Astronomical Sources of Gravitational Waves

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Abstract

Gravitational waves are propagating fluctuations of gravitational fields, that is, ”ripples” in space time, generated mainly by moving massive bodies. These distortions of space time travel with the speed of light. Every body in the path of such a wave feels a tidal gravitational force that acts perpendicular to the waves direction of propagation; these forces change the distance between points, and the size of the changes is proportional to the distance between the points. Gravitational waves can be detected by devices which measure the induced length changes. The frequencies and the amplitudes of the waves are related to the motion of the masses involved. Thus, the analysis of gravitational waveforms allows us to learn about their source and, if there are more than two detectors involved in observation, to estimate the distance and position of their source on the sky. The most promising design of gravitational wave detector, offering the possibility of very high sensitivities over a wide range of frequency, uses test masses a long distance apart and freely suspended as pendulums on earth or in drag free craft in space; laser interferometry provides a means of sensing the motion of the masses produced as they interact with a gravitational wave.

Keywords: Gravitational fields , Gravitational waves, Tidal Gravitational Force

1. THEORY OF GRAVITATIONAL WAVES

Newton’s theory of gravity has enjoyed great success in describing many aspects of our every-day life and additionally explains most of the motions of celestial bodies in the universe. General relativity corrected Newton’s theory and is recognized as one of the most ingenious creations of the human mind. The laws of general relativity, though, in the case of slowly moving bodies and weak gravitational fields reduce to the standard laws of Newtonian theory. Nevertheless, general relativity is conceptually different from Newton’s theory as it introduces the notion of space time and its geometry. One of the basic differences of the two theories concerns the speed of propagation of any change in a gravitational field. As the apple falls from the tree, we have a rearrangement of the distribution of mass of the earth, the gravitational field changes, and a distant observer with a high-precision instrument will detect this change. According to Newton, the changes of the field are instantaneous, i.e., they propagate with infinite speed; if this were true, however the principle of causality would break down. No information can travel faster than the speed of light. In Einstein’s theory there is no such ambiguity; the information of the varying gravitational field propagates with finite speed, the speed of light, as a ripple in the effects of a gravitational wave travelling perpendicular the plane of a circular ring of particles, is sketched as a series of snapshots. The deformations due the two polarizations are shown. fabric of space time. These are the gravitational waves. The existence of gravitational waves is an immediate consequence of any relativistic theory of gravity. However, the strength and the form of the waves depend on the details of the gravitational theory. This means that the detection of gravitational waves will also serve as a test of basic gravitational theory. The fundamental geometrical framework of relativistic metric theories of gravity is space time, which mathematically can be described as a four-dimensional manifold whose points are called events. Every event is labeled by four coordinates x^μ ($\mu = 0, 1, 2, 3$); the three coordinates x^i ($i = 1, 2, 3$) give the spatial position of the event, while x^0 is related to the coordinate time t ($x^0 = ct$, where c is the speed of light, which unless otherwise stated will be set equal to 1). The choice of the coordinate system is quite arbitrary and coordinate transformations of the form $\tilde{x}^\mu = f^\mu (x^\lambda)$ are allowed. The motion of a test particle is described by a curve in space time. The distance ds between two neighbouring events, one with coordinates x^μ and the other with coordinates $x^\mu + dx^\mu$, can be expressed as a function of the coordinates via a symmetric tensor $g_{\mu\nu}(x^\lambda) = g_{\nu\mu}(x^\lambda)$, i.e.,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \qquad (1)$$

This is a generalization of the standard measure of distance between two points in Euclidian space. For the Minkowski space time (the space time of special relativity),

$g_{\mu\nu} \equiv \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. The symmetric tensor is called the metric tensor or simply the metric of the space time. In general relativity the gravitational field is described by the metric tensor alone, but in many other theories one or more supplementary fields may be needed as well. In what follows, we will consider only the general relativistic description of gravitational fields, since most of the alternative theories fail to pass the experimental tests. The information about the degree of curvature (i.e., the deviation from flatness) of a space time is encoded in the metric of the space time. According to general relativity, any distribution of mass bends the space time fabric and the Riemann tensor $R_{\kappa\lambda\mu\nu}$ (that is a function of the metric tensor $g_{\mu\nu}$ and of its first and second derivatives) is a measure of the space time curvature. The Riemann tensor has 20 independent components. When it vanishes the corresponding space time is flat. In the following presentation, we will consider mass distributions, which we will describe by the stress-energy tensor $T^{\mu\nu}(x^\lambda)$. For a perfect fluid (a fluid or gas with isotropic pressure but without viscosity or shear stresses) the stress-energy tensor is given by the following expression

$$T^{\mu\nu}(x^\lambda) = (\rho + p)u^\mu u^\nu + pg^{\mu\nu} \qquad (2)$$

where $p(x^\lambda)$ is the local pressure, $\rho(x^\lambda)$ is the local energy density and $u^\mu(x^\lambda)$ is the four velocity of the infinitesimal fluid element characterized by the event x^λ . Einstein's gravitational field equations connect the curvature tensor (see below) and the stress-energy tensor through the fundamental relation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu} \qquad (3)$$

