleng [15] and Maharaj and Beesham [23].

Pradhan and Vishwakarma [3] have studied LRS Bianchi type-I cosmological models in self-creation theory. Pradhan and Pandey [4] have obtained a class of LRS Bianchi type-I models in Barber's self- creation theory in the presence of bulk viscous fluid for constant deceleration parameter. Recently Katore et al.[8] have obtained plane symmetric cosmological models with negative constant deceleration parameter in self-creation theory. Singh et al. [18] have obtained exact solutions of the field equations for orthogonal Bianchi type-I space time self-creation theory of gravitation. Very recently Jain et al. [24] have investigated Bianchi type-I cosmological model with varying Λ term in self-creation theory of gravitation.
Recently, Singh and Kumar [5] have studied LRS Bianchi type II models with perfect fluid in Barber's second self-creation theory of gravitation by using a special law of variation for Hubble parameter. Rao et al. [25] discussed Bianchi type-V cosmological model with perfect fluid using negative constant deceleration parameter in a scalar tensor theory based on Lyra manifold. Bali and Chandnani [22] described Bianchi type-I cosmological model for perfect fluid distribution in Lyra geometry. Singh [17] discussed exact solutions of some cosmological models in Lyra geometry. Very recently Pradhan and Singh [2] explored anisotropic Bianchi type-I string cosmological models in normal gauge for Lyra manifold with constant deceleration parameter.

2. Model and Field Equations

We consider the Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2, \tag{1}$$

where A, B, C are metric potentials depending on time alone.

The energy momentum tensor has the form

$$T_i^j = (\rho + p)v_i v^j + pg_i^j. \tag{2}$$

In equation (2) ρ is energy density, p is the pressure and v^i is the four velocity vector satisfying the relation

$$g_{ij}v^i v^j = -1. \tag{3}$$

Einstein's field equations in normal gauge for Lyra Manifold in Barber's theory of self-creation are given by

$$R_i^j - \frac{1}{2}Rg_i^j + \frac{3}{2}\alpha_i\alpha^j - \frac{3}{4}\alpha_k\alpha^k g_i^j = -8\pi\phi^{-1}T_i^j, \tag{4}$$

$$\text{and} \quad \phi_{;k}^{k} = \frac{8\pi}{3}\eta T, \tag{5}$$

where $v_i = (0,0,0,-1)$, $\alpha_i = (0,0,0,\beta(t))$, $v_4 = -1$ and β is the gauge function.

Here $\phi_{;k}^{k}$ is invariant d'Alembertian and the contracted tensor T is trace of energy momentum tensor describing all non-gravitational and non-scalar field matter and energy. Here η is a coupling constant to be determined from experiments. The measurements of the deflection of light restrict the value of coupling to $|\eta| < 10^{-1}$. In the limit $\eta \to 0$ the Barber's second theory approaches the standard general relativity in every respect. Because of the homogeneity condition imposed by the metric, the scalar field ϕ will be a function of t only. For the line element (1) field equations (4) and (5) lead to the following system of equations:

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{3}{4}\beta^2=-8\pi G\phi^{-1}p\,, \tag{6}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\frac{3}{4}\beta^2=-8\pi G\phi^{-1}p\,, \tag{7}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{3}{4}\beta^2=-8\pi G\phi^{-1}p\,, \tag{8}$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}-\frac{3}{4}\beta^2=8\pi G\phi^{-1}\rho\,, \tag{9}$$

$$\phi_{44}+\phi_4\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=\frac{8\pi}{3}\eta\left(\rho-3p\right), \tag{10}$$

where the suffix ‘4’ denotes differentiation with respect to time t.

Using correspondence to general relativity we define equivalent densities and pressure (Soleng [15]) as

$$\rho_{eq}=\frac{\rho}{\phi}\,, \tag{11}$$

$$p_{eq}=\frac{p}{\phi}, \tag{12}$$

If we use the equivalent energy conservation equation of general relativity on these quantities, we find that

$$\left(\frac{\rho}{\phi}\right)_4+\left(\frac{\rho+p}{\phi}\right)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0\,, \tag{13}$$

The conservation of left hand side of equation (4) leads to

$$\left(R_i^j-\frac{1}{2}Rg_i^j\right)_{;j}+\frac{3}{2}(\alpha_i\alpha^j)_{;j}-\frac{3}{4}(\alpha_k\alpha^k\,g_i^j)_{;j}=0\,, \tag{14}$$

which leads to

$$\frac{3}{2}\beta\beta_4+\frac{3}{2}\beta^2\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0\,. \tag{15}$$

