Abstract: This paper presents an efficient approach to estimate the thermal power plant fuel cost parameter which are usually modeled as a polynomial of desired order. The accuracy of parameters of fuel cost is in turn influences the economic load dispatch (ELD) where cost function of fuel is to be minimized. The fuel cost polynomial parameters in this paper are obtained first by popularly used and industry adopted Least square estimation method. Upon observing the local optimality of error by LSE this paper proposes a simple geometry based, non-stochastic and derivative free optimization, known as Nelder-Mead (NM) to further reduce the absolute error by LSE. This approach LSE-NM is tested on different fuel costs such as coal, oil and gas for obtaining parameters of polynomial which results in global minimum error between practical data points and mathematical model. The global optimality for situation considered is confirmed upon comparison with global optimizers such as Particle swarm optimization (PSO) and Ant Bee colony (ABC). The prime requirements of reliability of estimated parameters and time of computation along with global optimality is possible by LSE-NM for ELD parameter estimation.

Index Terms: Nelder-Mead, parameter estimation.

I. INTRODUCTION

The economic load dispatch or optimal power flow or compromised fuel cost and environmental dispatch of fossil fired power plant is an optimization problem. In these problems the cost functions of fossil power plants can be mathematically modeled as linear, quadratic or cubic or non-convex fuel cost curves. The objective function accurate value says that fuel cost depends on parameters of the cost function. Due to the ageing of electric power units, ambient operating temperatures of the machines, ever fluctuating fuel cost, labour cost and transportation cost, necessitates the parameter estimation during electrical power generation from fossil fuels. so, the frequent updation of parameter of cost curve that reflect fuel cost curve becomes necessary and vital. Hence researchers from industry/academics try to explore ways to reduce error between estimated and actual values [1].

In conventional approach the parameter estimation problem is optimization technique. In optimization technique an error function is to be minimized [1]. The error is difference between the actual and estimated fuel cost. One of the widely used traditional optimization technique to this type of parameter estimation is least square estimation (LSE) [2].For an nth order polynomial LSE calculates [3], first order partial derivatives of squared error w.r.t parameters to be estimated which results in (n+1) linear algebraic equations. The (n+1) linear algebraic equation can be solved by standard gauss elimination method. This method of ELD parameter estimation was shown to end up with large error [4]. To overcome this researches focused on applying Genetic Algorithms, Particle swarm optimization PSO, Ant Bee colony (ABC) non-derivative, nature inspired algorithms to ELD parameter estimation problem. In recent works on this topic of parameter estimation, it was shown [5,6] that ABC could reduce the error to a least value compared with GA, PSO and LSE. It is worth to observe that the authors who proposed Evolutionary optimization methods to polynomial curve fitting of ELD with smooth curves of quadratic and cubic polynomials did not mention number of function evolution per iteration and hence population size (which is key to the success of any EA[7]) and consequently time of computation, reliability and search range were not reported. Based only final minimum error one can’t conclude the applicability of EA to smooth parameter estimation problem.

Motivated by above facts authors of this paper aim to bring global minimum using LSE followed by Nelder-Mead NM[8]. The NM approach forms a simplex of (n+1) dimensions and evaluates the objective function and using the concept of reflection, expansion, and shrinking of points of simplex. The improved simplex is obtained on geometric basis rather than stochastic. Hence it was contemplated that such non-stochastic local search derivative free optimizer should definitely reduce the error and objective of reaching global optimality can be achieved. This approach referred in this paper as LSE-NM is found suitable for parameter estimation problem which is not that much non-convex. EA’s are advantageous for highly non-smooth, non-convex and mixed integer problems of power systems[9].

II. MATHEMATICAL MODEL OF FUEL COST:

In practical power system the heat rate curve or fuel cost function can be mathematically modeled as suitable polynomial by estimating the parameters of mathematical model that exactly suits the desired polynomial fit. The general mathematical model of polynomial is

\[ F = a_0 + \sum_{k=1}^{j} a_k P^k \] (1)
In equation \( \alpha_o \) is minimum cost operation. The coefficients \( a_j \) reflects the combined steam boiler process. \( P \) is the power output of the unit. \( j \) is the order of the polynomial or desired mathematical model. During the course of operation the re estimation of coefficients are required due to fluctuations in various costs and ageing of the power unit. The above formulation of fuel cost is for power units without load point effects. The least square estimation of above polynomial is explained below.

### III. LEAST SQUARE ESTIMATION (LSE):

Assuming the availability of practical data of fuel cost \( F_i \) power units as \((P_i,F_i)\) at various \( i \)th power outputs the squared error can be calculated using equation (2)

\[
\text{error} = [F_i - F]^2
\]  

(2)

In equation 2, \( F \) is shown in equation.

