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ORIGINAL ARTICLE

MIXED CONVECTION MHD FLOW PAST A SEMI-INFINITE VERTICAL POROUS PLATE UNDER THE EFFECT OF OSCILLATING PLATE TEMPERATURE

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ABSTRACT

In this paper, the effects on mixed convection flow past a semi-infinite vertical porous plate have been studied mathematically and numerically when the plate temperature oscillates about a non zero mean under the effect of transverse magnetic field. Only out of phase component of unsteady part of the temperature is shown graphically. The results show that these is always a phase lead in the rate of heat transfer at small values of ω *and the mean temperature is superimposed by shear wave type solution at large values of* ω.

INTRODUCTION

Over the years, many analytical or approximate and the solution of the magnetohydrodynamic (MHD) dusty fluid boundary layer solutions have been obtained. There have been numerous theoretical and experimental studies of heat and mass transfer induced by natural convection in fluids. These studies have many applications in physical system where heat transport by buoyancy induced convective motion takes place, such as chemical reactor, nuclear reactor, combustion systems, pneumatic transport etc. The effects of oscillating plate temperature on the mixed convection flow past a semi-infinite vertical plate was studied by Soundalgekar and Vighnesam (1997). They also studied this problem for a semi-infinite vertical porous plate when the plate temperature is oscillating about a non-zero plate temperature we are considering mixed convection MHD flow past a semi-infinite vertical porous plate under the effect of oscillating plate temperature. The function Ψ_i is shown in Fig.1 and 2.

MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

The flow of a viscous incompressible fluid past a semi-infinite vertical porous plate, with the x-axis along the plate and the y-axis normal to the plate is assumed. Let U_o be the uniform velocity of the fluid, and v_w be the suction or injection velocity at the plate. The temperature is assumed to be represented by an unsteady component θ_1 (x,y) to be superimposed on the steady temperature θs (x,y) as

$$\theta = \frac{T-T_\infty}{T_w - T_\infty} = \theta_s(x,y) + \in e^{i\omega t}\theta_1(x,y) \qquad \text{.....(1)}$$

Substituting equation (1) into unsteady Navier-Stokes equations, equating the harmonic and non-harmonic terms and introducing the transverse magnetic field we get

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + g\beta(T_w - T_\infty)\theta_s - M^2 u \qquad \text{.....(2)}$$

$$u\frac{\partial \theta_s}{\partial x} + v\frac{\partial \theta_s}{\partial y} = \frac{\partial^2 \theta_s}{\partial y^2} + \frac{v}{c_p(T_w - T_\infty)}\left(\frac{\partial u}{\partial y}\right)^2 \qquad \text{.....(3)}$$

$$i\omega\theta_1 + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta_1}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 \theta_1}{\partial y^2} \qquad \text{.....(4)}$$

where, $M^2 = -\frac{\sigma_o^2}{\rho}B_o^2$

under the following boundary conditions –

$u = 0,\quad v = v_w,\quad \theta_s = 1,\quad \theta_1 = 1$ at $y = 0$

$u = U_o,\quad \theta_s = 0,\quad \theta_1 = 0$ as $y = \infty$ .....(5)

Assuming low-frequency condition as Vighynesam (2001) the unsteady part of the temperature, θ_1 in terms of in-phase and out-of-phase components can be expressed as

$$\theta_1 = \theta_r + i\theta_i \qquad \text{.....(6)}$$

Substituting equation (6) in equation (4) and (5) and equating the real and imaginary parts, one gets -

$$-\omega\theta_1 + u\frac{\partial\theta_r}{\partial x} + v\frac{\partial\theta_r}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2\theta_r}{\partial y^2} \qquad \text{…..(7)}$$

$$\omega\theta_r + u\frac{\partial\theta_t}{\partial x} + v\frac{\partial\theta_t}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2\theta_r}{\partial y^2} \qquad \text{…..(8)}$$

With the following boundary conditions –

$\theta_r(0) = 1$, $\theta_t(0) = 0$ $\theta_r(\infty) = 0$, $\theta_t(\infty) = 0$ …..(9)

If β_1 is the phase-shift between the temperature changes within the boundary layer and at the plate, it can be represented by

$\beta_1 = \tan{-1}(\theta_t / \theta_r)$ …..(10)

