PAST A POROUS FLAT PLATE WITH CONSTANT HEAT FLUX MHD FREE CONVECTION FLOW THROUGH A ROTATING SYSTEM

Purushottam Singh
(Associate Professor)
(Department of Mathematics Govt.. College Gangapur City, Rajasthan, -322201 India)

ABSTRACT

The paper presents the problem of three-dimensional free convection flow past a vertical porous plate in a rotating system when the whole system is in a state of solid body rotating with a constant angular velocity about an axis normal to the plate. The applied magnetic field has considerable strength so that the Hall effect on the flow could not be neglected. The plate is subjected to constant heat flux. The coupled non-linear equations of velocity and temperature are solved by series method. The effect of Hall parameter, magnetic parameter and heat flux are show on velocity and temperature fields, skin-friction and Nusselt number graphically or tabular from and are discussed. It is being observed that the increase in Hall parameter decrease both primary as well as secondary velocity. On the other hand the increase in magnetic number increase secondary velocity and decrease primary velocity.

Keyword: Free Convection, Rotating system, Heat flux, Magnetic field.

1. INTRODUCTION

The Problems of free Convection heat temperature occupy an important place in heat transfer studies and hence attracted the attention of many research worker. Free convection flow is encountered in Aeronautics, Chemical Engineering, Nuclear Reactors etc. Eckert and Drack (1) have solved the free convection flow past a vertical plate. Sparrow and cess (2) investigated the effect of suction free convection. Soundalgekar and Gupta (3) have considered free convection effect past a moving vertical porous plate. Greespan for have studies the flow in rotating system. Devnath and Mukerjee (5) solved the problems in rotating system with uniform suction/injection at the plate unsteady flow problems in rotating system in electrically conducting fluid have been considered by Devnath (6-7) and Mukherajee (8) have solved a problem of free convection with mass transfer in rotating liquid.

where the strength of magnetic field is very strong Hall effect on the flow are to be considered. This is due to fact that the Ohm's law is modified [crawling (a)]. Hall effect on the hydromagnetic flow of a viscous fluid through a horizontal channel have been studied by sato (10).Datta and Jana (11) have discussed Hall currents on a fully developed laminar free convection in a vertical parallel plate channel. An exact solution for unsteady flow past an accelerated plate with Hall effects has been presented by Soundalgekar et.al. (12) and found that at high values of the Hall parameter, the velocity is found to be oscillatory near the plate.

Despite all these studies the influence of Hall current in rotating flows could not receive much attraction. Agrawal et. al. (13) considered a problem of this nature.

Brinkman (14) and Yamamato (15) have proposed the equation of motion for a class of flows in a highly store have proposed to the equations of motion for a class of flow in a highly porous medium. Yamamato and Iwamura (16) have generalized the Darcy's law when the flow velocity in porous medium is not small. Raptis et.al. (17) discussed the free convection terms in the equation of motion.

In the present era of high altitude flights the study of slip conditions at the surface of the body is to be considered as the continuum approach fails to field satisfactory result. Keeping this fact in mind Jain (18) studies the problem of viscous elastic flow an infinite flat plate.

In this study the effects of σ (Darcy number), t (Taylor number) and R (Rarefaction Parameter) are observed on velocity, temperature, skin-friction and Nusselt number.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Let us consider a three dimensional flow of an electrically conducting fluid on a porous vertical plate with the x'-axis along the plate in direction of flow, z'-axis perpendicular to the plate and y'-axis is chosen perpendicular to the (x', z') plane. The plate is taken to be with constant heat flux. It is further assumed that the fluid and the plate rotate in unison with constant angular velocity Ω about z'-axis, taken normal to the plate. A strong magnetic field H_o is acting along z'-axis and the plate is assume to be electrical non-conducting. Assuming the plate z' = 0 to be infinite, the physical quantities depends on z' only. The equation of continuity $\nabla q = 0$ gives on integration w'= $-w_o(w_o > 0)$ where $q = (U, V, W)$. It is assumed that the induced magnetic field is negligible so that $H = (0,0, H_o)$. The equation of conservation of electric charge $\nabla J = 0$ gives J_z = constant where J = (J_x, J_y J_z). This constant is zero since $J_z = 0$ at the plate which is electrically non-conducting. Thus, J_z =0 everywhere in the flow. In the rotating frame of reference the governing equations are:

Momentum Equations

$$-w_o \frac{dU}{dz'} - 2\Omega v = V\frac{d^2U}{dz'^2} + g\beta(T - T_\infty) + \frac{\mu_e H_o}{\rho} J_{y'} \quad \text{........(1)}$$

$$-w_o \frac{dV}{dz'} - 2\Omega U = V\frac{d^2v}{dz'^2} - \frac{\mu_e H_o}{\rho} J_{x'} \quad \text{........(2)}$$

