Impact of Heat Transfer on MHD Mixed Convective Flow over a Nonlinear Permeable Sheet

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Abstract

In this paper, the issue of consistent laminar two-dimensional limit layer MHD stream and warmth move of an incompressible thick liquid with the nearness of lightness power and gooey dispersal over a vertical nonlinear extending sheet with halfway slip is explored numerically. Numerical arrangements of the subsequent nonlinear limit esteem issue for the situation when the sheet extends with a speed differing nonlinearly with the separation is done. The impacts of for different estimations of suction parameter, attractive parameter, Prandtl number, Eckert number, lightness parameter, nonlinear extending parameter and slip parameter on stream and warmth move attributes is examined.
Keywords: MHD, Nonlinear Stretching Parameter, Joules Dissipation, Viscous Dissipation, Eckert Number, Slip Parameter

1. Introduction

The investigation of stream over an extending sheet has produced much enthusiasm for ongoing years in perspective on its various mechanical applications, for example, the streamlined expulsion of plastic sheets, the limit layer along a fluid film, buildup procedure of metallic plate in a cooling shower and glass, and furthermore in polymer ventures. Since the pio-neering work of Sakiadis [1] which concentrated the moving plate stream issue, wherein different parts of the prob-lem have been researched by numerous creators, for example, Cortell [2], Xu and Liao [3], Hayat et al. [4] and so forth.

The investigation of two-dimensional limit layer stream, warmth and mass exchange over a permeable extending surface is significant as it finds numerous viable applications in various territories. To be progressively explicit, it might be brought up that numerous metallurgical procedures include the cool-ing of constant strips or fibers by illustration them through a tranquil liquid and that during the time spent illustration these strips, are some of the time extended. Thick dissemination changes the temperature circulations by assuming a job like a vitality source,which prompts influence warmth move rates. The value of the impact of gooey dis-

sipation relies upon whether the sheet is being cooled or warmed.

Aside from the gooey dissemination, the Joules dissipa-tion additionally goes about as a volumetric warmth source. Warmth move investigation over permeable surface is of much viable enthusiasm because of its plentiful applications. To be progressively explicit, heat-treated materials going between a feed roll and wind-uproll or materials fabricated by expulsion, glass-fiber and paper generation, cooling of metallic sheets or electronic chips, gem developing are a couple of handy utilizations of stream over an extending sheet. In every one of these cases, the last result of wanted attributes relies upon the rate of cooling and furthermore the rate of extending. In perspective on every one of these viewpoints, the present work manages the impact of gooey and Joules dissemination on MHD stream, warmth and mass exchange over a permeable sheet, with incomplete slip. Examines in these fields have been directed by numerous specialists. For instance, analyti-cal results were completed by Vajravelu and Hadjinico-laou [5] who considered the impacts of thick dispersal and inside warmth age. An examination of warm limit layer in an electrically directing influenza id over a straightly extending sheet within the sight of a

steady transverse attractive field with suction or blow-ing at the sheet was completed by Chaim [6].

In all respects as of late, the thick and joules dissemination and inside warmth age was considered in the vitality condition. Sajid et al. [7] explored the non-comparative diagnostic answer for MHD stream and warmth move in a third-request liquid over an extending sheet. He found that the skin grinding coefficient diminishes as the magnet-ic parameter or the third grade parameter increments. A scientific investigation has been completed on momen-tum and warmth move attributes in an incompressible, electrically directing viscoelastic limit layer liquid stream over a straight extending sheet by Abel et al. [8].

A numerical reinvestigation of MHD limit layer stream over a warmed extending sheet with variable viscos-ity has been investigated by Pantokratoras [9].

Ishak et al. [10] concentrated blended convection limit layers in the stagnation-point stream of an incompressible gooey liquid over an extending vertical sheet.

Hossain and Takhar [11] have examined the radia-tion impact on blended convection limit layer stream of an optically thick gooey incompressible liquid along a vertical plate with uniform surface temperature.