The scalar expansion θ and components of shear tensor σ_{ij} are given by

$$\theta=\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\,, \tag{16}$$

$$\sigma_1^1 = \frac{1}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right], \tag{17}$$

$$\sigma_2^2 = \frac{1}{3}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right], \tag{18}$$

$$\sigma_3^3 = \frac{1}{3}\left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right], \tag{19}$$

$$\sigma_4^4 = 0\ , \tag{20}$$

therefore

$$\sigma^2 = \frac{1}{3}\left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{C_4 A_4}{AC}\right]. \tag{21}$$

The spatial volume (V) is given by

$$V = R^3. \tag{22}$$

The average Hubble's parameter H may be generalized in anisotropic cosmological models as

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{1}{3}\left(\log \mathrm{V}\right)_4 = \left(\log R\right)_4 = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \tag{23}$$

where $H_1 = \frac{A_4}{A}, H_2 = \frac{B_4}{B}, H_3 = \frac{C_4}{C}$ are Hubble's factors in the direction of *x, y* and *z* respectively.

3. Solution of Field Equations

There are six independent field equations having seven unknowns $A, B, C, \rho, \mathrm{p}, \phi$ and β. To find the deterministic solutions of the field equations (6)-(10), we assume the variation of the Hubble parameter given by Berman [21]:

$$H = k_1 R^{-m} = k_1\left(ABC\right)^{-\frac{m}{3}} \tag{24}$$

where $k_1 > 0$ and $m \geq 0$ are constants.

From equation (23) we have

$$R = \left(k_1 m t + c_1\right)^{\frac{1}{m}} \quad \text{when} \quad m \neq 0 \tag{25}$$

and

$$R = c_2 e^{k_1 t} \quad \text{when} \quad m = 0 \tag{26}$$

where c_1 and c_2 are constants of integration.

The deceleration parameter q is given as

$$q = -\frac{RR_{44}}{R_4^2} \tag{27}$$

From equations (23),(24) and (27) we get

$$q = m - 1 \tag{28}$$

which shows that the law (24) gives a constant deceleration parameter.

For deterministic solution, we use disordered radiating condition

$$\rho = 3p \tag{29}$$

Using equation (29) in (13) and integrating we get

$$\frac{\rho}{\phi} = \frac{k_2}{R^4} \tag{30}$$

where k_2 is constant of integration

Equations (8)-(10) on integration yield

$$\frac{A_4}{A} = \frac{R_4}{R} + \frac{(2a_2 + a_3)}{3R^3} \tag{31}$$

$$\frac{B_4}{B} = \frac{R_4}{R} + \frac{(a_3 - a_2)}{3R^3} \tag{32}$$

$$\frac{C_4}{C} = \frac{R_4}{R} - \frac{(a_2 + 2a_3)}{3R^3} \tag{33}$$

where a_2 and a_3 are constants of integration. We solve (31)-(33) by using power law cosmology $(m \neq 0)$ and exponential cosmology $(m = 0)$ given by (25) and (26) respectively.

4. Power Law Cosmology $(m \neq 0)$

Using equation (26) in equations (32)-(34), we obtain the line element (1) in the form

$$\begin{aligned} ds^2 = {} & -dt^2 + (k_1 mt + c_1)^{2/m} \exp\left[\frac{2(2a_2 + a_3)}{3k_1(\mathrm{m}-3)}(k_1 mt + c_1)^{\frac{m-3}{m}}\right] dx^2 + \\ & (k_1 mt + c_1)^{2/m} \exp\left[\frac{2(a_3 - a_2)}{3k_1(\mathrm{m}-3)}(k_1 mt + c_1)^{\frac{m-3}{m}}\right] dy^2 + \\ & (k_1 mt + c_1)^{2/m} \exp\left[-\frac{2(a_2 + 2a_3)}{3k_1(\mathrm{m}-3)}(k_1 mt + c_1)^{\frac{m-3}{m}}\right] dz^2 \end{aligned} \tag{34}$$

where $m \neq 3$.