The error minimization is done by finding partial derivatives of \( \text{error} \) with respect to each \( \alpha_k \) coefficient to be determined. Thus for \( j \)th coefficient the general formula is [2]

\[
\frac{\partial \text{error}}{\partial a_j} = -2 \sum_{i=1}^{n} [F_i - F] P_i^j = 0
\]  

(3)

The above partial derivatives can be put in standard algebraic form as follows.

\[
AX = B
\]  

(4)

In above equation

\[
A = \begin{bmatrix}
\sum P_i^j & \cdots & \sum P_i^n \\
\vdots & \ddots & \vdots \\
\sum P_i^j & \cdots & \sum P_i^n 
\end{bmatrix}, \quad B = \begin{bmatrix}
\sum F_i P_i^j \\
\vdots \\
\sum F_i P_i^n 
\end{bmatrix}, \quad X = \begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_j 
\end{bmatrix}
\]  

(5)

Using Gauss elimination method – can be used to solve the above equation. The LSE method can be invoked in MATLAB [10] environment using polyfit(), command. This general command is as follows. The usage form of polyfit command is as follows.

\[
x = \text{polyfit}(P,F,j) ;
\]

in the above command \( P \) is a row vector of real power outputs. \( F \) is a row vector of actual cost while \( j \) is power of polynomial to be fit for \( P \) and \( F \). The function returns \( x \) which are the coefficients of the polynomial.

### IV. NELDER-MEAD (NM) OPTIMIZATION METHOD:

The NM is a geometry based non-derivative, simplex local search function extrapolation approach developed by John Nelder et.al [1]. The prime requirement of this method is to have an initial simplex with \((N + 1)\) vertices. The \( N \) is number of search directions. The method [8] in brief sorts the vertices objective function values from best to worst. Then reflects the worst point towards centroid of simplex excluding the worst point. The reflected point \( x_r \) may be better than \( x_n \) or better than current best value \( x_1 \) or worse than second best point \( x_2 \) in case of option one \( x_r \), replaces \( x_{n+1} \). The option two i.e \( x_r \) better than \( x_1 \), the NM explores along the line centroid and the move is known as expansion and computes the point \( x_c \). The function value at \( x_c \) decides replacement of \( x_{n+1} \) with \( x_r \) or \( x_r \). And the third option of NM is contraction move, in this move contraction point \( x_c \) replaces the worst value upon comparison of worst vertex with \( x_c \). Every time when worst vertex is replaced.

The new simplex is reformed and procedure is repeated till termination conditions of optimization which may include shrink of simplex to down-hill search.

Assume \((N+1)\) simplex points with function values arranged from best to worst.

1. \( Itr=0 \) (initialize the iteration counter)

2. \( \text{step-A :} f(x_1) \leq f(x_2) \leq \cdots \leq f(x_{N+1}) \)

3. Till stopping criteria of the ALGORITHM repeat the following steps.

   - Compute centroid of \( x_o = \frac{1}{N} \sum_{i=1}^{N} x_i \)
   - Find reflection point \( x_r \) using \( x_r = x_o + \alpha (x_o - x_{N+1}) \).
   - Compute objective value at \( x_r \) i.e \( f(x_r) \)
If \( f(x_r) < f(x_n) \& f(x_r) > f(x_1) \) replace \( x_{N+1} \) with \( x_r \) go to step A.

If \( f(x_c) < f(x_1) \), obtain an expansion point \( x_e \) using \( x_e = x_o + y(x_r - x_o) \)

If \( f(x_e) < f(x_r) \), replace \( x_{N+1} \) with \( x_e \) go to step A. else replace with \( x_r \) go to step A.

If \( (x_r) \geq f(x_n) \), calculate contraction point \( x_c = x_o + \rho(x_{N+1} - x_o) \)

Replace \( x_c \) with \( x_{N+1} \) with \( x_c \) go to step 1 if \( (x_c) < f(x_{N+1}) \).

Shrink the simplex i.e replace all points except the best \( x_1 \), using

\[ x_i = x_1 + \sigma(x_i - x_1) \] go to step A.

\[ \text{Itr} = \text{Itr} + 1. \]

go to step A.

Typical values for \( \gamma, \rho, \sigma \) and \( \alpha \) are respectively 2, \( 1/2 \), \( 1/2 \) and 1.

The stopping criteria of the NM method may be number of iterations or number of function evaluations or distance norm of initial best point and current best point. Step A is called as initialization of the simplex. During Step A, function values are arranged from best to worst (descending) order. This algorithm can be simply implemented in MATLAB\[10\] by invoking a function known as "fminsearch()". Typically fminsearch() accepts address of the function to be minimized and initial search value and returns possibly an improved value and objective function value at current iterate. For detailed usage of this function is available \[10\].

The general command in MATLAB environment is \([x,fvals]=fminsearch(@objfunction,x);\)

Where fminsearch is NM solver in MATLAB environment and @objfunction is another matlab script file where objective function to be minimized is written.

The function returns \( x \) which are further improved quantities of \( x \) computed by polyfit() command.

The objective function for fminsearch is as follows

\[ \text{err} = |F_i - F| \] (5)

In the above equation \( F_i \) are data points and \( = x_o + \sum_{k=1}^{j} x_{k}p_{i}^{k} \). As stated earlier \( x \) are coefficients of polynomial to be estimated. As both solvers LSE and NM are used for parameter estimation the method is named as LSE-NM.

