An analysis of flow and heat transfer over a flat sheet in a permeable media with fluid-particles

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Abstract: In this article, the author investigates an impact of variable fluid properties on the flow and heat transfer characteristics of a dusty fluid over a elestic sheet. Temperature dependent fluid properties are assumed to vary as a function of the temperature. The governing coupled nonlinear partial differential equations along with the appropriate boundary conditions are transformed into coupled, nonlinear ordinary differential equations by a similarity transformation. The resultant coupled highly non-linear ordinary differential equations are solved numerically. The numerical solutions are compared with the approximate analytical solutions, obtained by a perturbation technique. The analysis reveals that even in the presence of variable fluid properties the transverse velocity of the fluid is to decrease with an increase in the fluid-particle interaction parameter. This observation holds even in the presence of magnetic field

***Keywords*:** fluid-particle interaction, MHD flow, stretching sheet, heat transfer,

1. INTRODUCTION

The investigation of two-dimensional limit layer stream and warmth exchange instigated by constant extending surfaces pulled in enthusiasm because of its different applications to designing and mechanical controls. These applications incorporate expulsion procedure, wire and fiber covering, polymer preparing, nourishment stuff handling, plan of warmth exchangers, and substance preparing gear. The idea of ceaseless extending will acquire a unidirectional introduction to expel; thusly the nature of the last item depends extensively on the stream and warmth exchange system. Keeping that in mind, the investigation of energy and warm transports inside the liquid on a ceaselessly extending surface is imperative for increasing some central comprehension of such procedures. Keeping these viable applications in view, Crane [1] started the investigation of enduring two-dimensional limit layer stream because of the extending of a versatile sheet. In this manner, a few expansions identified with Crane's [1] stream issue were made for various physical circumstances (see Gupta and Gupta [2], Grubka and Bobba [3], Siddappa and Abel [4] , Vleggaar [5], Chen [6], Dutta et al. [7], Ali [8], Cortell [9], and Liu [10]). In these examinations the limit layer estimation is considered and the limit conditions are recommended at the sheet and on the liquid at boundlessness. Burden of likeness change decreased the framework to a lot of conventional differential conditions (ODEs), which are then tackled diagnostically or numerically.

All the above authorities bind their examinations to the stream provoked by a direct broadening sheet without fluid atom suspension. The examination of two-organize streams in which solid round particles are scattered in a fluid are of excitement for a wide extent of particular issues, for instance, travel through stuffed beds, sedimentation, normal defilement, outspread segment of particles and blood rheology. The examination of the cutoff layer of fluid atom suspension stream is indispensable in choosing the particle accumulation and impingement of the particle externally. In context on these applications, Chakrabarti [11] separated the breaking point layer in a dusty gas. Datta and Mishra [12] investigated point of confinement layer stream of a dusty fluid over a semi-unlimited dimension plate. Further, investigate in this field has been finished by Agranat [13], Kumar and Sharma [14], Vajravelu and Nayfeh [15], Asmolov and Manuilovich [16], Palani et al. [17], and Gireesha et al. [18]. Kumar and Sharma [14] thought about the fluid atom suspension stream over a broadening sheet by using the least square constrained segment procedure. Starting late, Gireesha [18] dismembered the stream and warmth trade of a dusty fluid over a non-isothermal expanding sheet inside seeing non-uniform warmth source/sink.In every one of the papers over, the thermo-physical properties of the encompassing liquid molecule suspension were thought to be consistent. Be that as it may, it is notable that (Herwig and Wickern [19], Lai and Kulacki [20], Takhar et al. [21], Pop et al. [22], Hassanien [23], Subhas Abel et al. [24], Seedbeek [25], Ali [26], Prasad et al. [27]) these physical properties may change with temperature, particularly the consistency and the warm conductivity. For greasing up liquids, heat produced by inside grating and the relating ascend in the temperature influences the physical properties of the liquid, and the properties of the liquid are never again thought to be steady. The expansion in temperature prompts increment in the vehicle marvels by modifying the physical properties over the warm limit layer, thus the warmth exchange at the divider is additionally influenced. Thusly to anticipate the stream and warmth exchange rates, it is important to consider the variable liquid properties.

