# Intuitionistic (S,T)-Fuzzy $H_{\nu}$ -Ideals and their Characterizations

Manoj Kumar Dewangan M M College of Technology Raipur Chhattisgarh

**Abstract:** In this paper, we apply the concept of intuitionistic fuzzy set to  $H_{\nu}$ -rings. The notion of an intuitionistic (S, T)-fuzzy  $H_{\nu}$ -ideals of an  $H_{\nu}$ -ring is introduced and some related properties are investigated.

**Keywords:**  $H_{\nu}$ -ring,  $H_{\nu}$ -ideal, fuzzy  $H_{\nu}$ -ring, fuzzy  $H_{\nu}$ -ideal, intuitionistic (S, T)- fuzzy  $H_{\nu}$ -ideal.

**Mathematics Subject Classification: 20N20** 

### 1. Introduction

The concept of hyperstructure was introduced in 1934 by Marty [12]. Hyperstructures have many applications to several branches of pure and applied sciences. Vougiouklis [14] introduced the notion of  $H_{\nu}$ -structures, and Davvaz [5] surveyed the theory of  $H_{\nu}$ -structures. After the introduction of fuzzy sets by Zadeh [16], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. For more details on intuitionistic fuzzy sets, we refer the reader to [2, 3].

In [4] Biswas applied the concept of intuitoinistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group. In [10] Kim et al. introduced the notion of fuzzy subquasigroups of a quasigroup. In [11] Kim and Jun introduced the concept of fuzzy ideals of a semigroup. Zhan et al. [17] introduced the notion of an intuitionistic (S, T)-fuzzy  $H_{\nu}$ -submodule of an  $H_{\nu}$ -module. This paper continues this line of research for fuzzy  $H_{\nu}$ -ideal of  $H_{\nu}$ -ring. In this paper, the notion of an intuitionistic (S, T)-fuzzy  $H_{\nu}$ -ideal of an  $H_{\nu}$ -ring is

introduced and some related properties are investigated.

The paper is organized as follows: in section 2 some fundamental definitions on  $H_{\nu}$ -structures and fuzzy sets are explored, in section 3 we define intuitionistic (S, T)-fuzzy  $H_{\nu}$ -ideals and establish some useful theorems.

## 2. Basic Definitions

We first give some basic definitions for proving the further results.

**Definition 2.1** [6] Let X be a non-empty set. A mapping  $\mu: X \to [0,1]$  is called a fuzzy set in X. The complement of  $\mu$ , denoted by  $\mu^c$ , is the fuzzy set in X given by

$$\mu^{c}(x) = 1 - \mu(x) \quad \forall x \in X.$$

**Definition 2.2** [6] Let f be a mapping from a set X to a set Y. Let  $\mu$  be a fuzzy set in X and  $\lambda$  be a fuzzy set in Y. Then the inverse image  $f^{-1}(\lambda)$  of  $\lambda$  is a fuzzy set in X defined by

$$f^{-1}(\lambda)(x) = \lambda(f(x)) \quad \forall x \in X.$$

The image  $f(\mu)$  of  $\mu$  is the fuzzy set in Y defined by

$$f(\mu)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x), & f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

For all  $y \in Y$ .

**Definition 2.3** [6] An intuitionistic fuzzy set A in a non-empty set X is an object having the form  $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ , where the functions  $\mu_A : X \to [0, 1]$  and  $\lambda_A : X \to [0, 1]$  denote the degree of membership and degree of non membership of each element  $x \in X$  to the set A respectively and  $0 \le \mu_A(x) + \lambda_A(x) \le 1$  for all  $x \in X$ . We shall use the symbol  $A = \{\mu_A, \lambda_A\}$  for the intuitionistic fuzzy set  $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}.$ 

**Definition 2.4** [6] Let  $A = \{\mu_A, \lambda_A\}$  and  $B = \{\mu_B, \lambda_B\}$  be intuitionistic fuzzy sets in X. Then