Energy Equation

$$-w_o \frac{dT}{dz'} = \frac{k}{\rho C_p}\frac{d^2T}{dz'^2} + \frac{\mu}{\rho C_p}\left[\left(\frac{d^2U}{dz'}\right)^2 + \left(\frac{dv}{dz'}\right)^2\right] \quad \text{........(3)}$$

The boundary conditions of the problem are

$$U = 0, v = 0 \; and \; \frac{dT}{dz'} = -\frac{q'}{k} \; at \; z' = 0$$

$$U \to 0, v \to 0 \; and \; T \to 0 \; as \; z' \to \infty \quad \text{........(4)}$$

As mention above when the strength of the magnetic field is very large, Ohm’s law must be modified to include Hall currents as follows

$$\vec{J} + \frac{\omega_e \tau_e}{H_o}\vec{J} x \vec{H} = \sigma[\vec{E} + \mu_e \vec{q} x \vec{H}] \quad \text{.......(5)}$$

Where $\vec{E}$ is electric field, ω_e is the cyclotron frequency and τ_e is the collision time of electrons, σ is fluid conductivity and μ_e is magnetic permeability.

Under the usual assumptions the electron pressure, thermoelectric pressure and ionslip are negligible. We also assume that the electric field $\vec{E} = 0$ [Mayer(19)] using the above assumptions equation (5) give us

$J_{x'} + mJ_{y'} = \sigma\mu_e H_o V$ ....(6)

$Y_{y'} - mJ_{x'} = -\sigma\mu_e H_o U$ ....(7)

Where m = $\omega_e \tau_e$ is Hall parameter.

On solving (6) and (7), we get

$$J_{x'} = \frac{\sigma\mu_e H_o}{1+m^2}(U + mU) \quad \text{........(8)}$$

and

$$J_{y'} = \frac{\sigma\mu_e H_o}{1+m^2}(-U + mU) \quad \text{........(9)}$$

Now introducing the following non-dimensional quantities

$z = \frac{w_o z'}{V}$,

$(u, v) = \frac{1}{w_o}(U, V)$,

$\theta = \frac{T - T_\infty}{q'V} k w_o$,

$E = \frac{w_o{}^3}{q'V c_p}$ (Eckert number),

$Pr = \frac{\mu c_p}{k}$ (Prandtl number),

$G = \frac{q' g \beta V^2}{w_o{}^4 k}$ (Grashoff number),

$E_k = \frac{\Omega V}{w_o{}^2}$ (Ekman Number)

$M^2 = \frac{\mu_e^2 H_0^2 V\sigma}{\rho {w_\circ}^2}$ (Hartmann number) ……..(10)

We get,

$$-\frac{du}{dz} + 2E_k v = \frac{d^2u}{dz^2} + \frac{M^2}{1+m^2}(-u+mv) - G\theta \qquad \text{……..(11)}$$

$$-\frac{dv}{dz} + 2E_k u = \frac{d^2v}{dz^2} + \frac{M^2}{1+m^2}(v+mu) \qquad \text{……..(12)}$$

and

$$-\frac{d\theta}{dz} = \frac{1}{Pr}\frac{d^2\theta}{dz^2} + E\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right] \qquad \text{…….(13)}$$

and the boundary conditions are

$$u = 0\,, v = 0 \; and \; \frac{d\theta}{dz} = -1 \;\; at\; z = 0$$

$$u = 0\,, v = 0 \; and \;\; \theta = 0 \;\; as\; z \to \infty \qquad \text{……(14)}$$

introducing q = u + iv, equations (11) to (14) can be written as

$$\frac{d^2q}{dz^2} + \frac{dq}{dz} - \left(2iE_k + \frac{M^2}{1-im}\right)q = -G\theta \qquad \text{……(15)}$$

$$\frac{d^2\theta}{dz^2} + Pr\frac{d\theta}{dz} = -PrE\frac{dq}{dz}\frac{d\underline{q}}{dz} \qquad \text{……(16)}$$

Where $\underline{q}$ is the complex conjugate of q.