The issue of non-direct extending sheet for contrast ent instances of liquid stream has additionally been broke down by vary ent analysts. Vajravelu [12] inspected liquid stream over a nonlinearly extending sheet. Cortell [13] has chipped away at thick stream and warmth move over a non-directly stret-ching sheet. Cortell [14] further examined on the ef-fects of gooey dissemination and radiation on the warm limit layer, over a non-straightly extending sheet. Raptis et al. [15] examined gooey stream over a non-direct extending sheet within the sight of a substance response and attractive field. Abbas and Hayat [16] tended to the radiation impacts on MHD stream because of an extending sheet in permeable space. Cortell [17] explored the impact of

[23] Takhar and Ram [24], and Duwairi and Damseh [25]. Henceforth the present examination researches the impact of gooey and Joules dispersal on MHD stream over a por-ous nonlinear vertical extending sheet with thick and joules scattering with fractional slip.

2. Mathematical Analysis

Two-dimensional, nonlinear, steady, MHD laminar boun- dary similarity solution for flow and heat transfer of a quies-cent fluid over a non-linear stretching surface. Awang and Kechil [18] obtained the series solution for flow over nonlinearly stretching sheet with chemical reaction and

magnetic field. Cortell [19] investigated the influence of similarity solution for flow and heat transfer of a quies-cent fluid over a non-linear stretching surface.

The study of magnetohydrodynamics of a conducting

fluid finds applications in a variety of astrophysical and geophysical problems. The effects of magnetic field on

the natural convection heat transfer have been discussed by Romig [20], Elbashbeshy [21], considered heat trans-fer over a stretching surface with a variable surface heat flux. The convective heat transfer in an electrically con-

ducting fluid at a stretching surface has been studied by Vajravelu and Hadjinicolaou [22]. Other studies dealing with hydromagnetic flows can be found in Grandet *et al.*

where u and v are the velocity components along the x

and y axes, respectively. Further, μ, ρ, α, β, T, and g are the dynamic viscosity, fluid density, thermal diffusivity, thermal expansion coefficient, fluid temperature in the boundary layer, and acceleration due to gravity, respec-

tively.

A common feature of all these analyses is the assump-tion that the flow field obeys the conventional no-slip condition at the sheet that is the velocity component

$u(x, y)$ parallel with the sheet becomes equal to the

layer flow with heat transfer of a viscous, incom- pressible and electrically conducting fluid over a porous vertical stretching sheet embedded in the presence of transverse magnetic field including viscous and Joules dissipation is considered for investigation. An uniform transverse magnetic field of strength B is applied parallel to y-axis. Consider a stretching sheet that emerges out of a slit at $x = 0$, $y = 0$ and subsequently being stretched, as in a polymer extrusion process. Let us assume that the speed at a point on the plate is proportional to the power of its distance from the slit and the boundary layer ap-proximations are applicable. In writing the following equations, it is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible.

Consider a steady, two-dimensional free convection flow adjacent to a nonlinear stretching vertical sheet immersed in an incompressible electrically conducting viscous fluid of temperature T_∞. The stretching velocity $U_w(x)$ and the surface temperature $T_w(x)$ are where a and b are constants with $a > 0$ and $b \geq 0$.

The sheet is assumed to vary nonlinearly with the dis-tance x from the leading edge, *i.e.* $U_w(x) = ax^m$ and $T_w(x) = T_\infty + bx$. Under these conditions, the governing boundary layer equations of momentum, energy with buoyancy, viscous and Joules dissipation, with partial slip are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\sigma B^2}{\rho c_p}\right)u^2, \tag{3}$$

and are subjected to the following boundary conditions

$$u(x,y) = L\frac{\partial u}{\partial y} + ax^m,\ v = v_w(x),\ T = T_w + bx^s \text{ at } y = 0,$$

where $v_w(x) = -f_w\sqrt{\frac{\nu a(m+1)}{2}}\,x^{\frac{m-1}{2}}$

$$u \to 0,\ T \to T_\infty \text{ as } y \to \infty, \tag{4}$$

$$\theta' = P_r f\theta' - \left(\frac{2s}{m+1}f'\right)\theta - \text{Ec.Pr}\left(f'^2 + Mf'^2\right) \tag{8}$$

