Designing An Instructional Strategy For Training In Physics Problem Solving

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Abstract

This study focuses on the design of an instructional framework to train students on itemization of math and physics elements in the problem-solving process with layers of scaffolding. We formulated physics problems based on our earlier study that investigated student understanding of vector concepts without a physics context. In this article, we analyse the solution-methods adopted by students in the open-ended problem and misconstrues while choosing the incorrect option of multiple-choice problems. We found that students experienced difficulties in the use of vector concepts related to dot and cross product in physics contexts. Based on the evidence of inadequate grasp of solution method, we illustrate a framework that suggests a possible sequence of instructions that would help students perform math manipulation in the problem–space by decoding physics and math elements.

Keywords: vector dot and cross product, instruction, scaffolding & problem-solving framework

1. Introduction

Research on problem solving in physics has principally focused on 'how students solve it' and on developing instructional strategies that address student difficulties and help the learner develop problem solving skills [1-4]. Identifying student difficulties in physics problem solving is arduous for teachers as they could stem from inadequate grasp of physics concepts and/or lack of math procedural skills, in addition to various other factors. An effective outcome of problem solving depends on how physics teachers guide students to adopt a solution-framework for solving physics problems. However, the question that needs to be addressed is what instructional practices promote problem solving abilities. Quite often, the problem-solving process is hindered as students do not know how to initiate the problem solution.

Providing solution-pointers to lead students into the problem–solving process is a possible approach, though the pointers are problem-specific. An important aspect that needs attention is the nature of the pointers that would provide scaffolding for students during problem–solving with added pedagogical attributes and without impeding the students' ability to think. The scaffolding may be configured from an understanding of the underlying student difficulties.

One deterrent for effective problem solving is the need to use mathematics knowledge in a physics context. Studies by physics education researchers have revealed that students often do not effectively activate the math elements in a given physics context, specifically in problem solving [5-7]. According to Redish [8], in most instances, blending of physical meaning with math equation is a significantly more complex cognitive process than that experienced by students in a math class. Hegde and Meera [9] probed the microstructure of student thought processes in the course of problem solving.

In this study, we have chosen vector that is often used in physics learning, as a representative example to investigate the physics problem solution-methods students adopt. We illustrate an instructional strategy to train students solve problems and help them perform math manipulation in the problem–space by decoding physics and math elements.

Physics education researchers have focused on studies related to operational and interpretational aspects of vector products, with and without physics context. Knight [10] tested students' understanding of math procedures related to dot product that involved vectors in *ijk* format and in addition, required students to use two equations comprising of the cosine and sine trigonometric functions. Zavala and Barniol [11] investigated students' understanding of dot product as a projection in three isomorphic multiple-choice problems. In the test of Understanding of Vectors (TUV) developed by Barniol and Zavala [12], the authors framed test questions based on interpretation and mathematical procedures of vector dot and cross products without physics context. Mikula and Heckler [13] found that students had difficulty with the concepts related to the two vector products and employed computer-based instruction to improve the required skills. Literature review reveals students' difficulties with concepts of vector multiplication in both the contexts. A vector has many semiotic representations that add to the complexity of understanding. Switching between representations is a key aspect of problem solving [14].

In this paper, we report on students' understanding of two vector operations - dot and cross product, both in mathematics and physics contexts. We analyse the responses to the problems and follow the analysis with an illustration of an instructional strategy that may be sought for either a multiple-choice or free-ended question in a physics context.

2. Methodology

This investigation was prompted from the analysis of results of our previous study. We had analysed students' responses to the multiple-choice Test of Understanding Vectors (TUV) developed by Barniol and Zavala [12] to 217 students who were pursuing Master's Course in Physics. For the current study, we also presented students with three problems formulated by us based on vector operations of dot and cross product in a physics context to get a perspective of methods used and types of errors made in solving problems. Physics problem-solving difficulties, more often than not, have their underpinnings in math difficulties. In the present study, we kept the problem structure simple with less interference arising from math difficulties. The reduced complexity and finite identifiable steps would make the analysis of solution process explicable. We administered the test to 67 students (a subset of 217 students who were respondents to TUV). However, there was no pre-determined criterion to administer the test to the subset of respondents. We analysed the solution methods adopted by students in the open-ended problem and misconstrues while choosing the incorrect option of the multiple-choice problems. As an instructional strategy, we elucidated the solution steps required in solving physics problem using a systematic framework.