This means that the gravitational field, which is directly connected to the geometry of space time, is related to the distribution of matter and radiation in the universe. By solving the field equations, both the gravitational field (the $g^{\mu\nu}$) and the motion of matter is determined. $R_{\mu\nu}$ is the so- called Ricci tensor and comes from a contraction of the Riemann tensor

($R_{\mu\nu}=g^{\alpha\sigma}R_{\sigma\mu\rho\nu}$), R is the scalar curvature ($R = g^{\rho\sigma}R_{\rho\sigma}$), while $G_{\mu\nu}$ is the so-called Einstein tensor, $k = 8\pi G/c^4$ is the coupling constant of the theory and is the gravitational constant, which, unless otherwise stated will be considered equal to 1. The vanishing of the Ricci tensor corresponds to a space time free of any matter distribution. However, this does not imply that the Riemann tensor is zero. As a consequence, in the empty space far from any matter distribution, the Ricci tensor will vanish while the Riemann tensor can be nonzero; this means that the effects of a propagating gravitational wave in an empty space time will be described via the Riemann tensor.

2. ASTRONOMICAL SOURCES OF GRAVITATIONAL WAVES

The new generation of gravitational wave detectors (LIGO, VIRGO) have very good chances of detecting gravitational waves, but until these expectations are fulfilled, we can only make educated guesses as to the possible astronomical sources of gravitational waves. The detectability of these sources depends on three parameters: their intrinsic gravitational wave luminosity, their event rate, and their distance from the Earth. The luminosity can be approximately estimated via the quadrupole formula that we discussed earlier. Even though there are certain restrictions in its applicability (weak field, slow motion), it provides a very good order-of-magnitude estimate for the expected gravitational wave flux on Earth. The rate, at which various events with high luminosity in gravitational waves take place is extrapolated from astronomical observations in the electromagnetic spectrum. Still, there might be a number of gravitationally luminous sources, for example binary black holes, for which we have no direct observations in the electromagnetic spectrum. Finally, the amplitude of gravitational wave signals decreases as one over the distance to the source. Thus, a signal from a supernova explosion might be clearly detectable if the event takes place in our galaxy (2-3 events per century), but it is highly unlikely to be detected if the supernova explosion occurs at far greater distances, of order 100 Mpc, where the event rate is high and at least a few events per day take place. All three factors have to be taken into account when discussing sources of gravitational waves. It was mentioned earlier that the frequency of gravitational waves is proportional to the square root of the mean density of the emitting system; this is approximately