Boundary conditions (4) becomes

$$\begin{gathered} f(0) = f_w\ f'(0) = 1 + \gamma f''(0), \\ \theta(0) = 1,\ f'(\infty) = 0,\ \theta(\infty) = 0. \end{gathered} \tag{9}$$

sheet velocity ax^m at the sheet. In certain situations, however, the assumption of no-slip does no longer apply and should be replaced by a partial slip boundary condition which relates the fluid velocity u to the shear rate $\frac{\partial u}{\partial y}$ at the boundary. Here L is the slip length, and y denotes the coordinate perpendicular to the surface. This slip-flow condition was first introduced by C-L. M. H. Navier more than a century ago and has more recently been used in studies of fluid flow past permeable walls, slotted plates, rough and coated surfaces, and gas and liquid flow in micro devices. The no-slip boundary condition is known as the central tenets of the Navier-Stokes theory. But there are situations wherein such condition is not appropriate. Especially, no slip condition is inadequate for most non-Newtonian fluids. For example polymer melts often exhibit macroscopic wall slip and that in general is governed by a non-linear and monotone relation between the slip velocity and traction. The fluids exhibiting boundary slip find applications in technology such as in the polishing of artificial heart valves and internal cavities. Navier suggested a slip boundary condition in terms of linear shear stress.

The momentum, and energy Equations (2), (3), and (4) can be transformed into the corresponding nonlinear ordinary differential equations by the following similarity transformation:

$$\eta = \left(\frac{(m-1)a}{2\nu}\right)^{1/2} x^{\frac{m-1}{2}} y,\quad u(x,y) = ax^m f'(\eta),$$

$$v(x,y) = \left(\frac{(m+1)\nu a}{-2}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}} \left(\frac{m-1}{m+1}\eta f'(\eta) + f(\eta)\right) \quad (5)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \quad (6)$$

where $T_w(x) = T_\infty + bx^s$, b is dimensional constant and s is the index of power law variation of temperature.

The transformed nonlinear ordinary differential equations are

where

$$\gamma = L_x\sqrt{\mathrm{Re}_x(1+m)}, \quad \mathrm{Re}_x = \frac{ax^{m+1}}{\nu}, \quad \lambda = \frac{Gr_x}{(\mathrm{Re}_x)^2},$$

and $Gr_x = \frac{g\beta(T_w - T_\infty)}{\nu^2}$.

3. Numerical Solution

The nonlinear boundary value problem represented by Equations (7) to (9) is solved numerically using Fourth-order Runge Kutta shooting technique.

The system of non-linear ordinary differential Equations (7) and (8) together with the boundary conditions Equation (9) are similar and are solved numerically by using the fourth order of Runge Kutta integration scheme accompanied with the Shooting scheme. Making an initial guess for the values of $f''(0)$ and $\theta'(0)$ to initiate the shooting process is very crucial in this process. The success of the procedure depends very much on how good this guess is. Numerical solutions are obtained for several values of the physical parameters *i.e.* magnetic parameter M, stretching parameter m, Prandtl number Pr, slip parameter γ Buoyancy parameter λ, Eckert number (Ec) and, suction/injection parameter f_w. We have chosen a step size of $\Delta\eta = 0.01$ to satisfy the convergence criterion of 10^{-6} in all cases. The maximum value of η_∞ was found to each iteration loop by $\eta_\infty = \eta_\infty + \Delta\eta$ The maximum value of η_∞ to each group of parameter is determined when the value of the unknown boundary conditions at $\eta = 0$ is not changed to successful loop with error less than 10^{-6}.

4. Results and Discussion

In order to gain physical insight, the velocity, and temperature profiles have been discussed by assigning numerical values to the parameter, encountered in the problem *i.e.* numerical calculations were carried out for different values of suction parameter f_w, magnetic parameter *M*, power law stretching parameter m, Prandtl number Pr, Eckert number Ec, buoyancy parameter λ, slip parameter γ, and their effects on flow and heat transfer characteristics are analysed graphically.