The boundary conditions are

$$q = 0\,,\; \frac{d\theta}{dz} = -1 \;\; at\; z = 0$$

$$q = 0\,,\;\; \theta \to 0 \;\; as\; z \to \infty \qquad \text{…….(17)}$$

3. SOLUTION OF THE PROBLEM

To solve the non-linear coupled equations (15) and (16) we consider a series solution in powers of the Eckert number E, assuming to be small as follows

$$q = q_\circ + Eq_1$$

$$\theta = \theta_\circ + E\theta_1 \qquad \text{……(18)}$$

Substituting (18) in the equations (15) and (16) and equating the coefficient of different power of E

$$\frac{d^2q_\circ}{dz^2} + \frac{dq_\circ}{dz} - Sq_\circ = -G\theta_\circ \qquad \text{……..(19)}$$

$$\frac{d^2\theta_\circ}{dz^2} + Pr\frac{d\theta_\circ}{dz} = 0 \qquad \text{…….(20)}$$

$$\frac{d^2q_1}{dz^2} + \frac{dq_1}{dz} - Sq_1 = -G\theta_1 \qquad \text{…….(21)}$$

$$\frac{d^2\theta_1}{dz^2} + Pr\frac{d\theta_1}{dz} = -Pr\frac{dq_\circ}{dz}\frac{d\underline{q_\circ}}{dz} \qquad \text{…….(22)}$$

Where $S = -2iE_k + \frac{M^2}{1-im}$.

The boundary conditions reduce to

$$q_\circ = 0, q_1 = 0, \frac{d\theta_\circ}{dz} = -1, \frac{d\theta_1}{dz} = 0 \; at\; z = 0$$

$$q_\circ = q_1 \to 0, \theta_\circ = \theta_1 \to 0 \; as\; z \to \infty \qquad \text{……(23)}$$

the solutions of the equations (19) to (22) under boundary conditions (23) are obtained as

$$q_o = \frac{G(n+iT)}{PrPr\,(n^2+T^2)}(e^{-h_1 z} - e^{-Prz}) \quad \text{......(24)}$$

$$\theta_o = \frac{1}{Pr}e^{-Prz} \quad \text{......(25)}$$

$$q_1 = -\frac{G^3}{(n^2+T^2)}[e^{h_1 z}F_2(0) - F_2(z)] \quad \text{......(26)}$$

$$\theta_1 = -\frac{G^2}{(n^2+T^2)}[e^{Prz}F'_1(0) + PrF_1(z)] \quad \text{.......(27)}$$

where

$\alpha_o = \frac{M^2}{1+m^2}$,

$h_1 = \alpha_1 + i\beta_1$

$\underline{h}_1 = \alpha_1 - i\beta,$

$\alpha_1 = \frac{1+\alpha}{2}$,

$\beta_1 = \frac{\beta}{2}$,

$$\alpha = \left[\frac{\sqrt{(1+4\alpha_o)^2+(8E_k+\alpha_o m)^2}+(1+4\alpha_o)}{2}\right]^{\frac{1}{2}}$$

$$\beta = \left[\frac{\sqrt{(1+4\alpha_o)^2+(8E_k+\alpha_o m)^2}-(1+4\alpha_o)}{2}\right]^{\frac{1}{2}}$$

$n = Pr^2 - Pr - \alpha_o,$

$l = 4Pr^2 - 2Pr - \alpha_o,$

$T = 2E_k + \alpha_o m,$

$m = 4\alpha_1^2 - 2\alpha_1 - \alpha_o$,

$Y = \alpha_1^2 - \beta_1^2 + \alpha_1 Pr,$

$x_1 = 2\alpha_1 y + 2\beta_1(2\alpha_1 + Pr)$

$y_1 = 2\beta_1 y - 2\beta_1(2\alpha_1 + Pr)$

$z_1 = y^2 + \beta_1^2(2\alpha_1 + Pr)^2$

$x_2 = a\,cos\beta_1 z + \delta\beta_1\,sin\beta_1 z$

$y_2 = a\,sin\,sin\,\beta_1 z - \delta\beta_1\,cos\beta_1 z$

$z_2 = j - 2\alpha_1 iT$

$x_3 = j(x_1x_2 + y_1y_2) + 2aT^2(x_1\,cos\beta_1 z + y_1\,sin\,sin\,\beta_1 z\,)$