For low frequency, this shift is very small and hence, $\theta_t \approx \theta_r$ so for small frequency, $-\omega\theta_r$ is very small and hence, this term can be neglected in equation (7), which is then reduced to -

$$u\frac{\partial\theta_r}{\partial x} + v\frac{\partial\theta_r}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2\theta_r}{\partial y^2} \qquad \text{…..(11)}$$

The following transformations have been introduced in equation (2), (3), (8) and (11)

$$\left.\begin{aligned} &\eta = y\sqrt{\frac{U_o}{vx}},\ \psi = \sqrt{U_o vx}.f(\eta),\ u = U_o f'(\eta) \\ &v = \frac{1}{2}\sqrt{\frac{U_o v}{x}}\cdot(f - \eta'),\ \theta_x = \varphi(\eta) \\ &\theta_r = \psi_r(\eta),\ \theta_r = \left(\frac{U_o x}{v}\right)\psi_t(\eta) \end{aligned}\right\} \qquad \text{…..(12)}$$

and taking into account the equation of continuity, we get

$$f''' + \frac{1}{2}f\ f'' + \left(Gr/\mathrm{Re}^2\right)\phi - Hf' = 0 \qquad \text{…..(13)}$$

$$\phi'' + \frac{1}{2}Pr\ f\phi' + \mathrm{Pr}\,Ec\ f''^2 = 0 \qquad \text{…..(14)}$$

$$\psi_r'' + \frac{1}{2}\ Pr\ f\ \psi_r' = 0 \qquad \text{…..(15)}$$

$$\psi_r'' + \frac{1}{2}\ Pr\ f\ \psi_r' - Pr\ f\ \psi_i - \beta Pr\,\psi_r - = 0 \qquad \text{…..(16)}$$

With the following boundary conditions

$$\left.\begin{aligned} &f(0) = f_w,\quad f(0) = 0,\quad \phi(0) = 1,\quad \psi_i(0) = 0 \\ &f'(\infty) = 1,\ \phi(\infty) = 0,\ \psi_r(\infty) = 0,\ \psi_i(\infty) = 0 \end{aligned}\right\} \qquad \text{…..(17)}$$

Here the non-dimensional quantities are defined –

$$\left.\begin{aligned} &Gr = \frac{g\beta(T_w - T_\infty)x^3}{v^2},\ \mathrm{Re} = \frac{U_o x}{v},\ \mathrm{Pr} = \frac{\mu c_p}{K} \\ &Ec = \frac{U_o^2}{c_p(T_w - T_\infty)},\quad \beta = \frac{\omega v}{U_O^2},\quad f_w = -2v\sqrt{\frac{x}{U_o v}},\ H = \frac{M^2}{U_o}x \end{aligned}\right\} \qquad \text{…..(18)}$$

The rate of heat transfer is given by

$$\left(U_o / 2vx\right)^{-1/2}\cdot q = \phi'(0) + \in |Q|\cos(\omega t + \alpha) \qquad \text{…..(19)}$$

where, $Q = \left\{\psi_r^2(0)\right\}^2 + \left\{\frac{U_o x}{v}, \psi_i'(0)\right\}^2$ …..(20)

and $\tan\alpha = \dfrac{U_o x\,\psi_t'(0)}{V\,\psi_r'(0)}$

At high frequency, equation (4) reduces to

$$i\omega\theta_1 = \frac{K}{\rho c_p}\frac{\partial^2\theta_1}{\partial y^2}$$

which has a shear wave-type solution, Viqhensam et al. (2001).

RESULTS

Table 5.1 shows the numerical values of $\{-\phi'(0)\}$, $\{-\psi_r'(0)\}$, $\{-\psi_i'(0)\}$, at Pr=0.71 and when H=0.5 Table 5.2 shows the numerical value of $\{-\phi'(0)\}$, $\{-\psi_r'(0)\}$, $\{-\psi_i'(0)\}$ at Pr=0.71 and when H=0. An increase in Gr/Re^2 or β or Ec leads to increase in the amplitude of the rate of heat transfer at small values of ω, while the magnetic field acts to decrease. But an increase in suction velocity leads to an increase in the amplitude of the rate of heat transfer, whereas an increase in injection velocity leads to a decrease in the amplitude of the rate of heat transfer. At large values of frequency ω, the mean temperature is superimposed by a shear wave types solution which is not affected by ω or suction – injection velocity. The function ψ_i is shown in Fig.1 and 2.