After taking suitable transforms it takes the form

$$dS^2 = -\frac{dT^2}{k_1^2 m^2} + T^{2/m} \exp\left[\frac{2(2a_2 + a_3)}{3k_1(\mathrm{m}-3)} T^{\frac{m-3}{m}}\right] dX^2 + T^{2/m} \exp\left[\frac{2(a_3 - a_2)}{3k_1(\mathrm{m}-3)} T^{\frac{m-3}{m}}\right] dY^2 + T^{2/m} \exp\left[-\frac{2(a_2 + 2a_3)}{3k_1(\mathrm{m}-3)} T^{\frac{m-3}{m}}\right] dZ^2 \tag{35}$$

5. Some Physical Properties:

In equation (10) using equation (29) and integrating twice, we get

$$\phi = \frac{c_3 m}{k_1(m-3)} T^{m-3/m} + c_4 \tag{36}$$

where c_3 and c_4 are constants of integration.

Equation (15) gives either $\beta = 0$ or

$$\frac{\beta_4}{\beta} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{37}$$

which leads to

$$\beta = \frac{a_1}{R^3} = \frac{a_1}{T^{3/m}} \tag{38}$$

Using (36) in (30) we get

$$\rho = \frac{k_2}{T^{4/m}}\left[\frac{c_3 m}{k_1(m-3)} T^{m-3/m} + c_4\right] \tag{39}$$

From (29) and (39) we get

$$p = \frac{k_2}{3T^{4/m}}\left[\frac{c_3 m}{k_1(m-3)} T^{m-3/m} + c_4\right] \tag{40}$$

Scalar expansion (θ) is given as

$$\theta = \frac{3k_1}{T} \tag{41}$$

Components of shear tensor (σ_i^j) are given as

$$\sigma_1^1 = \frac{(2a_2 + a_3)}{3T^{3/m}} \tag{42}$$

$$\sigma_2^2 = \frac{(a_3 - a_2)}{3T^{3/m}} \tag{43}$$

$$\sigma_3^3 = -\frac{(a_2 + 2a_3)}{3T^{3/m}} \tag{44}$$

$$\sigma_4^4 = 0 \tag{45}$$

Hence shear tensor (σ) is given as

$$\sigma^2 = \frac{(a_2^2 + a_3^2 + a_2 a_3)}{3T^{6/m}} \tag{46}$$

The ratio of shear tensor (σ) and scalar expansion (θ) is calculated as

$$\frac{\sigma}{\theta} = \frac{(a_2^2 + a_3^2 + a_2 a_3)}{3\sqrt{3}k_1 T^{3/m-1}} \tag{47}$$

The components of Hubble parameter are given as

$$H_1 = \frac{A_4}{A} = \frac{k_1}{T} + \frac{(2a_2 + a_3)}{3T^{\frac{3}{m}}} \tag{48}$$

$$H_2 = \frac{B_4}{B} = \frac{k_1}{T} + \frac{(a_3 - a_2)}{3T^{\frac{3}{m}}} \tag{49}$$

$$H_3 = \frac{C_4}{C} = \frac{k_1}{T} - \frac{(a_2 + 2a_3)}{3T^{\frac{3}{m}}} \tag{50}$$

Hence the Hubble parameter (H) is given as

$$H = \frac{k_1}{T} \tag{51}$$

The anisotropy parameter $\overline{A}$ is defined as

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{52}$$

In our model the anisotropy parameter $(\overline{A})$ is evaluated as

$$\bar{A} = \frac{2\left(a_2^2 + a_3^2 + a_2 a_3\right)}{9k_1^2 T^{6-2m/m}} \tag{53}$$

The spatial volume (V) is given by

$$V = T^{\frac{1}{m}} \tag{54}$$

The energy conditions given by Hawking and Ellis [16] $\rho + p > 0$, $\rho + 3p > 0$ are satisfied if c_2, c_3 and c_4 are all positive and $m \neq 3$.

It is possible to discuss entropy. To solve entropy we have $dS > 0$ necessarily.

The conservation equation $T_{i;j}^{j} = 0$ for the metric (1) is

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{55}$$

In our case $S^3 = ABC$

$$\text{Since } \rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) > 0 \tag{56}$$

It is evident that

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) < 0 \tag{57}$$

Which leads to $\beta < 0$.

Thus the displacement vector β affects entropy because for entropy $dS > 0$ which leads to $\beta < 0$. In this case the universe starts with an infinite rate of expansion from $t = t_0$ where $t_0 = -\frac{c_1}{k_1 m}$. The energy density (ρ), scalar field (ϕ) and pressure (p) are infinite at the initial singularity provided $m < 3$. The space time exhibits 'POINT TYPE' singularity at $t = t_0$. As t increases, the spatial volume V increases, but the rate of expansion slows down. All physical parameters decrease with time. Spatial volume V becomes infinitely large as $T \to \infty$. The energy density, pressure and directional Hubble factors tend to zero as $T \to \infty$. The scalar field (ϕ) is constant for $m < 3$, but anisotropic parameter $(\bar{A})$ vanishes at $T \to \infty$. The shear σ is infinite at T = 0 and tends to zero as $T \to \infty$. Since $\lim_{T \to \infty} \frac{\sigma}{\theta} \to 0$, which shows that the model approaches isotropy for large values of T. Therefore the model represents non-rotating, shearing and expanding universe with a big-bang start.