- (1)  $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$  and  $\lambda_A(x) \le \lambda_B(x)$ ,
- (2)  $A^c = \{(x, \lambda_A(x), \mu_A(x)) : x \in X\}$

$$(3) A \cap B = \begin{cases} (x, \min\{\mu_A(x), \mu_B(x)\}, \\ \max\{\lambda_A(x), \lambda_B(x)\} : x \in X \end{cases},$$

$$(4) A \cup B = \begin{cases} (x, \max\{\mu_A(x), \mu_B(x)\}, \\ \min\{\lambda_A(x), \lambda_B(x)\} : x \in X \end{cases},$$

$$(4) A \cup B = \begin{cases} (x, \max\{\mu_A(x), \mu_B(x)\}, \\ \min\{\lambda_A(x), \lambda_B(x)\} : x \in X \end{cases},$$

$$(5) \Box A = \{(x, \mu_A(x), \mu_A^c(x)) : x \in X\},\$$

$$(6) \Diamond A = \{(x, \lambda_A^c(x), \lambda_A(x)) : x \in X\}.$$

**Definition 2.5** [15] Let G be a non-empty set and  $*: G \times G \to \wp^*(G)$  be a hyperoperation, where  $\wp^*(G)$  is the set of all the non-empty subsets of G. Where  $A*B = \bigcup_{a \in A, b \in B} a*b, \forall A, B \subseteq G$ .

The \* is called weak commutative if  $x * y \cap y * x \neq \emptyset$ ,  $\forall x, y \in G$ .

The \* is called weak associative if  $(x * y) * z \cap x * (y * z) \neq \phi$ ,  $\forall x, y, z \in G$ .

A hyperstructure (G, \*) is called an  $H_{\nu}$ -group if

- (i) \* is weak associative.
- (ii) a \* G = G \* a = G,  $\forall a \in G$  (Reproduction axiom).

**Definition 2.6** [7] Let G be a hypergroup (or  $H_{\nu}$ -group) and let  $\mu$  be a fuzzy subset of G. Then  $\mu$  is said to be a fuzzy subhypergroup (or fuzzy  $H_{\nu}$ -subgroup) of G if the following axioms hold:

(i)  $\min\{\mu(x), \mu(y)\} \le \inf_{\alpha \in x^*y} \{\mu(\alpha)\}, \quad \forall x, y \in G \ (ii) \ \text{For all} \ x, a \in G \ \text{there exists} \ y \in G \ \text{such that} \ x \in a * y \ \text{and}$  $\min\{\mu(a), \mu(x)\} \le \{\mu(y)\}.$ 

**Definition 2.7** [13] Let G be a hypergroup (or  $H_{\nu}$ -group). An intuitionistic fuzzy set  $A = \{\mu_{A}, \lambda_{A}\}$  of G is called intuitionistic fuzzy subhypergroup (or intuitionistic fuzzy  $H_{\nu}$ -subgroup) of G if the following axioms hold:

- (i)  $\min\{\mu_A(x), \mu_A(y)\} \le \inf_{\alpha \in \mathbb{R}^n} \{\mu_A(\alpha)\}, \quad \forall x, y \in G.$
- (ii) For all  $x, a \in G$  there exists  $y \in G$  such that  $x \in a * y$  and  $\min\{\mu_A(a), \mu_A(x)\} \le \{\mu_A(y)\}$ .
- (iii)  $\sup \{\lambda_A(\alpha)\} \le \max \{\lambda_A(x), \lambda_A(y)\}, \forall x, y \in G.$
- (iv) For all  $x, a \in G$  there exists  $y \in G$  such that  $x \in a * y$  and  $\{\lambda_A(y)\} \le \max\{\lambda_A(a), \lambda_A(x)\}$ .