The influences of the magnetic parameter M on the longitudinal velocity profile is depicted in **Figure 1**. It can be seen that increasing M is to reduce the velocity distribution in the boundary layer which results in thin- ning of the boundary layer thickness, and hence induces an increase in the absolute value of the velocity gradient at the surface.

The influence of suction parameter f_w ($f_w < 0$), over the non-dimensional longitudinal velocity profiles are shown in **Figure 2**. It is seen that the effect of suction parameter decelerates the longitudinal velocity.

The influence of injection parameter f_w ($f_w > 0$), over the dimensionless longitudinal velocity profile is shown in **Figure 3** and it is noticed that longitudinal velocity increases with injection. It should be noted that in **Figure 2**, the boundary-layer assumptions do not permit a solution of the boundary-layer equation for large f_w, because it will approach a constant value of 1, and the boundary layer is almost literally blown off the surface, similar to that of stationary plate with injection (Burmeister [26]; Kays and Crawford [27]).

Figure 3 shows the effect of suction/injection on dimensionless temperature profile and it is observed that there is decrease in temperature in the thermal boundary layer resulting in thinning of thermal boundary layer thickness in the case of suction and the reverse trend is observed for injection. Further it is clear that suction ($f_w <$ 0) enhances the heat transfer coefficient much better than injection ($f_w > 0$), and the thickness of the thermal boundary layer is reduced. Thus, suction can be used as a means for cooling the surface much faster than injection.

Figures 4 and **5**, describe respectively the behaviors of the longitudinal velocity profile and temperature profile for different values of power law stretching parameter m and it is noticed that increase in m results in decrease of longitudinal velocity profile which is more pronounced for small values of m, where as temperature profile increases with the increase of stretching parameter m. It is observed that the variation of the sheet temperature has a substantial effect on the thermal boundary layer. This effect is more pronounced when sheet temperature varies in the direction of highest stretching rate.

An increase in Prandtl number Pr is associated with a decrease in the temperature distribution which is displayed in **Figure 6**, which is consistent with the fact that thermal boundary layer thickness decreases with increase in the values of Prandtl number. The rate of heat transfer increases with the increasing values of Prandtl number. The boundary layer edge is reached faster as Pr increases.

Dimensionless velocity profile $f'(\eta)$ is presented in **Figure 7** for some different values of the slip parameter γ. It is readily seen that γ has a substantial effect on the

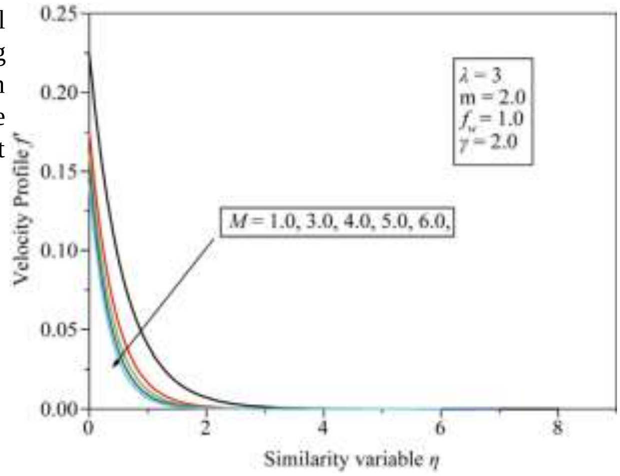

Figure 1. Velocity profile f' versus similarity variable η for different values of magnetic parameter.