$y_3 = j(x_1\,cos\beta_1 z + y_1\,sin\beta_1 z - 2a(x_1x_2 + y_1y_2)$

$z_3 = z_2\underline{z_2},$

$a = (\alpha_1 + Pr)^2 - (\alpha_1 + Pr) - \beta_1^2$

$j = a^2 - T^2 + \delta^2\beta_1^2$

$\delta = 1 - 2(\alpha_1 + Pr)$

$$F_1(z) - \frac{(\alpha_1^2+\beta_1^2)}{2\alpha_1(2\alpha_1 - Pr)}e^{-2\alpha_1 z} + \frac{e^{-2Prz}}{2} - Pr\frac{e^{-(\alpha_1+Pr)z}}{z_1}[x_1\,cos\,cos\,\beta_{1z} + y_1\,\,sin\,sin\,\beta_1 z]$$

$$F_2(z) = \frac{F_1(0)(n+iT)}{(n^2+T^2)} e^{-Prz} - Pr[\frac{(\alpha_1^2+\beta_1^2)(m+iT)}{2\alpha_1(2\alpha_1-Pr)(m^2+T^2)} e^{-2\alpha_1 z} + \frac{1}{2}\frac{l+iT}{(l^2+T^2)} e^{-2\alpha_1 z} \frac{Pre^{-(\alpha_1+Pr)z}}{z_1 z_3}(x_3 - iy_3)$$

3. DISCUSSIONS

To study the effect of Hall current and viscos dissipation on the hydromagnetic free convective flow in a rotating fluid, We have carried out the numerical calculations for different values of m (Hall Parameter), M (magnetic Parameter). we have plotted the non-dimensional velocity and temperature fields for E = 0.01, E_k = 0.4, G = 5.0 and Pr = 1.0 in figure 1 and 2.

From figure 1 it is seen that with increase in m, both the primary velocity u and secondary velocity v decreases. However, for fixed m primary velocity decreases while secondary velocity increases with the increase of M. Figure 2 show that the non-dimensional temperature θ increase with m and decreases with M.

The components of the skin-friction at the plate in x and y-directions are

$$\tau_u = (\frac{du}{dz})_{z=0} = (\frac{du_\circ}{dz} + E\frac{du_1}{dz})_{z=0} \quad .....(28)$$

and

$$\tau_v = (\frac{dv}{dz})_{z=0} = (\frac{dv_\circ}{dz} + E\frac{dv_1}{dz})_{z=0} \quad .....(29)$$

Dimensionless heat transfer coefficient Nu (Nusselt Number) is given by

$$Nu = \frac{1}{\theta(0)} \quad ...(30)$$

Table 1

m	$M^2 = 3.0$ Nu	τ_u	$-(\tau_v)$	$M^2 = 10.0$ Nu	τ_u	$-(\tau_v)$
0.5	0.9899	2.08	0.55	0.9975	1.35	0.31
1.0	0.9843	2.15	0.81	0.9959	1.43	0.53
2.0	0.9700	2.33	1.09	0.9909	1.62	0.81

From the table 1, it is seen that with the increase in m the skin-friction τ_u increases while the skin-friction τ_v decreases. Further, τ_u decrease and τ_v increase with increase in M. It is also observe that the Nusselt number decreases with the increase in Hall parameter m and increases with increase in magnetic parameter M. It is important to note that the effect of constant heat flux is to decrease the Nusselt number as compared with constant temperature at the plate [Agrawal et.al. (13)].

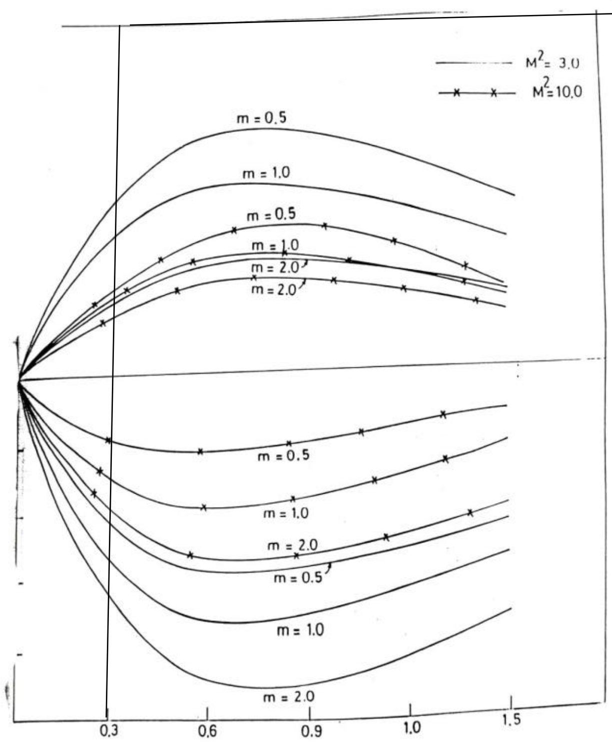

Fig -1 …f primary (u) and secondary (v) velocity plotted against z …ferent values of m and M ($E = 0.01$, $E_k = 0.4$, $G = 5.0$ and $Pr = 1.0$)