**Definition 2.8** [15] An H<sub> $\nu$ </sub>-ring is a system  $(R, +, \cdot)$  with two hyperoperations satisfying the ring-like axioms:

(i)  $(R,+,\cdot)$  is an  $H_{\nu}$ -group, that is,

$$((x+y)+z)\cap(x+(y+z))\neq\phi\quad\forall x,y\in R,$$

 $a + R = R + a = R \quad \forall a \in R;$ 

- (ii)  $(R,\cdot)$  is an  $H_{\nu}$ -semigroup;
- (iii) (·) is weak distributive with respect to (+), that is, for all  $x, y, z \in R$ ,

$$(x \cdot (y+z)) \cap (x \cdot y + x \cdot z) \neq \phi,$$
  
$$((x+y) \cdot z) \cap (x \cdot z + y \cdot z) \neq \phi.$$

**Definition 2.9** [9] Let R be an  $H_{\nu}$ -ring. A nonempty subset I of R is called a left (resp., right)  $H_{\nu}$ -ideal if the following axioms hold:

- (i) (I,+) is an  $H_{\nu}$ -subgroup of (R,+),
- (ii)  $R \cdot I \subseteq I$  (resp.,  $I \cdot R \subseteq I$ ).

**Definition 2.10** [9] Let  $(R,+,\cdot)$  be an  $H_{\nu}$ -ring and  $\mu$  a fuzzy subset of R. Then  $\mu$  is said to be a left (resp., right) fuzzy  $H_{\nu}$ -ideal of R if the following axioms hold:

$$(1) \min\{\mu(x), \mu(y)\} \le \inf\{\mu(z) : z \in x + y\}$$
$$\forall x, y \in R,$$

- (2) For all  $x, a \in R$  there exists  $y \in R$  such that  $x \in a + y$  and  $\min\{\mu(a), \mu(x)\} \le \mu(y)$ ,
- (3) For all  $x, a \in R$  there exists  $z \in R$  such that  $x \in z + a$  and  $\min\{\mu(a), \mu(x)\} \le \mu(z)$ ,

$$(4)\mu(y) \le \inf\{\mu(z): z \in x \cdot y\}$$
 [respectively  $\mu(x) \le \inf\{\mu(z): z \in x \cdot y\} \quad \forall x, y \in R$ ].

**Definition 2.11** [9] An intuitionistic fuzzy set  $A = \{\mu_A, \lambda_A\}$  in R is called a left (resp., right) intuitionistic fuzzy  $H_V$ -ideal of R if

$$(1) \min\{\mu_A(x), \mu_A(y)\} \le \inf\{\mu_A(z) : z \in x + y\}$$

$$\max\{\lambda_A(x), \mu_A(y)\} \ge \sup\{\lambda_A(z) : z \in x + y\}$$

$$\forall x, y \in R;$$

- (2) For all  $x, a \in R$  there exists  $y \in R$  such that  $x \in a + y$  and  $\min\{\mu_A(a), \mu_A(x)\} \le \mu_A(y)$  and  $\max\{\lambda_A(a), \lambda_A(x)\} \ge \lambda_A(y)$ ;
- (3) For all  $x, a \in R$  there exists  $z \in R$  such that  $x \in z + a$  and  $\min\{\mu_A(a), \mu_A(x)\} \le \mu_A(z)$  and  $\max\{\lambda_A(a), \lambda_A(x)\} \ge \lambda_A(z)$ ;

$$(4)\mu_{A}(y) \leq \inf\{\mu_{A}(z) : z \in x \cdot y\} \text{ [respectively } \mu_{A}(x) \leq \inf\{\mu_{A}(z) : z \in x \cdot y\} \quad \forall x, y \in R \text{ ]}$$
 and 
$$\lambda_{A}(y) \geq \sup\{\lambda_{A}(z) : z \in x \cdot y\} \text{ [respectively } \lambda_{A}(x) \geq \sup\{\lambda_{A}(z) : z \in x \cdot y\} \quad \forall x, y \in R \text{ ]}.$$

**Definition 2.12** [17] By a t-norm T, we mean a function  $T:[0,1]\times[0,1]\to[0,1]$  satisfying the following conditions:

$$(i)T(x,1) = x,$$

$$(ii)T(x,y) \le T(x,z) \quad \text{if } y \le z,$$

$$(iii)T(x,y) = T(y,x),$$

$$(iv)T(x,T(y,z)) = T(T(x,y),z)$$
For all  $x, y, z \in [0,1]$ .