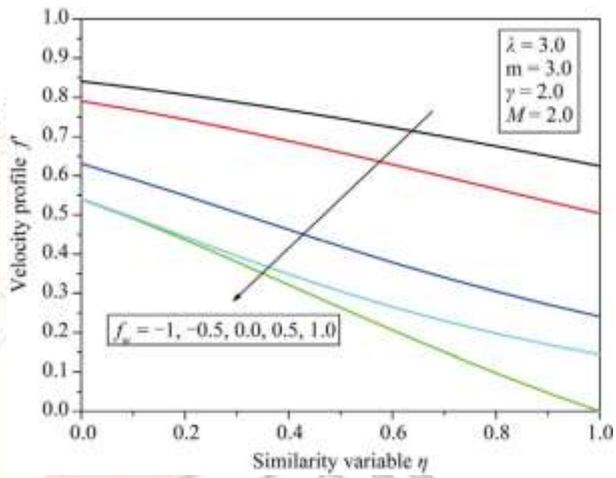

Figure 2. Dimensionless velocity profile f' versus similarity variable η for different values of suction/injection parameter.

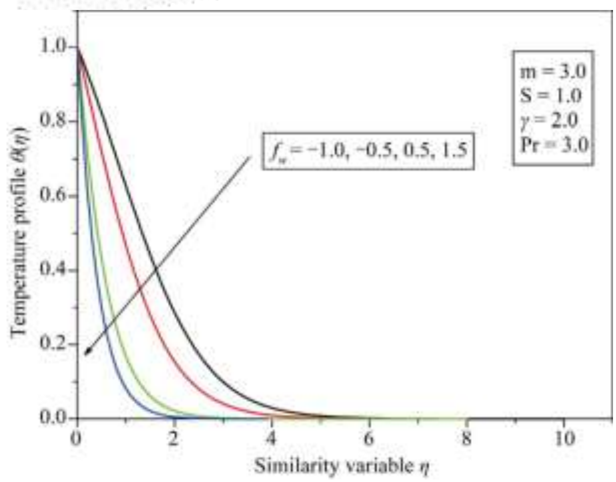

Figure 3. Temperature profile $\theta(\eta)$ vs similarity variable η for different values of suction/injection.

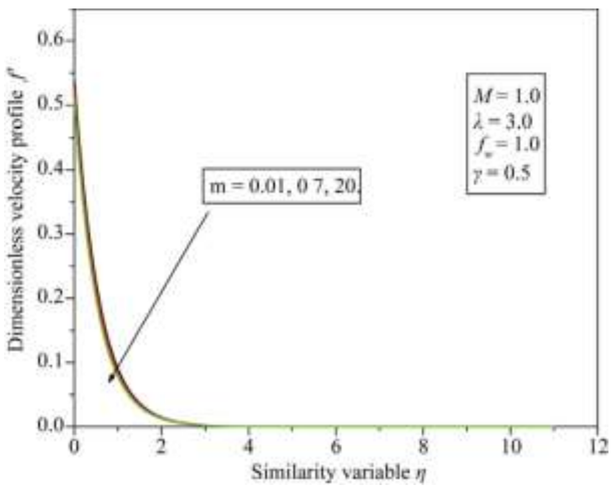

Figure 4. Dimensionless velocity profile f' vs similarity variable η for different values of m.

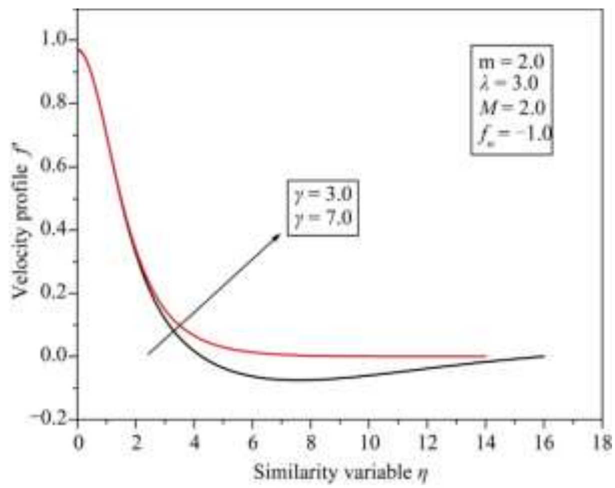

Figure 7. Dimensionless velocity profile f' versus similarity variable η for different values of slip parameter γ.