Concept	Item number in TUV	Item Description	Percentage of responses				
			A	B	C	D	E
Cross product	12	Geometric interpretation of cross product	11	28	22	6	33
	18	Mathematical definition of cross product	7	5	10	60	18
	15	Calculation of cross product of vectors written in *ijk* format	61	8	14	15	2

3. Results and Discussion

3.1 Analysis of TUV test item responses related to dot and cross product

In our study on students' understanding of vector concepts identified in TUV, we found the following results related to test items on dot and cross products.

Concept	Item number in TUV	Item Description	Percentage of responses				
			A	B	C	D	E
Dot product	3	Geometric interpretation of dot product	20	47	9	21	3
	6	Mathematical definition of dot product	3	85	2	9	1
	8	Calculation of dot product of vectors written in *ijk* format	83	1	9	1	6

The numbers in colour indicate the percentage of students who selected the right option for a particular test item. Results revealed that students were quite adept at performing mathematical procedure of dot product of vectors written in the *ijk* format in comparison to the operation of cross product. Excepting the confusion between the sine/cosine conventions, the performance on the tasks of recalling the mathematical definitions of dot and cross products was also satisfactory. However, the responses to item which required the interpretation of the dot and cross product of two vectors in the arrow format are often faulty. The dominant incorrect answers were as follows. The arrow representation of vectors in the arrangement provided in figures of item 3 and item 12 prompted students to try and perform vector addition-like operations! This is reflected in their choice of incorrect answer options: A & C in item 3 (both in almost equal proportions) and similarly, A & C in item 12. Students did not pay attention to the aspect whether the resulting product is a scalar or a vector. The arrow representation in a two-dimensional example is an aid to assign a direction to the cross product using physical mnemonic, the right-hand rule. Our analysis showed that students have confusion about direction of the resulting vector. The accustomed way of naming of the first vector in the cross product along the positive *x* axis in our classroom teaching probably prompts a response which is a vector out of page. Yet again, cross product of two vectors having a direction in addition to magnitude is a hard learning for the students.

Overall, our results showed a hierarchy of performance by students on test items based on different dot product concepts in all representational formats in comparison to the same on cross product concepts. The fairly positive responses to questions based on the calculation of dot and cross product steered us to formulate physics problems that require the same.

3.2 Physics problems related to dot and cross product

We presented three physics problems that involved vector multiplication procedures. Since our TUV analysis suggested that cross product determination was more difficult than dot product determination, we kept the cross-product problem in physics context to be fairly simple. We then devised a dot product problem and the mixed concept-problem in the order of increasing complexity.

3.2.1. Analysis of responses to problem 1

Test item 15 in TUV required students to calculate the cross product of the two vectors in *ijk* representation. As 61% of students chose the right answer, we expected the question formulated by us to be reasonably straightforward in the aspect of vector operation. Moreover, we lowered the difficulty level by stating both the vectors in one dimension. Following is the problem statement and multiple options.

1. A force $12\,\hat{\imath}$ N acts on a particle whose position vector is $6\hat{\jmath}$ m. Choose the correct option that shows the torque produced.
   a) 72 Nm
   b) $72\,\hat{k}$ Nm
   c) $12\,\hat{\imath}+6\hat{\jmath}$ Nm
   d) $-72\,\hat{k}$ Nm

About 46% of test respondents chose the incorrect option 'b', i.e. $72\hat{k}$. The high percentage of respondents picking option 'b' suggests confusion in recalling the correct mathematical equation for torque $\vec{\tau}$ as the cross product of position vector and force or equally well, a flawed understanding of the unit vector multiplication ($\hat{\jmath}$ x $\hat{\imath}$). There could be another likely reason for the choice of this incorrect option. The mention of the force vector represented along the direction of unit vector $\hat{\imath}$ and position vector along the direction of unit vector $\hat{\jmath}$ may have prompted students to consider force as the first vector in the cross product. Twenty-eight per cent of students chose option 'c' which suggests that they have used vector addition tool. About 15% of students who chose option 'a, primarily do not understand that a cross product of a vector yields a vector. About 11% of students chose the right option which is not satisfactory for a fairly simple calculation. Students perceived cross product of vectors in *ijk* format in a physics context to be difficult.