**Definition 2.13** [17] By a *s*-norm *S*, we mean a function  $S:[0,1]\times[0,1]\to[0,1]$  satisfying the following conditions:

$$(i) S(x,0) = x,$$

$$(ii)S(x,y) \le S(x,z)$$
 if  $y \le z$ ,

$$(iii)S(x,y) = S(y,x),$$

$$(iv)S(x,S(y,z))=S(S(x,y),z)$$

For all  $x, y, z \in [0,1]$ .

It is clear that

 $T(\alpha, \beta) \le \min\{\alpha, \beta\} \le \max\{\alpha, \beta\} \le S(\alpha, \beta)$  For all  $\alpha, \beta \in [0, 1]$ .

# 3. Intuitionistic (S, T)-fuzzy $H_{\nu}$ -ideal

In this section we give the definition of intuitionistic (S, T)-fuzzy  $H_{\nu}$ -ideal and prove some related results.

**Definition 3.1** Let R be a  $H_{\nu}$ -ring. An intuitionistic fuzzy set  $A = \{\mu_A, \lambda_A\}$  of R is called intuitionistic (S, T)-fuzzy  $H_{\nu}$ -ideal of R if the following axioms hold:

 $(1)T\{\mu_{A}(x), \mu_{A}(y)\} \le \inf\{\mu_{A}(z) : z \in x + y\}$ 

 $S\{\lambda_A(x), \mu_A(y)\} \ge \sup\{\lambda_A(z): z \in x+y\}$  (2) For all  $x, a \in R$  there exists  $y \in R$  such that  $x \in a+y$  and  $\forall x, y \in R$ ;

 $T\{\mu_{A}(a), \mu_{A}(x)\} \leq \mu_{A}(y) \text{ and } S\{\lambda_{A}(a), \lambda_{A}(x)\} \geq \lambda_{A}(y); \qquad (3) \text{ For all } x, a \in R \text{ there exists } z \in R \text{ such that } x \in z + a \text{ and } T\{\mu_{A}(a), \mu_{A}(x)\} \leq \mu_{A}(z) \text{ and } S\{\lambda_{A}(a), \lambda_{A}(x)\} \geq \lambda_{A}(z); \\ (4)\mu_{A}(y) \leq \inf\{\mu_{A}(z): z \in x \cdot y\} \text{ [respectively } \mu_{A}(x) \leq \inf\{\mu_{A}(z): z \in x \cdot y\} \quad \forall x, y \in R \text{ ]} \\ \lambda_{A}(y) \geq \sup\{\lambda_{A}(z): z \in x \cdot y\} \text{ [respectively } \lambda_{A}(x) \geq \sup\{\lambda_{A}(z): z \in x \cdot y\} \quad \forall x, y \in R \text{ ]}.$ 

**Definition 3.2** The norms T and S are called dual if for all  $a,b \in [0,1]$ ,  $\overline{T(a,b)} = S(\overline{a},\overline{b})$ . **Lemma 3.3** Let T and S be dual norms. If  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic (S,T)-fuzzy  $H_V$ -ideal of R, then so is  $\Box A = \{\mu_A, \mu_A^c\}$ .

**Proof** It is sufficient to show that  $\mu_A^c$  satisfies the conditions of Definition 3.1.

For 
$$x, y \in R$$
 we have  $T\{\mu_A(x), \mu_A(y)\} \le \inf\{\mu_A(z) : z \in x + y\}$  and so  $T\{1-\mu_A^c(x), 1-\mu_A^c(y)\} \le \inf\{1-\mu_A^c(z) : z \in x + y\}$ 

Hence 
$$T\{1-\mu_A^c(x), 1-\mu_A^c(y)\} \le 1-\sup\{\mu_A^c(z): z \in x+y\}$$

Which implies  $\sup \{ \mu_A^c(z) : z \in x + y \} \le 1 - T \{ 1 - \mu_A^c(x), 1 - \mu_A^c(y) \}$ 

Since T and S are dual. Therefore  $\sup \{\mu_A^c(z): z \in x + y\} \le S\{\mu_A^c(x), \mu_A^c(y)\}$ 

Now, let  $a, x \in R$ . Then there exists  $y \in R$  such that  $x \in a + y$  and  $T\{\mu_A(a), \mu_A(x)\} \le \{\mu_A(y)\}$ 