6. Exponential Cosmology $(m = 0)$

From equation (26) and equations (31)-(33), we obtain the line element (1) in the form

$$ds^2 = -dt^2 + \exp\left[2k_1 t - \frac{2(2a_2 + a_3)}{9k_1 c_2^3} e^{-3k_1 t}\right] dx^2 + \exp\left[2k_1 t - \frac{2(a_3 - a_2)}{9k_1 c_2^3} e^{-3k_1 t}\right] dy^2 + \exp\left[2k_1 t + \frac{2(a_2 + 2a_3)}{9k_1 c_2^3} e^{-3k_1 t}\right] dz^2 \tag{58}$$

After taking suitable transformation of coordinates line element (58) reduces to

$$ds^2 = -\frac{dT^2}{k_1^2} + \exp\left[2T - \frac{2(2a_2 + a_3)}{9k_1 c_2^3} e^{-3T}\right] dX^2 + \exp\left[2T - \frac{2(a_3 - a_2)}{9k_1 c_2^3} e^{-3T}\right] dY^2 + \exp\left[2T + \frac{2(a_2 + 2a_3)}{9k_1 c_2^3} e^{-3T}\right] dZ^2 \tag{59}$$

7. Some Physical Properties

Using (26) in (10) and integrating twice we get

$$\phi = \frac{c_5}{-3k_1} e^{-3T} + c_6 , \tag{60}$$

where c_5 and c_6 are constants of integration.

Displacement vector (β) is given as

$$\beta = \frac{a_1}{c_2^3 e^{3T}} \tag{61}$$

Using (60) in (30) we get

$$\rho = \frac{k_2}{c_2^4 e^{4T}} \left[\frac{c_5}{-3k_1} e^{-3T} + c_6\right] \tag{62}$$

Using (29) and (62) we get

$$p = \frac{k_2}{3c_2^4 e^{4T}} \left[\frac{c_5}{-3k_1} e^{-3T} + c_6\right] \tag{63}$$

Scalar expansion (θ) is given as

$$\theta = 3k_1 \tag{64}$$

Components of shear tensor (σ_i^j) are given as

$$\sigma_1^1 = \frac{(2a_2 + a_3)}{3c_2^3 e^{3T}} \tag{65}$$

$$\sigma_2^2 = \frac{(a_3 - a_2)}{3c_2^3 e^{3T}} \tag{66}$$

$$\sigma_3^3 = -\frac{(a_2 + 2a_3)}{3c_2^3 e^{3T}} \tag{67}$$

$$\sigma_4^4 = 0 \tag{68}$$

Hence shear tensor (σ) is given as

$$\sigma^2 = \frac{(a_2^2 + a_3^2 + a_2 a_3)}{3c_2^6 e^{6T}} \tag{69}$$

The ratio $\frac{\sigma}{\theta}$ is given as

$$\frac{\sigma}{\theta} = \frac{(a_2^2 + a_3^2 + a_2 a_3)}{3\sqrt{3}k_1 c_2^2 e^{2T}} \tag{70}$$

The components of Hubble parameter are given as

$$H_1 = \frac{A_4}{A} = \frac{k_1}{c_2 e^T} + \frac{(2a_2 + a_3)}{3c_2^3 e^{3T}} \tag{71}$$

$$H_2 = \frac{B_4}{B} = \frac{k_1}{c_2 e^T} + \frac{(a_3 - a_2)}{3c_2^3 e^{3T}} \tag{72}$$

$$H_3 = \frac{C_4}{C} = \frac{k_1}{c_2 e^T} - \frac{(a_2 + 2a_3)}{3c_2^3 e^{3T}} \tag{73}$$

Hubble parameter (H) is given as

$$H = \frac{k_1}{c_2 e^T} \tag{74}$$

The anisotropy parameter $\overline{A}$ is defined as

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{75}$$

$$\overline{A} = \frac{2(a_2^2 + a_3^2 + a_2 a_3)}{9k_1^2 c_2^6 e^{6T}} \tag{76}$$

The spatial volume (V) is given by

$$V = ABC = c_2 e^T \tag{77}$$

We must have c_2, c_4 and c_5 positive for the energy density to be positive. The model has no initial singularity. Initially spatial volume, energy density, pressure and other cosmological parameters are constant. The universe starts evolving with a constant volume and expands with exponential rate. When T increases the scalar field, energy density and pressure decrease. As $T \to \infty$, density and pressure tend to zero. The scalar field and Hubble factors tend to a constant and anisotropy parameter tends to zero as $T \to \infty$. The scalar field remains finite during the whole span of evolution. The model expands uniformly and approaches isotropy for large values of T.