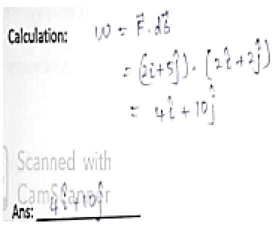
3.2.2. Analysis of responses to problem 2

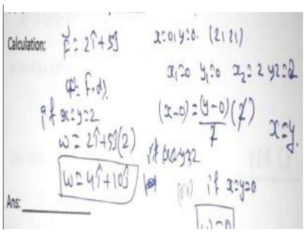
Sequence	*Context*	*Suggested scaffolding*	*Pre-requisite/Key aspect*
1	*Physics*	To write the equation for work done	To recall the equation for work done as dot product of force vector and displacement vector
2	*Math*	To write the displacement vector as the sum of vector components in two dimensions	To identify the displacement vector as vector sum of *changes* in position coordinates both along *+x* and *+y* axes
3		To recognise *dx* and *dy* from given data	VECTOR REPRESENTATIONAL FLUENCY
4		To write the displacement vector from step 3	
5		To write the equation of work done as a dot product	Procedure to calculate dot
6		To execute dot product operation	
		To check for the scalar/vector property of the solution.	To recognise that dot product of vectors is a SCALAR
7	*Physics*	To write the relevant unit for work done	

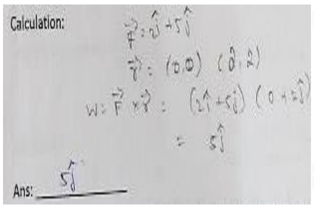
Our results of TUV revealed that students are quite adept at performing habitual mathematical procedure related to dot product. To investigate the transfer of procedural math knowledge to a physics context and vector representational fluency, we framed the following free-ended problem.

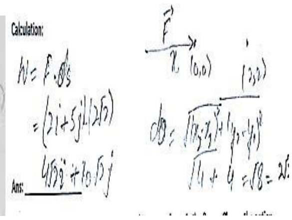
2. A force $\vec{F} = 2\hat{\imath} + 5\hat{\jmath}$ N acts on a particle which is displaced from a point (0, 0) to another point (2, 2). What is the work done by the force?

The preliminary steps to work the solution involves recalling the mathematical equation for the work done and also writing the displacement vector $d\vec{r}$ in *ijk* format from the component representation *(x, y)*. Subsequently, students are required to calculate the dot product. The process calls for a sequential thinking where each step in the process must be methodical and correct. However, we observed that students were unable to activate the appropriate math element/s in the context of the physics problem. Following are a few written responses.









The illustrated responses indicate that students think that dot product of two vectors yields a sum of two vectors. While all the students recalled the equation for dot product, majority were unable to translate the vector representation from the coordinate format to the *ijk* format. Students who were able to write the displacement vector in the *ijk* format performed an incorrect mathematical operation (illustrated responses). Also, among the students who answered incorrectly; about 21% assigned a vector property to dot product of vectors. Sixty-six per cent of students calculated the correct value for the work done. A few students did not represent the physical quantities in the mathematical equation as vectors. Also, the notation 's' used for the displacement vector by physics teachers, does not appear to strongly convey the displacement vector meaning. Use of the notation $d\vec{r}$ (or $\Delta\vec{r}$) in the equation of work done is probably more appropriate as it strengthens the understanding of 'change' in position along a specified direction. Pedagogically, this would be significant as the notation is a generic form, applicable for both, motion along a straight line or motion along a curve. Analysing the difficulties students experienced during problem solving, we developed the following solution-framework that can function as instructional scaffolding. (Table1).