It follows that

$$T\left\{1-\mu_{A}^{c}(a),1-\mu_{A}^{c}(x)\right\} \leq \left\{1-\mu_{A}^{c}(y)\right\} \mu_{A}^{c}(y) \leq 1-T\left\{1-\mu_{A}^{c}(a),1-\mu_{A}^{c}(x)\right\} = S\left\{\mu_{A}^{c}(a),\mu_{A}^{c}(x)\right\}$$

So that

$$\{\mu_A^c(y)\} \leq S\{\mu_A^c(a), \mu_A^c(x)\}$$

Similarly, let  $a, x \in R$  then there exists  $z \in R$  such that  $x \in z + a$  and  $\{\mu_A^c(z)\} \le S\{\mu_A^c(a), \mu_A^c(x)\}$ 

Now, let  $x, y \in R$ , we have  $\mu_A(y) \le \inf\{\mu_A(z) : z \in x \cdot y\}$  since  $\mu_A$  is a T fuzzy  $H_v$ -ideal of R. Hence  $1 - \mu_A^c(y) \le \inf\{1 - \mu_A^c(z) : z \in x \cdot y\}$  which implies  $\sup\{\mu_A^c(z) : z \in x \cdot y\} \le \mu_A^c(y)$ . Therefore  $\square A = \{\mu_A, \mu_A^c\}$  is an intuitionistic (S, T)-fuzzy  $H_v$ -ideal of R

**Lemma 3.4** Let T and S be dual norms. If  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic (S, T)-fuzzy  $H_v$ -ideal of R, then so is  $\Diamond A = \{\lambda_A^c, \lambda_A\}$ .

**Proof** The proof is similar to the proof of Theorem 3.2.

Combining the above two lemmas it is not difficult to verify that the following theorem is valid.

**Theorem 3.5** Let T and S be dual norms. Then  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic (S, T) - fuzzy  $H_V$ -ideal of R if and only if  $\Box A$  and  $\Diamond A$  are intuitionistic (S, T) - fuzzy  $H_V$ -ideal of R

**Corollary 3.6** Let T and S be dual norms. Then  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic (S, T) - fuzzy  $H_v$ -ideal of R if and only if  $\mu_A$  and  $\lambda_A^c$  are intuitionistic (S, T) -fuzzy  $H_v$ -ideal of R

**Definition 3.7** For any  $t \in [0,1]$  and a fuzzy set  $\mu$  in R the set  $U(\mu;t) = \{x \in R : \mu(x) \ge t\}$   $L(\mu;t) = \{x \in R : \mu(x) \le t\}$ 

is called an upper (respectively, lower) t-level cut of  $\mu$ .

**Definition 3.8** An intuitionistic (S, T)-fuzzy  $H_v$ -ideal  $A = \{\mu_A, \lambda_A\}$  of R is said to be imaginable if  $\mu_A$  and  $\lambda_A$  satisfy the imaginable property.

The following are obvious.

**Lemma 3.9** Every imaginable intuitionistic (S, T) - fuzzy  $H_v$ -ideal of R is intuitionistic fuzzy  $H_v$ -ideal.

**Lemma 3.10** [8] A fuzzy set  $\mu$  in R is a fuzzy  $H_{\nu}$ -ideal of R if and only if the non empty set  $U(\mu;t), t \in [0,1]$  is an  $H_{\nu}$ -ideal of R

**Lemma 3.11** [8] A fuzzy set  $\mu$  in R is a fuzzy  $H_{\nu}$ -ideal of R if and only if the non empty set  $\mu^{c}$  is an antifuzzy  $H_{\nu}$ -ideal of R

By the above Lemmas, we can give the following results.

**Theorem 3.12** If  $A = \{\mu_A, \lambda_A\}$  is an imaginable intuitionistic fuzzy set in R Then  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic (S, T)- fuzzy  $H_v$ -ideal of R if and only if the non-empty sets  $U(\mu_A; t)$  and  $L(\lambda_A; t)$  are  $H_v$ -ideal of R for every  $t \in [0,1]$ .