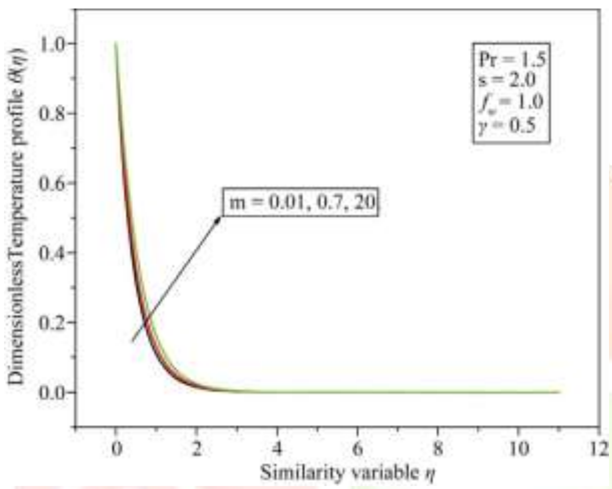

Figure 5. Dimensionless temperature profile $\theta(\eta)$ vs similarity variable η for different values of m.

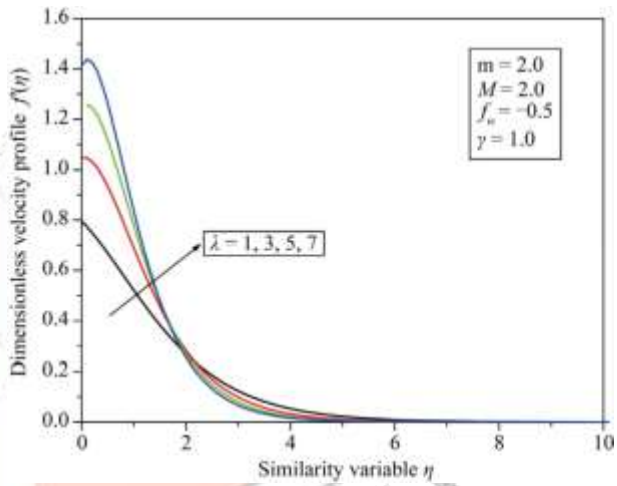

Figure 8. Dimensionless velocity profile vs similarity varia- ble η for different values of buoyancy parameter λ.

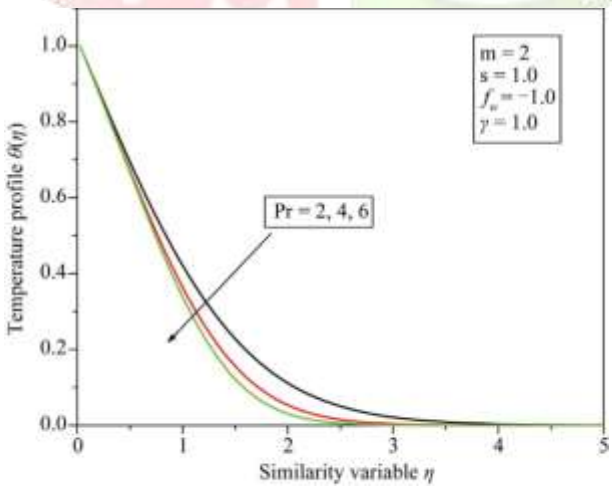

Figure 6. Temperature profile $\theta(\eta)$ versus similarity varia- ble η for different values of Pr.

solutions. In fact, the amount of slip $1+\gamma f'(0)$ in- creases monotonically with γ from the no-slip solution for $\gamma = 0$ and towards full slip as γ tends to infinity. The latter limiting case implies that the frictional resis- tance between the viscous fluid and the surface is elimi- nated, and the stretching of the sheet does no longer im- pose any motion of the fluid.

In **Figure 8**, the effects of buoyancy parameter λ on dimensionless longitudinal velocity is shown graphically and the effects of buoyancy force is found to be more pronounced for a fluid with a small Pr. Thus, fluid with smaller Pr is more susceptible to buoyancy force effects.

The velocity and temperature profiles presented in **Figures 1-9**, show that the far field boundary conditions are satisfied asymptotically, which support the validity of the numerical results presented.

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