Table 1 : Values of $\{-\phi'(0)\}$, $\{-\psi_r'(0)\}$, $\{-\psi_i'(0)\}$ at Pr=0.71, H=0.5

Gr/Re^2	**fw**	**Ec**	β	$\{-\phi'(0)\}$	$\{-\psi_r'(0)\}$	$\{-\psi_i'(0)\}$
0.5	**0.5**	**0.1**	**0.2**	**0.4433**	**0.4827**	**0.1084**
0.5	**0.5**	**0.4**	**0.2**	**0.3190**	**0.4851**	**0.1078**
0.5	**0.7**	**0.1**	**0.2**	**0.4921**	**0.5332**	**0.1061**
0.8	**-0.5**	**0.1**	**0.2**	**0.4439**	**0.5043**	**0.1028**
0.5	**-0.5**	**0.1**	**0.2**	**0.2322**	**0.2634**	**0.1174**
0.5	**-0.5**	**0.4**	**0.2**	**0.1371**	**0.2667**	**0.1163**
0.8	**-0.5**	**0.1**	**0.2**	**0.2442**	**0.2924**	**0.1085**
0.5	**-0.7**	**0.1**	**0.2**	**0.1979**	**0.2273**	**0.1185**
0.5	**-0.7**	**0.4**	**0.2**	**0.1083**	**0.2308**	**0.1172**
0.8	**-0.7**	**0.1**	**0.2**	**0.2105**	**0.2571**	**0.1090**

Table 2 : Values of $\{-\phi'(0)\}$, $\{-\psi_r'(0)\}$, $\{-\psi_i'(0)\}$ at Pr=0.71, H=0

Gr/Re^2	**f_x**	**Ec**	**H**	β	$\{-\phi'(0)\}$	$\{-\psi_r'(0)\}$	$\{-\psi_i'(0)\}$
0.5	**0.5**	**0.1**	**0.5**	**0.2**	**0.4323**	**0.4654**	**0.1082**
0.5	**0.5**	**0.4**	**0.5**	**0.2**	**0.3082**	**0.4632**	**0.1079**
0.5	**0.7**	**0.1**	**0.5**	**0.2**	**0.4831**	**0.5213**	**0.1058**
0.8	**-0.5**	**0.1**	**0.5**	**0.2**	**0.4379**	**0.5013**	**0.1023**
0.5	**-0.5**	**0.1**	**0.5**	**0.2**	**0.2132**	**0.2415**	**0.1099**
0.5	**-0.5**	**0.4**	**0.5**	**0.2**	**0.1278**	**0.2563**	**0.1078**
0.8	**-0.5**	**0.1**	**0.5**	**0.2**	**0.2238**	**0.2725**	**0.1082**
0.5	**-0.7**	**0.1**	**0.5**	**0.2**	**0.1878**	**0.2189**	**0.1044**
0.5	**-0.7**	**0.4**	**0.5**	**0.2**	**0.1075**	**0.2257**	**0.1075**
0.8	**-0.7**	**0.1**	**0.5**	**0.2**	**0.2077**	**0.2498**	**0.1084**

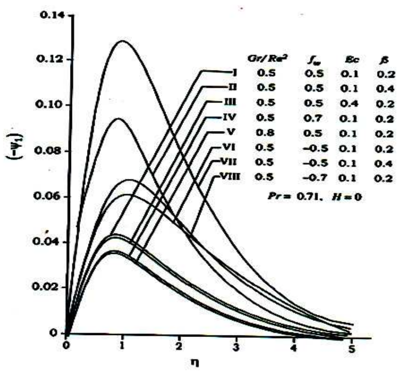

Fig. 1 : Function ψ_i

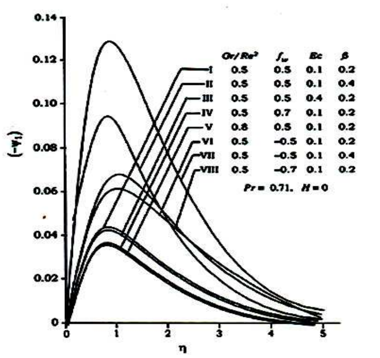

Fig. 2 : Function ψ_i

CONCLUSION

Some conclusions of the study are as below –

At small values of ω, the increase in Gr/Re^2 or β or Ec leads to increase in the amplitude of the rate of heat transfer while the magnetic field acts to decrease. These is always a phase lead in case of the rate of heat transfer. At large values of frequency ω, the mean temperature is superimposed by a shear-wave type solution which is not affected by ω.

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