Propelled by these investigations, we broaden crafted by Vajravelu and Nayfeh [15] by considering the temperature-subordinate variable liquid properties. In this way in the present paper, we contemplate the impacts of variable consistency and variable warm conductivity on the hydromagnetic, liquid molecule suspension stream and warmth exchange over an extending sheet. The coupled non-direct halfway differential conditions administering the issue are decreased to an arrangement of coupled non-straight common differential conditions by applying an appropriate closeness change. These non-straight coupled differential conditions are explained numerically by the Keller-Box strategy for various estimations of the relevant parameters. The numerical outcomes are displayed through tables and charts. Further, the striking highlights of the stream and warmth exchange qualities are talked about.

2. MATHEMATICAL FORMULATION

Consider the steady flow of a viscous, incompressible and electrically conducting dusty fluid over a horizontal stretching sheet with a stretching linear velocity. The thermo-physical fluid properties are assumed to be isotropic and constant, except for the fluid viscosity and the fluid thermal conductivity which are assumed to vary as a function of temperature in the following forms:

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma\left(T-T_\infty\right)\right], \tag{1}$$

$$K\left(T\right)=K_\infty\left(1+\varepsilon\frac{T-T_\infty}{\Delta T}\right), \tag{2}$$

where μ_∞ and K_∞ are the ambient fluid viscosity and thermal conductivity respectively. ε is a small parameter known as the variable thermal conductivity parameter, T is the temperature of the fluid and $\Delta T=\left(T_w-T_\infty\right)$. Equation (1) can be written as,

$$\frac{1}{\mu}=a\left(T-T_r\right), \tag{3}$$

where

$$a=\frac{\gamma}{\mu_\infty}\text{ and } T_r=T_\infty-\frac{1}{\gamma}. \tag{4}$$

Both a and T_r are constants and their values depend on the reference state and the thermal property of the fluid, i.e. γ (a constant). In general, $a>0$ for liquids and $a<0$ for gases, when $T_w>T_\infty$. The correlations between the viscosity and the temperature for air and water are given as follows:

For air: $\frac{1}{\mu}=-123.2\left(T-742.6\right)$, based on $T_\infty = 293$ K (20^0C),

and for water : $\frac{1}{\mu}=-29.83\left(T-258.6\right)$, based on $T_\infty = 288$ K (15^0C). Also, let θ_r be the constant which is defined by

$$\theta_r=\frac{T_r-T_\infty}{\Delta T}=-\frac{1}{\gamma\,\Delta T}. \tag{5}$$

It is worth mentioning here that for $\gamma\to 0$ i.e. $\mu=\mu_\infty$ (constant), $\theta_r\to\infty$. It is also important to note that θ_r is negative for liquids and positive for gases. This is due to the fact that viscosity of a liquid usually decreases with increasing temperature while it increases for gases. The reference temperatures selected here for the correlations are very useful for most applications (see for details Refs. [28-29]). The flow region is exposed under uniform transverse magnetic field $\mathbf{B}=(0,B_0,0)$ and the imposition of such a magnetic field, stabilizes the boundary layer flow. It is assumed that the flow is generated by stretching of an elastic sheet from a slit by imposing two equal and opposite forces in such a way that sheet is intact. It is also assumed that the magnetic Reynolds number is very small and the electric field due to polarization of charges is negligible. Under these conditions, the basic boundary-layer equations for continuity, conservation of mass (with no pressure gradient) and energy can be written as,

$$\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}=0, \tag{6}$$

$$\rho_\infty\left(u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\sigma B_0^2 u-\frac{\rho_p}{\tau}\left(u-u_p\right), \tag{7}$$

$$\left(u_p\frac{\partial u_p}{\partial x}+\mathrm{v}_p\frac{\partial u_p}{\partial y}\right)=\frac{1}{\tau}\left(u-u_p\right), \tag{8}$$

$$\left(u_p\frac{\partial \mathrm{v}_p}{\partial x}+\mathrm{v}_p\frac{\partial \mathrm{v}_p}{\partial y}\right)=\frac{1}{\tau}\left(\mathrm{v}-\mathrm{v}_p\right), \tag{9}$$

$$\frac{\partial}{\partial x}\left(\rho_p u_p\right)+\frac{\partial}{\partial y}\left(\rho_p \mathrm{v}_p\right)=0, \tag{10}$$

$$u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha(T)\frac{\partial T}{\partial y}\right)+\frac{\rho_p c_s}{\gamma_T\rho_\infty c_p}\left(T_p-T\right), \tag{11}$$