8. Conclusion

In this paper we have obtained exact solutions of the field equations for Bianchi type –I space time in Barber's second self-creation theory of gravitation and Lyra geometry. Cosmological models with constant deceleration parameter have been presented for $m \neq 0$ and m = 0 cosmologies. There are two solutions: one is power law solution and other is exponential solution. We have discussed both solutions for disordered radiating case. We have also discussed geometrical and kinematical properties of different parameters in detail in each phase. The nature of singularities of the models has been clarified and explicit forms of the scale factors have been obtained in each case. For $m \neq 0$ the spatial volume V grows linearly with cosmic time. It has been observed that the model represents shearing, non-rotating and expanding universe with a big bang start. If the deceleration parameter q is positive $(m > 3)$, the model decelerates and for q to be negative $(m < 3)$, the model inflates. The model has singular origin for $m \neq 0$ and nonsingular origin for m = 0. For m = 0 the model represents a shearing, non-rotating and expanding universe with a finite start. The universe becomes isotropic for large values of T. it is found that if $\eta \to 0$ the Barber's self-creation theory tends to general theory of relativity in all respects.

References

1. A Pradhan, L. Yadav and A.K. Yadav, Astrophys. Space Sci. 299, 31-42 (2005).
2. A. Pradhan, A.K. Singh, Int. J. Theor. Phys. 50, 916- 933 (20f11).
3. A. Pradhan, A.K. Vishwakarma, J. Geom. Phys. 49, 332–342 (2004).
4. A. Pradhan, P. Pandey, K. Jotania, and M.K. Yadav, Int. J. Theor. Phys. 46, 2774-2787 (2007).
5. C. P. Singh, Suresh Kumar, Astrophysics and Space Science 310, 31-39 (2007).
6. C.B.Collins, J. Math. Phys. 18, http://dx.doi.org/10.1063/1.523191 (1977).
7. D. K. Sen, Zeitschrift für Physik 149, 311-323 (1957).
8. D. Katore, R.S. Rane, and V.B. Kurkure, Astrophys. Space Sci. 315, 347-352 (2008).
9. D.R.K. Reddy, Astrophys. Space Sci. 300, 381-386 (2005).
10. D.R.K. Reddy, M.V. Subba Rao, Astrophys. Space Sci. 302, 157-160 (2006).
11. F. Rahaman, S. Mal, Astrophys. Space Sci. 302, 3-5 (2006).
12. G. A. Barber, Gen. Rel. Grav., 14(2), 117-136 (1982).
13. G. Lyra, Mathematische Zeitschrift, 54, 52-64 (1951).
14. H. Weyl, Sber. Preuss. Akad. Wiss., 465 (1918).
15. H.H. Soleng, Astrophys. Space Sci. 136, 109-111 (1987a).
16. S.W. Hawking, G.F.R. Ellis, Cambridge Univ. Press, 91(1973).
17. J.K. Singh, Astrophys. Space Sci. 314, 361-365 (2008).
18. J.P. Singh, R.K. Tiwari and S. Kumar, Astrophys. Space Sci. 314, 145-150 (2008).
19. K. S. Bhamra, Australian Journal of Physics 27, 541 – 548 (1974).
20. L.O. Pimentel, Astrophys. Space Sci. 116(2), 395-399 (1985).
21. M.S. Berman, Nuovo Cimento B74, 182 (1983).
22. R. Bali, N. K. Chandnani, Math. Phys. 49, 032502-032502-8 (2008).

23. S.D. Maharaj, A. Beesham, Astrophys. Space Sci. 140, 33-37 (1988).
24. V.C. Jain, M.K. Yadav and P.K. Mishra, Int. J. Theor. Phys. 48, 2205, 343-349 (2009).
25. V.U.M. Rao, T. Vinutha and M.V. Santhi, Astrophys. Space Sci. 314, 213-216 (2008).
26. W. D. Halford, Australian Journal of Physics 23, 863 – 870 (1970).