2.1 Radiation from Gravitational Collapse

type II supernovae are associated with the core collapse of a massive star together with a shock-driven expansion of a luminous shell which leaves behind a rapidly rotating neutron star or a black hole, if the core has mass of >2-3M The typical signal from such an explosion is broadband and peaked at around 1 kHz. Detection of such a signal was the goal of detector development over the last three decades. However, we still know little about the efficiency with which this process produces gravitational waves. For example, an exactly spherical collapse will not produce any gravitational radiation at all. The key question is what is the kinetic energy of the nonspherical motions, since the gravitational wave amplitude is proportional to this equation. After 30 years of theoretical and numerical attempts to simulate gravitational collapse, there is still no great progress in understanding the efficiency of this process in producing gravitational waves. For a conservative estimate of the energy in non-spherical motions during the collapse, relation leads to events of an amplitude detectable in our galaxy, even by bar detectors. The next generation of laser interferometers would be able to detect such signals from Virgo cluster at a rate of a few events per month. The main source for deviations from spherical or axial symmetry during the collapse is the angular momentum. During the contraction phase, the angular momentum is conserved, and the star spins up to rotational periods of the order of 1 msec. In this case, consequent processes with large luminosity might take place in this newly born neutron star. A number of instabilities, such as the so-called bar mode instability and the r-mode instability, may occur which radiate copious amounts of gravitational radiation immediately after the initial burst. Gravitational wave signals from these rotationally induced stellar instabilities are detectable from sources in our galaxy and are marginally detectable if the event takes place in the nearby cluster of about 2500 galaxies, the Virgo cluster, 15 Mpc away from the Earth. Additionally, there will be weaker but extremely useful signals due to subsequent oscillations of the neutron star; f, p and w modes are some of the main patterns of oscillations (normal modes) of the neutron star that observers might search for These modes have been studied in detail and once detected in the signal, they would provide a sensitive probe of the neutron star structure and its supranuclear equation of state. Detectors with high sensitivity in the kHz band will be needed in order to fully develop this so-called gravitational wave asteroseismology. If the collapsing central core is unable to drive off its surrounding envelope, then the collapse continues and finally a black hole forms. In this case the instabilities and oscillations that we discussed above are absent and the newly formed black hole radiates away, within a few milliseconds, any deviations from axisymmetry and ends up as a rotating or Kerr black hole. The characteristic oscillations of black holes (normal modes) are well studied, and this unique ringing down of a black hole could be used as a direct probe of their existence. The frequency of the signal is inversely proportional to the black hole mass. For example, it has been stated earlier that a 100Mblack hole will oscillate at a frequency of ∼100 Hz (an ideal source for LIGO), while a supermassive one with mass 107M which might be excited by an infalling star, will ring down at a frequency of 10−3 Hz (an ideal source for LISA). The analysis of such a signal should reveal directly the two parameters that characterize any (uncharged) black hole; namely its mass and angular momentum.

2.2 Radiation from binary systems

Binary systems are the best sources of gravitational waves because they emit copious amounts or gravitational radiation, and for a given system we know exactly what is the amplitude and frequency of the gravitational waves in terms of the masses of the two bodies and their separation. If a binary system emits detectable gravitational radiation in the bandwidth of our detectors, we can easily identify the parameters of the system. According to the formula the observed frequency change will be

$$\dot{f} \sim f^{11/3}\mathcal{M}^{5/3},$$

and the corresponding amplitude will be

$$h \sim \frac{\mathcal{M}^{5/3} f^{2/3}}{r} = \frac{\dot{f}}{r f^3},$$

Where $\mathcal{M}^{5/3} = \mu M^{2/3}$ is a combination of the total and reduced mass of the system, called chirp mass. Since both frequency f and its rate of change ˙f are measurable quantities, we can immediately compute the chirp mass (from the first relation), thus obtaining a measure of the masses involved. The second relation provides a direct estimate of the distance of the source. These relations have been derived using the Newtonian theory to describe the orbit of the system and the quadrupole formula for the emission of gravitational waves. Post-Newtonian theory inclusion of the most important relativistic corrections in the description of the orbit can provide more accurate estimates of the individual masses of the components of the binary system. When analysing the data of periodic signals the effective amplitude is not the amplitude of the signal alone but $h_c = \sqrt{n} \cdot h$, where n is the number of cycles of the signal within the frequency range where the detector is sensitive. A system consisting of two typical neutron stars will be detectable by LIGO when the frequency of the gravitational waves is ∼10Hz until the final coalescence around 1000Hz. This process will last for about 15 min and the total number of observed cycles will be of the order of 10^4 , which leads to an enhancement of the detectability by a factor 100. Binary neutron star systems and binary black hole systems with masses of the order of 50M are the primary sources for LIGO. Given the anticipated sensitivity of LIGO, binary black hole systems are the most promising sources and could be detected as far as 200 Mpc away. The event rate with the present estimated sensitivity of LIGO is probably a few events per year, but future improvement of detector sensitivity (the LIGO II phase) could lead to the detection of at least one event per month. Supermassive black hole systems of a few million solar masses are the primary source for LISA. These binary systems are rare, but due to the huge amount of energy released, they should be detectable from as far as the boundaries of the observable universe.

2.3 Radiation from spinning neutron stars

A perfectly axisymmetric rotating body does not emit any gravitational radiation. Neutron stars are axisymmetric configurations, but small deviations cannot be ruled out. Irregularities in the crust (perhaps imprinted at the time of crust formation), strains that have built up as the stars have spun down, off-axis magnetic fields, and/or accretion could distort the axisymmetry. A bump that might be created at the surface of a neutron star spinning with frequency f will produce gravitational waves at a frequency of 2f and such a neutron star will be a weak but continuous and almost monochromatic source of gravitational waves. The radiated energy comes at the expense of the rotational energy of the star, which leads to a spin down of the star. If gravitational wave emission contributes considerably to the observed spin down of pulsars, then we can estimate the amount of the emitted energy. The corresponding amplitude of gravitational waves from nearby pulsars (a few kpc away) is of the order of $h \sim 10^{-25} - 10^{-26}$, which is extremely small. If we accumulate data for sufficiently long time, e.g., 1 month, then the effective amplitude, which increases as the square root of the number of cycles, could easily go up to the order of $h_c \sim 10^{-22}$. We must admit that at present we are extremely ignorant of the degree of asymmetry in rotating neutron stars, and these estimates are probably very optimistic. On the other hand, if we do not observe gravitational radiation from a given pulsar we can place a constraint on the amount of non axisymmetry of the star.

