BAYESIAN MATHEMATICS IN RANKING OF MULTI-CRITERIA ALTERNATIVES.

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Abstract:

Bayesian Mathematics occupies an important place in decision making. However, even there exist many different ranking methods use of this method is scarce in ranking of multi-criteria alternatives. In this paper, we consider Bayesian decision analysis and then discuss prospective ranking of multi-criteria alternative performances using Bayesian Mathematical methods.

Keywords: Bayesian, Multi-critera Alternatives, Ranking.

1.Introduction: Bayesian mathematics is a mathematical procedure that applies probabilities to statistical problem. Bayesian Mathematics compares to the classical frequentist approach to statistics and the potential applications in both quantitative financial data science. Bayesian Mathematics is a very natural way to think about probabilistic events The root of the classical Bayesian Mathematics goes back to the revolutionary Bayes' theorem devised by Thomas Bayes (1701-1761) in his famous work, ''An Essay towards Solving a Problem in the Doctrine of Chance,'' was communicated by Richard Price to the Royal Society in a letter dated November 10, 1763, more than two years after Bayes' death, and it was read at the meeting of the society on 23rd December .Classical statistical decision making involves the notion that the uncertainty in the future can be characterised probabilistically. When we want to make a decision among various alternatives, our choice is redirected on information about the future, which is normally discretised into various ''states of nature.''. Classical Bayesian decision methods presume that future states of nature can be characterized as probability events. In this paper based on these probabilities, ranking of multi-criteria alternatives is attempted.

2. Bayes' Theorem: First we shall consider the formation of probabilistic decision analysis. Let S = $\{s_1, s_2, \ldots, s_n\}$ be a set of possible states of nature; and the probabilities that these states will occur are listed in a vector,

$$\mathbf{P} = \{p(s_1), p(s_2), \ldots, p(s_n)\} \text{ where } \sum_{i=1}^{n} p(s_i) = 1 \qquad (1)$$

The probabilities expressed in Eq. (1) are called ''prior probabilities'' in Bayesian jargon.

This information can be used in the Bayesian approach to update the prior probabilities, $p(s_i)$, in the following manner. First, the new information is expressed in the form of conditional probabilities $p(s_i / x_k)$ where the probability of each piece of data, x_k, where $k = 1, 2, \ldots, r$, is assessed according to whether the true state of nature, s_i , is known. In the literature these conditional probabilities, are also called likelihood values. The likelihood values are then used as weights on the previous information, the prior probabilities $p(s_i)$, to find updated probabilities, known as posterior probabilities, denoted by $p(s_i / x_k)$. These updated probabilities are determined by Bayes's theorem

$$p(s_i / x_k) = \frac{p(s_i)\, p(x_k / s_i)}{\sum_{i=1}^{n} p(s_i)\, p(x_k / s_i)} \qquad (2)$$

From these probabilities we can obtain the ranks of the alternatives. Higher is the probability higher is the rank.

3. Methodology: The methodology for ranking of multi-criteria alternatives by Bayesian Statistics is outlined bellow:

The structure of the alternative performance matrix is expressed as shown in Table 1:

Table 1: Structure of the Alternative Performance Matrix

	Criterion1	Criterion 2	…	Criterion n
Alternative 1	X_{11}	X_{12}	…	X_{1n}
Alternative 2	X_{21}	X_{22}	…	X_{2n}
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
Alternative m	X_{m1}	X_{m2}	…	X_{mn}

Let us first refresh some core concepts of the Bayes' theorem.

Priors: Bayesian decision methods requires prior probabilities which in the present case, are obtained as follows

$$p(s_i) = \frac{X_{i1}}{\sum_{i=1}^{m} X_{i1}} \qquad (3)$$

Likelihood: Likelihoods are calculated using the following relation

$$p(x_i / s_i) = \frac{X_{ij}}{\sum_{i=1}^{m} Xij} \qquad j = 2,3,\dots,n \qquad (4)$$

Posterior probabilities: Posterior probabilities are calculated by the relation (2)

Higher the posterior probability higher is the rank of the corresponding alternative.

4. Example: Following table shows scores obtained by four candidates four tests viz Written Test (WT), Intelligence Quotient (IT), Physical Test (PT) and Health Test (HT).

Table 2: Scores in tests

Candidates	WT	IT	PT	HT
1	45	24	36	20
2	35	41	39	27
3	36	34	27	45
4	20	29	42	47
5	37	41	38	34
6	45	28	29	29
7	43	39	43	41

Now we will try to rank the candidates on application of methodology described above.

Prior probabilities, likelihood values and posterior probabilities obtained using the above methodology are presented in the following Table 3.

Table 3: Probabilities and Ranks of Candidates

(1) Candidates	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9) Ranks
	P_1	L_1	P_2	L_2	P_3	L_3	P_4	
1	0.1724	0.0992	0.1202	0.1417	0.1221	0.0823	0.0714	7
2	0.1341	0.1694	0.1597	0.1535	0.1757	0.1111	0.1387	3
3	0.1379	0.1405	0.1363	0.1063	0.1037	0.1852	0.1365	4
4	0.0766	0.1198	0.0646	0.1654	0.0765	0.1934	0.1051	5
5	0.1418	0.1694	0.1689	0.1496	0.1810	0.1399	0.1799	2
6	0.1724	0.1157	0.1403	0.1142	0.1147	0.1193	0.0972	6
7	0.1648	0.1612	0.1867	0.1693	0.2264	0.1687	0.2713	1

The prior probabilities listed in column (2) of above Table 3 are obtained on application of equation (3) for the scores of WT of Table 2. Likelihood values listed in column (3), (5) and (7) are obtained on using equation (4) for scores obtained by students in IT, PT and HT respectively. Posterior probability values P_2, P_3, and P_4 are obtained using equation (2). Ranks of the candidates are listed in column (9) of the Table 3.

5. Conclusion: Bayesian Statistics occupies an important place in decision making. However, use of this method is rare in ranking of multi-criteria alternatives. The example given above reveals that Bayesian Statistics can effectively be used in ranking of multi-criteria alternatives.

References:

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