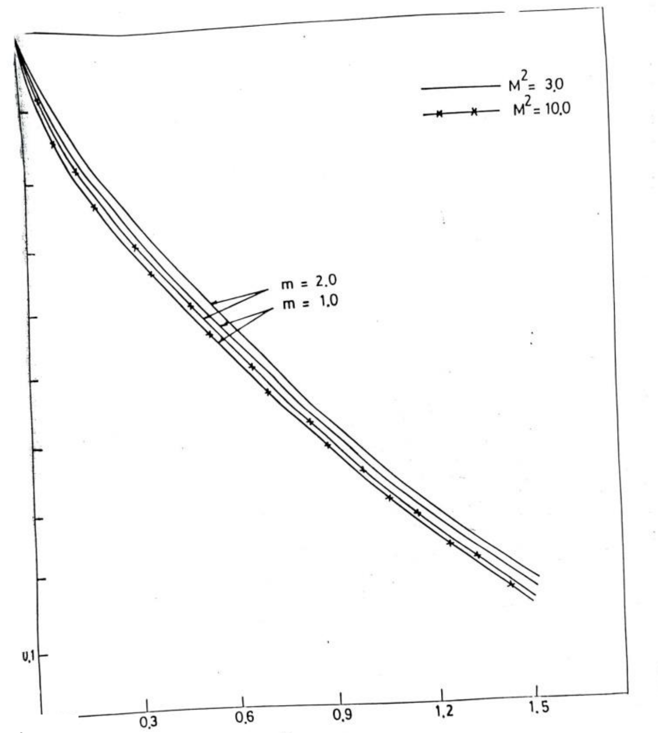

Fig-2

Temprature plotted against z for different values of m and M ($E = 0.01$, $E_k = 0.4$, $G = 5.0$ and $Pr = 1.0$)

REFERENCES

1. Eckert, E.R.C and Drack R.M., Heat and mass Transfer. MC grow Hill, New York (1959)
2. Sparrow, E.M. and Cess. R.D., Free Convection with blowing or suction, J. Heat Transfer, 83, p,387 (1961)
3. Soundalgekar, V.M. and Gupta S.K., Free convection on oscillating flow of a viscous incompressible fluid past a steady moving vertical plate with constant suction, Ind. J. Heat and Mass Transfer, 18.p.1083 (1975)
4. Greenspan, H.P., the theory of relating fluids, Cambridge University Press, London (1968).
5. Debnath, Land Mukherjee, S., Unsteady multiple boundary layers on a porous plate in a rotating system, Phys., fluids 16(9), p.1418. (1973).
6. Debnath, L., On unsteady megnatohydro-dynamics boundary layer in rotating flow, ZAMM 52, p.623(1972)
7. Debnath, L., Effect of Hall current on unsteady hydromagnetic flow past a porous plate in a rotating system, ZAMM, 59, p.469. (1979).
8. Mandal, G.C., and Mukherjee, S., Mass transfer and free convection flow past a vertical porous plate in a rotating liquid, Acta Ciencia Indica, XVII, M.1., p.161 (1991)
9. Cowling, T.G. Magnetohydro-dynamics, Inter Science Publishers, Inc. New York (1957).
10. Sato, H., The Hall effect in the viscous flow of ionized gas between parallel plates under transvers magnetic field, J. Phy. Soc. Japan, 16, p.1427 (1961).
11. Datta, N. and Jana, R.N., Hall effects on free convection between vertical parallel plates, MECCANICA, No.4, X,p.239 (1975).
12. Soundalgekar, V.M., Ravi, S. and Hiremath, S.B., Hall effects and MHD flow past an accelerated plate, J. Plasma Phys. 23, part 3, p.495 (1980)
13. Agrawal, H.L., Ram, P.C. and Singh, V., Combine influence of dissipation and Hall effects on free convective flow in a rotating fluid, Ind., J.Pure Appl. Math., 14(3), p.314 (1983).
14. Brinkmana, H.C,. A calculation of the viscous force extend by a flowing fluid on a dense swarm of particles, Appl., Sci. Res., A1, p.27 (1947).
15. Yamaoto, K. and Yoshida, Z. Flow through a porous wall with convective acceleration, J. Phys., Soc. Japan, 37, p.774 (1974).
16. Yamaoto, K. and Iwamura, N., Flow with convective acceleration through a porous medium, J. Engg. Math., 10, p.41(1976).
17. Raptis, A. Perdikis, C. and Tzivanidis, G., Free convection flow through a porous medium bounded by a vertical surface, J. Phys. D. Appl, Phys., 14, p.99(1981).
18. Jain N.C., Visco-elastic flow past an infinite flat plate in slip flow regime with constant heat flux, the Mathematical Education, XXIV, No.3 p.144(1990).