**Theorem 3.13** If  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic (S, T)-fuzzy  $H_v$ -ideal of R then  $\mu_A(x) = \sup\{\alpha \in [0,1]: x \in U(\mu_A; \alpha)\}$  And  $\lambda_A(x) = \inf\{\alpha \in [0,1]: x \in L(\lambda_A; \alpha)\}$  For all  $x \in R$ .

**Definition 3.14** Let  $f: R \to R'$  be a strong epimorphism of  $H_v$ -rings. If  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic fuzzy set in R', then the inverse image of A under f, denoted by  $f^{-1}(A)$ , is an intuitionistic fuzzy set in R defined by  $f^{-1}(A) = (f^{-1}(\mu_A), f^{-1}(\lambda_A))$ .

By the above Definition, we can give the following result.

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**Theorem 3.15** Let  $f: R \to R'$  be a strong epimorphism of  $H_v$ -ring. If  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic (S, T)-fuzzy  $H_v$ -ideal of R'. Then the inverse image  $f^{-1}(A) = (f^{-1}(\mu_A), f^{-1}(\lambda_A))$  of A under f is an intuitionistic (S, T)-fuzzy  $H_v$ -ideal of R

**Definition 3.16** A fuzzy set  $\mu$  in a set X is said to have sup property if for every non-empty subset S of X, there exists  $x_0 \in S$  such that

$$\mu(x_0) = \sup_{x \in S} \left\{ \mu(x) \right\}$$

**Proposition 3.17** Let R and R' be two  $H_v$ -rings and  $f: R \to R'$  be a surjection. If  $A = \{\mu_A, \lambda_A\}$  is an intuitionistic (S, T)-fuzzy  $H_v$ -ideal of R such that  $\mu_A$  and  $\lambda_A$  have sup property, then

$$(i) f(U(\mu_A;t)) = U(f(\mu_A);t);$$

$$(ii) f(L(\lambda_A;t)) \supseteq L(f(\lambda_A);t).$$

# **Proof**

$$(i) y \in f(U(\mu_{A};t)) \Leftrightarrow f(\mu_{A})(y) \geq t$$

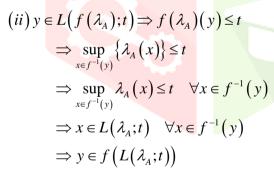
$$\Leftrightarrow \sup_{x \in f^{-1}(y)} \{\mu_{A}(x)\} \geq t$$

$$\Leftrightarrow \exists x_{0} \in f^{-1}(y), \mu_{A}(x_{0}) \geq t$$

$$\Leftrightarrow \exists x_{0} \in f^{-1}(y), x_{0} \in U(\mu_{A};t)$$

$$\Leftrightarrow f(x_{0}) = y, x_{0} \in U(\mu_{A};t)$$

$$\Leftrightarrow y \in f(U(\mu_{A};t))$$



**Proposition 3.18** Let R and R' be two  $H_v$ -rings and  $f: R \to R'$  be a map. If  $B = \{\mu_B, \lambda_B\}$  is an intuitionistic (S, T)-fuzzy  $H_v$ -ideal of R' then

$$(i) f^{-1}(U(\mu_B;t)) = U(f^{-1}(\mu_B);t);$$

$$(ii) f^{-1} \left( L(\lambda_B; t) \right) = L(f^{-1}(\lambda_B); t).$$

For every  $t \in [0,1]$ .

#### **Proof**

$$(i) x \in U (f^{-1}(\mu_B);t) \Leftrightarrow f^{-1}(\mu_B)(x) \geq t$$

$$\Leftrightarrow \mu_B (f(x)) \geq t$$

$$\Leftrightarrow f(x) \in U (\mu_B;t)$$

$$\Leftrightarrow x \in f^{-1} (U (\mu_B;t))$$

$$(ii) x \in L (f^{-1}(\lambda_B);t) \Leftrightarrow f^{-1}(\lambda_B)(x) \leq t$$

$$\Leftrightarrow \lambda_B (f(x)) \leq t$$

$$\Leftrightarrow f(x) \in L(\lambda_B;t)$$

$$\Leftrightarrow x \in f^{-1} (L(\lambda_B;t))$$

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