$$u_p\frac{\partial T_p}{\partial x}+\mathrm{v}_p\frac{\partial T_p}{\partial y}=-\frac{1}{\gamma_T}\left(T_p-T\right), \tag{12}$$

where (u,v) and (u_p,v_p) are the velocities components of the fluid and particle phases along the x and y axes respectively. Furthermore μ and ρ_∞ are the coefficients of viscosity of the fluid and the density of the fluid. Here $\tau=1/k$ is the relaxation time of particles, $k=6\pi\mu_\infty D$ is the Stokes' constant, and D is the average radius of the dust particles. Further, σ is the electrical conductivity, B_0 is the uniform magnetic field, and ρ_p is the mass of the dust particles per unit volume of the fluid: T and T_p are respectively the temperatures of the fluid and the dust particles. Further, c_p and c_s are respectively the specific heat capacity of the fluid and specific heat capacity of the dust particles, γ_T is the temperature relaxation time $(=3\,\mathrm{Pr}\,\gamma_p c_s/2c_p)$, γ_p is the velocity relaxation time $(=1/k)$, and Pr is the usual Prandtl number.

The last term in equation (7) represents the force due to the relative motion between the fluid and the dust particles. In such a case the force between dust and fluid is proportional to the relative velocity. $\alpha(T)=K(T)/\rho_\infty c_p$ is the thermal diffusivity of the fluid: It varies as a linear function of temperature. In deriving these equations the Stokesian drag force is considered for the interaction between the fluid and the particle phases. The appropriate boundary condition on velocity and temperature are

$$\begin{aligned}&u=U_w(x)=bx,\quad \mathrm{v}=0,\quad T=T_w=A_1\left(x/l\right)\quad \text{at}\quad y=0,\\&u\to 0,\quad u_p\to 0,\quad \mathrm{v}_p\to\mathrm{v},\quad \rho_p\to k\rho_\infty,\quad T\to T_\infty,\quad T_p\to T_\infty\quad \text{as}\quad y\to\infty.\end{aligned} \tag{13}$$

Here b is a constant known as stretching rate, A_1 is a constant and l is the characteristic length. Now, let the dimensionless similarity variable be

$$\eta=\sqrt{\frac{b}{\nu_\infty}}\;y \tag{14}$$

and the dimensionless similarity functions are

$$u = bx\, f'(\eta), \quad \mathrm{v} = -\sqrt{b\nu_\infty}\; f(\eta),\; u_p = bx\; F(\eta),\; \mathrm{v}_p = \sqrt{b\nu_\infty}\; G(\eta).$$
$$\rho_r = H(\eta),\; T - T_\infty = (T_w - T_\infty)\,\theta(\eta),\; T_p - T_\infty = (T_w - T_\infty)\,\theta_p(\eta),\; (T_w - T_\infty) = A(x/l). \tag{15}$$

Substituting the expressions for variable fluid viscosity and the variable fluid thermal conductivity from the equations (1) and (2) into equations (6) to (13) and making use of similarity equations from (14)-(15), we obtain

$$\left(\frac{f''}{(1-\theta/\theta_r)}\right)' + f\,f'' - f'^2 - Mn\,f' + \beta\,H(F - f') = 0,$$
$$GF' + F^2 + \beta(F - f') = 0$$
$$GG' + \beta(f + G) = 0$$
$$GH' + HG' + FH = 0 \tag{16}$$
$$\left((1+\varepsilon\theta)\theta'\right)' + \Pr(f\theta' - f'\theta) + \frac{2}{3}\beta H(\theta_p - \theta) = 0$$
$$2F\,\theta_p + G\,\theta_p{}' + L_0\;\beta(\theta_p - \theta) = 0$$

and

$$f' = 1, \quad f = 0, \quad \theta = 1 \qquad y = 0,$$
$$f' \to 0, \quad F \to 0, \quad G \to -f, \quad H \to k, \quad \theta \to 0, \quad \theta_p \to 0 \quad \text{as} \quad y \to \infty. \tag{17}$$

