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Openly Generated Boolean Algebra: An Overview

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ABSTRACT

In this paper, the main thrust is on developing generalised structure of subalgebra of free Boolean algebra. We have extended the results derived by. P.Ekloffs for a family of countable set. S.Shelah applied construction principle to discuss different aspects of Boolean algebra related to its length and height. We have derived some lemma and theorems to establish supremacy of strong reshuffling properties as introduced by Ekloffs. The main thrust is on extending the results of

S. Shelah and Magidor in respect of its compactness. For every infinite regular cardinal k, we construct 2^k Pairwise non-ismorphic Boolean algebaras of size k that are tightly σ-filtered and c.c.c as derived by Koppelberg. We extend the results which show that for every uncountable regular cardinal k there are only 2^k Isomorphism types of projective Boolean algebras of size k.

Key Words : uncountable, reshuffling, inclusion, abelian, filtration

Introduction

We derive here conditions so that the openly generated Boolean algebras are almost free. Unlike in the case of abelian groups, subalgebras of free Boolean algebras do not have to be free. We intro- duce here slightly generalized definition of almost freeness for Bool- ean algebras. We take help of P. Ekloffs. for notations.

σ Filtration:-

Suppose A be a Boolean algebra of size K. A filtration. $(A\alpha)\alpha<k$ of A is a σ filtration if for each $\propto <k$, $IA\alpha I<k$.

A is almost free if it has a σ—filtration $(A\propto)\propto <k$. consisting of free Boolean algebras.

Definition

Let A and B be Boolean algebras such taht $B\leq A$. Then $B\leq_{free}$ A if there is a free Boolean algebra F such that A is isomorphic to $B\oplus$ F

over B. Here $\oplus$denotes the coproduct is the category of Boolean algebras.

Hence, we find B $\leq_{free}$ A if there is a set $X\subseteq A$ such that X is independent over B and A= BUX.

Definition (λ -system)

The set $\lambda^{<w}$ ordered by set inclusion is a tree. Let (β) denote the sequence of length one with value β and let _ denote

concatenation of sequence. IF S is a Subtree of $\lambda^{<w}$ an element η of S is called a final node of S if in S there is no proper extension of η. ϕ

Let S_f denote the set of final nodes of S.

1. A λ-set S is a sub tree of $\lambda^{<w}$together with a cardinal $\lambda\eta$ for each $\eta\in s$ such that $\lambda\phi=\lambda$ and

(a) for all $\eta\in S$, $\eta\in S_f$ if and only if $\lambda\eta=N_0$ and

(b) if $\eta\phi S/S_f$, then $n\text{-}(\beta)\in S$ implies $\beta\in\lambda\eta$ and $\lambda\eta\text{-}(\beta)<\lambda\eta$ and $E\eta=\{\beta<\lambda\eta:\eta\_(\beta)\in S\}$ is stationary in $\lambda\eta$.

2. A λ- system is a λ-set together a set $\beta\eta$ for $\eta\in S$. such that $\beta\phi=\phi$ and for all $\eta\in S\backslash S_f$.

(a) for all $\beta\in E\eta,\lambda\eta\_(\beta)\leq I\beta\eta I<\lambda\eta$ (b) $\{\beta\eta\_(\beta)\}\beta\in E\eta$ is increasing and continuous i.e. if $\sigma\in E\eta$ is a limit point of $E\eta$, then $\beta\eta\_(\sigma)$

$=\bigcup\{\beta\eta\_(\beta):\beta\in E\eta\cap\sigma\}$

Theorem :

For every uncountable cardinnal λ there is an almost free, non- free abelian group of size λ if and only if NPT (λ, N_0) holds.

Proof We construct here a Boolean algebra A (s) for any given family S of countable sets. We suitable apply strong construcion Principle (CP+) for Boolean algebras. If the family S is sufficiently good, A (s) is openly generated.

Theorem

NPT (λ, N_0) implies the existence of almost free, non-free oblects of size λ in every variety V which satisfies (CP+). S. Shelah established the following lemma. We state it for its further applications.

Lemma

The variety of Boolean algebras satisfies the following strong construction Principle (CP+)

For each $\eta \in$ w\1 there are countably generated free Boolean al-gebras that $H \leq K \leq L$ and a partiton of w into η infinite block S^1, ... S^n such

H is freely generated by $\{h_m : m \in w\}$ and for each $J \subseteq w$ if for some $k \in Z$,

$J \cap S_k$ is finite, then $<\{hm:m \in J\}> \leq_{free} L$ and

(i) $L = K \oplus Fr(!)$ and $H \leq_{free} L$

Lemma

NPT (λ, N_0) implies that there is a family of countable sets based on family of countable sets on a λ-system. Shelah and Vaisanen has shown that $\lambda > N_1$, the λ- system in Lemma may be chosen such that its height is at least 2, i.e., such that the underlying set is really more than just a stationay subset of λ.

Lemma

If $\lambda > N_1$ and NPT(λ,No) holds, then there is a family of countable sets based on a λ-systgem of height η for some η>I.

Proof

S. Shelah considered a paricular kind of families of countable sets based on λ-system known as NPT (λ,N_0)- structure. NRT (λ,N_0)- structrue are applied for building another class of NPT-skeletons. Let λ>N_1 and sup- pose NPT (λ, N_o) holds. We show that there is an NPT (λ,N_o)-skeleton of height>I. We construct an NPT (λ, N_0)- skeleton of the same height. Hence, it is sufficient to show the existence of an NRT (λ,N_o)- skeleton.of height >1. By NPT (λ,N_o), there is an NRT (λ, N_0)- skeleton. If this NRT (λ, N_0)- skeleton is of height>1, we assume it is of height 1. Since the NRT (λ, N_o) skeleton is of height 1, its type is (λ). We know that there is an almost free, non-free abelian group of size N_1. It implies NPT (N_1, N_0). It thus follows that there is an NRT (N_1, N_0) skeleton of height 1. Since the NRT (λ, N_0)- skeleton is of type (λ) and since λ>N_1, the two skeleton are compatible Hence the two skeletons can be combined to an NRT (λ, N_0)- Skeleton of height 2 and of type (λ,N_1).

We thus conclude that free Boolean algebra from S_1 are based a λ-system such that (S,) has the strong reshuffling property.

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