2.4 Cosmological Gravitational Waves

One of the strongest pieces of evidence in favour of the Big Bang scenario is the 2.7 K cosmic microwave background radiation. This thermal radiation first bathed the universe around 380,000 years after the Big Bang. By contrast, the gravitational radiation background anticipated by theorists was produced at Planck times, i.e., at 10^{-32} sec or earlier after the Big Bang. Such gravitational waves have travelled almost unimpeded through the universe since they were generated. The observation of cosmological gravitational waves will be one of the most important contributions of gravitational wave astronomy. These primordial gravitational waves will be, in a sense, another source of noise for our detectors and so they will have to be much stronger than any other internal detector noise in order to be detected. Otherwise, confidence in detecting such primordial gravitational waves could be gained by using a system of two detectors and cross-correlating their outputs. The two LIGO detectors are well placed for such a correlation.

Conclusion

This paper as astronomical sources of gravitational waves with its application has been focussed. These primordial gravitational waves will be, in a sense, another source of noise for our detectors and so they will have to be much stronger than any other internal detector noise in order to be detected. Otherwise, confidence in detecting such primordial gravitational waves could be gained by using a system of two detectors and cross-correlating their outputs. The two LIGO detectors are well placed for such a correlation.The new generation of gravitational wave detectors (LIGO, VIRGO) have very good chances of detecting gravitational waves, but until these expectations are fulfilled, we can only make educated guesses as to the possible astronomical sources of gravitational waves. The detectability of these sources depends on three parameters: their intrinsic gravitational wave luminosity, their event rate, and their distance from the Earth. The luminosity can be approximately estimated via the quadrupole formula. Even though there are certain restrictions in its applicability (weak field, slow motion), it provides a very good order-of-magnitude estimate for the expected gravitational wave flux on Earth. The rate, at which various events with high luminosity in gravitational waves take place is extrapolated from astronomical observations in the electromagnetic spectrum. Still, there might be a number of gravitationally luminous sources, for example binary black holes, for which we have no direct observations in the electromagnetic spectrum. Finally, the amplitude of gravitational wave signals decreases as one over the distance to the source.

References

[1] Fundamentals of Interferometric Gravitational Wave Detectors, P.R. Saulson, World Scientific (1994).
[2] The Detection of Gravitational Waves, D.G. Blair, Cambridge University Press (1991). [3] Relativistic Gravitation and Gravitational Radiation, Cambridge University Press, Editors J-A. Marck and J-P. Lasota (1997).
[4] Gravitational Radiation, K.S. Thorne, in S.W. Hawking and W. Israel (eds), 300 Years of Gravitation, (Cambridge University Press, Cambridge).
[5] Gravitational Wave Physics, K.D. Kokkotas, Encyclopedia of Physical Science and Technology, 3rd Edition, Volume 7 Academic Press, (2002)
[6] An Overview of Gravitational-Wave Sources C. Cutler and K.P. Thorne gr-qc/0204090 April 30th 2002.
[7] mini-GRAIL http://www.minigrail.nl
[8] LIGO http://www.ligo.caltech.edu
[9] VIRGO http://www.pi.infn.it/virgo/virgoHome.html
[10] GEO http://www.geo600.uni-hannover.de
[11] J.R. Smith et al., Class. Quant. Grav. 26 (2009) 114013
[12] P. Fritschel, LIGO presentation G030460 (August 2003); S. Hild et al., Class. Quant. Grav. 26 (2009) 055012 ; T.T. Fricke et al., Class. Quant. Grav. 29 (2012) 065005
[13] R. Weiss, LIGO technical note T0900061 (February 2008)
[14] R. Schofield, LIGO presentation G1000923 (September 2010)
[15] B. Abbott et al., Phys. Rev. D 85 (2012) 082002
[16] F. Acernese et al., Astropart. Phys. 33 (2010) 182
[17] B. Abbott et al., Phys. Rev. D 73 (2006) 102002