Table 1- Problem solving instruction

We posed problem 3 to the students that is based on a mixed-concept (both dot and cross product) in a physics context. Following is the problem statement and multiple options.

3.The magnetic force $\vec{F}$ on a charged particle moving with an uniform velocity $\vec{v}$ in an uniform magnetic field $\vec{B}$ is given by the equation $\vec{F}$ = q ($\vec{v}$ x $\vec{B}$). If the direction of particle motion is perpendicular to the direction of the magnetic field, the work done by the magnetic force is

a) Zero
b) qvB
c) $\vec{v}$ x $\vec{B}$
d) $\vec{F} . \vec{B}$

3.2.3. Analysis of responses to problem 3

Analysis of responses to problem 3 revealed that 38% of students chose the correct option 'a'. The choice of incorrect option 'b' by 42% of students seems to suggest that they did not use the equation of work done. Seventeen per cent of students chose option 'c' and 3% chose option'd'.

One of the possible solution- methods for problem 3 is to use the definition of work done and the appropriate triple product vector identity (which the students are familiar with) to establish work done to be zero. We developed the following solution-framework that can function as an instructional scaffolding (table 2).

Table 2- Problem solving instruction

Sequence	*Context*	*Suggested scaffolding*	*Pre-requisite /Key aspect*
1	***Physics***	**To write the mathematical equation for work done**	**To recall the equation for work done as dot product of force vector and displacement vector.** $\boldsymbol{W = \vec{F}.d\vec{r}}$
2	***Physics***	**To write the equation for work done in the given physics context**	
3	***Math***	**To use the relevant vector identity**	**Vector triple product**
4		**To write displacement vector in terms of velocity**	$\boldsymbol{d\vec{r} = \vec{v}}$ **dt**
5		**To rewrite the equation for work**	
6		**To use the appropriate vector identity**	
		To evaluate work done	**To use the identity that cross product of a vector with an identical vector the yields Zero**
7	***Physics***	**To generalize the result**	**Work done by magnetic force is ZERO when force and velocity of a charged particle are mutually perpendicular.**

We recommend the administration of the suggested instruction method as follows. The students are presented with the problem statement. They are allowed to internalise the problem context and the scaffolding is provided layer by layer (weakest to the strongest). Research studies conducted earlier have explored the role of scaffolding in solving physics problems [9,15,16,]. The 'pre-requisite' indicated in tables 1 & 2 are the math elements that students need to activate and the 'key aspect' is related to the learning components both in mathematics and physics.

4. Conclusion

A significant outcome of our investigation on physics problem-solving revealed that students had difficulties in solving problems that are fairly simple. Our previous study with TUV revealed that among different vector concepts, the performance on test items based on dot and cross product to be fairly good. The intent of this study was to investigate students' ability to use vector concepts to physics situations that require vector multiplication in solving problems. From analysis of responses, we found that students are not adequately equipped with vector-math skills required for vector operations of dot and cross products in altered contexts. Students' reasoning about the use of vector multiplication procedures ascribed to physical quantities is not deep enough to relate to the physics context. The question is whether there are means to identify the origins of the student difficulties in physics problem solving. An effort in this direction would be train students to solve problems by providing scaffolding discretely in the math and physics domains. We developed an instructional strategy that focuses on training the students on itemization of math and physics elements in the problem-solving process.

Research has shown that problem solving is not algorithmic but heuristic [17]. Use of the solution navigator discussed in this paper involves student engagement far more than by mere observation of the solution methods provided in worked examples

either in classroom teaching or in text books. Hence, we do believe that there exists a strong learning component in the method suggested for the end-of chapter type problems. Training students on itemisation of math and physics elements in the problem-solving process coupled with scaffolding layers would assist them in the navigation of solution-steps of advanced problems too. When students are familiar with an approach to problem solving illustrated in the frameworks, integrating maths with physics would be an easy progression.

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