where a prime denotes differentiation with respect to η. Here $\rho_r = \rho_p/\rho_\infty$ is the relative density, $Mn = \sigma B_0^2/\rho_\infty b$ is the magnetic parameter, $\beta = 1/b\tau$ is the fluid particle interaction parameter, $\theta_r = 1/\gamma(\Delta T)$ is the fluid viscosity parameter, which is negative for liquids and positive for gases, $\Pr = \nu_\infty/\alpha_\infty$ is the Prandtl number, and ε is a small parameter known as the variable thermal conductivity parameter and $L_0 = \tau/\gamma_T$ is the temperature relaxation parameter. The value of θ_r is determined by the viscosity of the fluid under consideration and the operating temperature difference. If θ_r is large, in other words, if $(T_\infty - T_w)$ is small, the effects of variable viscosity on the flow can be neglected. On other hand, for smaller values of θ_r, either the fluid viscosity changes markedly with temperature or the operating temperature difference is high. In either case, the effect of the variable fluid viscosity is expected to be very important. Also let us keep in mind that the liquid viscosity varies differently with temperature compared to the gas viscosity. Therefore it is important to note that θ_r is negative for liquids and positive for gases.

3. SOLUTIONS FOR SOME SPECIAL CASES

In the limiting case of $\theta_r \to \infty$ and $\varepsilon = 0$, the system of equations (16) reduces to those of Gireesha et al. [18] and with those of Vajravelu and Nayefh [15], when no heat transfer is considered; also, for □ = 0 to those of Chakrabarthi and Gupta [30]. In the presence of variable fluid properties, when there is no fluid interaction and no magnetic field, the system of equations (16) reduces to those of Pop et al. [22]. Further, when the variable thermo-physical properties, fluid particle interaction and the magnetic field are absent, equations are similar to the ones studied by Crane [1], and Grubka and Bobba [4].

In the absence of variable fluid properties, the hydromagnetic boundary layer flow and heat transfer problem degenerates. In this case, the approximate analytical solutions for the velocity field and temperature fields are obtained via perturbation analysis. These solutions are useful and serve as a baseline for comparison with the solutions obtained via numerical schemes.

4. NUMERICAL PROCEDURE

The system of equations (16) is coupled and highly non-linear. Exact analytical solutions are not possible for the complete set of equations and therefore we use the efficient numerical method with second order finite difference scheme known as the Keller-Box method [27, 1-32]. The coupled boundary value problem (16,17); third order in $f(\eta)$, first order in $F(\eta), G(\eta), H(\eta), \theta_p(\eta)$ and second order in $\theta(\eta)$, respectively; is reduced to a system of nine simultaneous ordinary differential equations of first order with nine unknowns, by assuming $f = f_1, f' = f_2, f'' = f_3, \theta = \theta_1, \theta' = \theta_2$. To solve this system of equations we require nine initial conditions whilst we have only two initial conditions $f(0), f'(0)$ on $f(\eta)$ and one initial condition $\theta(0)$ on $\theta(\eta)$. The other six initial conditions $f''(0), F(0), G(0), H(0), \theta'(0)$ and $\theta_p(0)$ are not prescribed: However, the values of $f'(\eta), F(\eta), G(\eta), H(\eta), \theta(\eta)$ and $\theta_p(\eta)$ are known as $\eta \to \infty$. Now, we employ the Keller-Box scheme where these six boundary conditions are utilized to produce six unknown initial conditions at $\eta = 0$. To select η_∞, we begin with some initial guess values and solve the boundary value problem with some particular set of parameters to obtain $f''(0),\ F(0), G(0), H(0),\ \theta'(0)$ and $\theta_p(0)$. Thus, we start with the initial approximations as $f''(0) = \delta_1,\ F(0) = \delta_2,\ G(0) = \delta_3,\ H(0) = \delta_4$, and $\theta_p(0) = \delta_6$.

Let $\delta_i^*\ (i = 1,2,3,4,5,6)$ be the correct values of $f''(0),\ F(0),\ G(0), H(0),\ \theta'(0)$ and $\theta_p(0)$. We integrate the resulting system of nine ordinary differential equations using fourth order Runge-Kutta method and obtain the values of $f''(0),\ F(0), G(0), H(0),\ \theta'(0)$ and $\theta_p(0)$. The solution process is repeated with another larger value of η_∞ until two successive values of $f''(0),\ F(0), G(0), H(0),\ \theta'(0)$ and $\theta_p(0)$ differ only after desired digit signifying the limit of the boundary along η. The last value of η_∞ is chosen as appropriate value for that particular set of parameters. Finally, the problem can be solved numerically using a second order finite difference scheme known as the Keller-Box method (for details see Prasad et al. [27]). The numerical solutions are obtained in four steps as follows:

- reduce the systems of equations (16) and (17) to a system of first-order equations;
- write the difference equations using central differences;
- linearize the algebraic equations by Newton's method, and write them in matrix-vector form; and
- solve the linear system by the block tri-diagonal elimination technique.

For the sake of brevity, the details of the numerical procedure are not presented here. It is also important to note that the computational time for each set of input parameters should be sort. Because physical domain in this problem is unbounded, whereas the computational domain has to be finite, we apply the far field boundary conditions for the similarity variable η at finite value denoted by $\eta_{\max}$. We ran our bulk of computations with the value $\eta_{\max} = 7$, which is sufficient to achieve the far field boundary conditions asymptotically for all values of the parameters considered. For numerical calculations, a uniform step size of $\Delta\eta = 0.01$ is found to be satisfactory and the solutions are obtained with an error tolerance of 10^{-6} in all the cases. To assess the accuracy of the present method, comparison of the skin friction $f''(0)$ and the wall-temperature gradient $\theta'(0)$ between the present results and previously published results are made, for several special cases in which the fluid-particle interaction parameter and thermo-physical fluid properties are neglected (see Table 1). It was found from Tables 1 and 2 that the present results agree very well with those of analytical solutions given by Andersson et al. [31], Grubka and Bobba [4], Chen [9] and Ali [8].

5. RESULTS AND DISCUSSION

In this section, we analyze the effects of the pertinent parameters, namely, the fluid-particle interaction parameter β, the magnetic parameter Mn, the variable fluid viscosity parameter θ_r, the variable thermal conductivity parameter ε, and the Prandtl number Pr on the flow and heat transfer of fluid-particle suspension over a horizontal stretching sheet. Also, in order to get a clear insight into the physical problem, we present the numerical results graphically in Figs. 1-6. These figures depict respectively the velocity profiles (f, f'); the particle-suspension velocity profiles (F, G); and the temperature of the fluid and the dust phase profiles (θ, θ_p). The computed numerical results are recorded in table 3 to show the behavior of the skin friction; the particle velocity and the density components; the temperature gradient and the dust-phase temperature at the sheet for different values of the governing parameters.

The transverse velocity f, the horizontal velocity f', and the particle transverse velocity and horizontal velocity $(F(\eta), G(\eta))$ profiles are shown graphically in Figs. 1(a)-1(d) for different values of the magnetic parameter Mn and the fluid-particle interaction parameter β. The general trend is that f', F and G decrease monotonically, whereas f increases as the distance increases from stretching sheet. It is observed from these figures that the horizontal velocity and transverse velocity profiles decrease with an increase in the magnetic parameter. This observation holds true even with particle velocity component $F(\eta)$, but quite opposite is true with $G(\eta)$. Physically it means that, the induction of transverse magnetic field (normal to the flow direction) has a tendency to induce a drag, known as the Lorentz force which tends to resist the flow. It is noticed that the effect of increasing values of fluid-particle interaction parameter β is to reduce the fluid velocity in the boundary layer and increase the dust phase transverse velocity, as well as the horizontal velocity $F(\eta)$.

Figs. 2(a) and 2(b) exhibit the velocity profiles for several sets of values of the fluid viscosity parameter θ_r, and the fluid-particle interaction parameter β. From the graphical representation we infer that the effect of increasing values of the fluid viscosity parameter θ_r, is to decrease the momentum boundary layer thickness. Also, as θ_r, approaches zero the boundary layer thickness decreases and the horizontal velocity distribution tends to zero [see Fig. 2(b)] asymptotically. This is due to the fact that for a given fluid (air or water), when δ is fixed, smaller θ_r, implies higher temperature difference between the wall and the ambient fluid. The results presented here demonstrate clearly that θ_r, the indicator of the variation of fluid viscosity with temperature, has a substantial effect on the horizontal velocity components f', as well as the transverse velocity f and hence on the skin friction. This phenomenon is true with zero and non-zero values of the fluid-particle interaction parameter β.

In Figs. 3–6, the numerical results for the fluid temperature and the dust-phase temperature $(\theta(\eta), \theta_p(\eta))$ are presented for several sets of values of the governing parameters. The general trend is that the fluid-temperature distribution is unity at the wall; whereas the dust-phase temperature is not. However, with the changes in the governing parameters both asymptotically tend to zero as the distance increases from the boundary. Fig. 3 illustrates the effect of the magnetic parameter and the fluid-interaction parameter on $\theta(\eta)$. The effect of increasing values of the magnetic parameter Mn is to increase the fluid temperature $\theta(\eta)$ and also the dust-phase temperature $\theta_p(\eta)$. As explained above, the induction of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive force known as the Lorentz force. This force makes the fluid experience a resistance by increasing the friction between its layers. Hence, there is an increase in the temperature profile $\theta(\eta)$ as well as the dust-phase profile. The effect of fluid interaction parameter is to decrease the temperature profile which in turn reduces the thermal boundary layer thickness; whereas it enhances the dust phase temperature at the wall and hence increases the thickness of the dust phase temperature.

Figs. 4(a) and 4(b) exhibit the fluid-temperature distribution and dust-phase temperature distribution for several sets of values of the variable viscosity parameter θ_r, and the fluid-particle interaction parameter. From the graphical representation, we observe that the effect of increasing values of the variabe viscosity parameter θ_r is to enhance both the fluid-temperature as well as the dust-phase temperature. This is due to the fact that an increase in the variable viscosity parameter θ_r results in an increase in the thermal boundary layer thickness. This is very much noticeable for zero values of fluid-particle interaction parameter as compared to the larger values. The graphs for the fluid-temperature profile $\theta(\eta)$ and dust-phase temperature $\theta_p(\eta)$ for different values of the variable thermal conductivity parameter ε are respectively shown in Figs. 5(a) and 5(b). These figures demonstrate that an increase in the value of thermal conductivity parameter ε results in increasing the temperature profile $\theta(\eta)$. This is due to the fact that the assumption of temperature-dependent thermal conductivity (linear form) implies a reduction in the magnitude of the transverse velocity by a quantity $\partial K(T)/\partial y$ as can be seen from heat transfer equation.

Figs. 6(a) and 6(b) are drawn to display the fluid-temperature profile $\theta(\eta)$ and dust-phase temperature $\theta_p(\eta)$ for different values of the Prandtl number in the absence of the fluid-interaction parameter, respectively. We observe that the effect of increasing values of the Prandtl number Pr is to decrease both $\theta(\eta)$ as well as $\theta_p(\eta)$. Physically it means that an increase in the Prandtl number means a decrease in the thermal conductivity K_∞: Hence, there is a decrease in the thermal boundary layer thickness. This behavior can be seen even in the presence of fluid interaction parameter.

6. CONCLUSIONS

In this paper, the impacts of temperature subordinate thermo-physical properties on the MHD limit layer stream and warmth exchange of a liquid molecule suspension over an extending sheet are examined. The overseeing fractional differential conditions are changed over into common differential conditions by closeness changes.

- In the nearness of temperature-subordinate thermo-physical properties, the impact of expanding estimations of the liquid collaboration parameter and the attractive parameter is to diminish the speed all through the limit layer. In any case, a remarkable inverse is valid with residue stage speed profiles.

- The impact of expanding estimations of liquid consistency parameter is to diminish the speed limit layer thickness. Be that as it may, it improves the warm limit layer thickness. This marvel is genuine even with the liquid molecule suspension parameter.

- The impact of variable warm conductivity parameter is to upgrade the liquid temperature just as the molecule stage temperature in the stream area.

- The warm limit layers of the liquid and the residue stage are exceptionally affected by the Prandtl number. The impact of Pr is to diminish the warm limit layer thickness.

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Table 1.

Mn	Present results	Andersson et al. [31]	Exact Solution
0.0	-1.0001	-1.0000	-1.0000
0.5	-1.2249	-1.2247	-1.2247
1.0	-1.414	-1.414	-1.414
1.5	-1.581	-1.581	-1.582
2.0	-1.73205	-1.73350	-1.73205

Table 2.

Pr	Present results	Grubka and Bobba [4]	Chen [6]	Ali [8]
0.01	-0.01017936	-0.0099	-0.0091	
0.72	-0.4631462	-0.4631	-0.46315	-0.4617
1.0	-0.5826707	-0.5820	-0.58199	-0.5801
3.0	-1.16517091	-1.1652	-1.16523	-1.1599
5.0	-1.56800866	-------------------------	------------	---------
10.0	-2.308029	-2.3080	-2.30796	-2.2960
100.0	-7.769667	----------------------------	---------	----------