V.IMPLEMENTATION USING IN MATLAB:

Read the data such as j, \( F_i \) and \( P_i \).

Call polyfit command as explained above. This returns \( x \) the coefficients for polynomial order j.

Use fminsearch() command using \( x \) returned by polyfit.

Output the \( x \) and absolute error.

5.1 Test results and discussion:

In this paper the data of fuel cost at real power output available for coal, oil and gas \[6\] are taken for comparison with PSO and Ant Bee colony (ABC) algorithms.

Test case-1 pertains to j=2, i.e quadratic cost modeling. The comparative results are shown in Table 1.

Table 1: comparative table of error estimated for quadratic parameter estimation.
The following are the observation on quadratic estimation

1) Absolute error by LSE-NM is less than ABC and PSO for all power plants.

2) The error at 10, 30 and 50 MW is zero by the LSE-NM while the error by other approaches are not exactly zeros.

3) The percentage reduction in total error compared with LSE is shown in table 2 below.

<table>
<thead>
<tr>
<th>Unit</th>
<th>LSE-NM</th>
<th>ABC</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>31.3063</td>
<td>30.9544</td>
<td>28.7936</td>
</tr>
<tr>
<td>Oil</td>
<td>31.0357</td>
<td>29.9433</td>
<td>18.0725</td>
</tr>
<tr>
<td>Gas</td>
<td>31.4153</td>
<td>27.1173</td>
<td>10.3686</td>
</tr>
</tbody>
</table>

It is very clear from the above table that NM has reduced the error compared to ABC and PSO with reference to LSE.

4) The total iteration by NM is only 132.

5) The time of parameter estimation is 0.312s.

6) The polynomial coefficients by proposed approach is shown in table 3

<table>
<thead>
<tr>
<th>Unit</th>
<th>ao</th>
<th>a1</th>
<th>a2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>96.6000</td>
<td>7.5880</td>
<td>0.0414</td>
</tr>
<tr>
<td>Oil</td>
<td>101.5312</td>
<td>7.8800</td>
<td>0.0442</td>
</tr>
<tr>
<td>Gas</td>
<td>101.8125</td>
<td>8.1000</td>
<td>0.0439</td>
</tr>
</tbody>
</table>

Test case 2: In this case for power outputs and cost given in table 1 is used to estimate parameters with j=3 i.e cubic polynomial cost function. The error of individual units and total error are shown in table 4 below.

<table>
<thead>
<tr>
<th>Unit</th>
<th>P(MW)</th>
<th>F(actual)(GJ/h) F1</th>
<th>F_estimated (GJ/h)</th>
<th>Error(F_estimated-F_actual)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSE-NM</td>
<td>ABC(6)</td>
<td>PSO(5)</td>
<td>LSE(5)</td>
<td>LSE-NM</td>
</tr>
<tr>
<td>Coal</td>
<td>20</td>
<td>256.4</td>
<td>256.413</td>
<td>257.134</td>
</tr>
<tr>
<td>Oil</td>
<td>30</td>
<td>361.5</td>
<td>356.651</td>
<td>357.093</td>
</tr>
<tr>
<td>Gas</td>
<td>40</td>
<td>467.6</td>
<td>467.6</td>
<td>467.492</td>
</tr>
</tbody>
</table>
The following are the observations from the above table 4:

1) This case also the total error and individual errors by proposed LSENM is less compared to other approaches.

2) At certain power outputs error is exactly driven to zero by NM while other approaches could not achieve this performance.

3) The percentage reduction in total error compared with LSE is shown in table below.

### Table 5: % reduction in total error for cubic polynomial parameter estimation.

<table>
<thead>
<tr>
<th>unit</th>
<th>LSE-NM</th>
<th>ABC</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>COAL</td>
<td>52.9693</td>
<td>47.5031</td>
<td>16.3423</td>
</tr>
<tr>
<td>oil</td>
<td>54.8458</td>
<td>52.6114</td>
<td>49.8418</td>
</tr>
<tr>
<td>Gas</td>
<td>51.5432</td>
<td>43.0755</td>
<td>42.8557</td>
</tr>
</tbody>
</table>

A significant percentage reduction is definite by LSE-NM.

4) In this case total iteration after which no further reduction in error is 154.

5) The time of parameter estimation is 0.45s

6) The polynomial coefficients by proposed approach is shown in table 6

### Table 6: Parameters for cubic polynomial fitting

<table>
<thead>
<tr>
<th>unit</th>
<th>ao</th>
<th>a1</th>
<th>a2</th>
<th>a3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>7.0226</td>
<td>3.1251</td>
<td>0.1997</td>
<td>-0.0016</td>
</tr>
<tr>
<td>oil</td>
<td>0.8137</td>
<td>3.5770</td>
<td>0.1974</td>
<td>-0.0016</td>
</tr>
<tr>
<td>Gas</td>
<td>2.3367</td>
<td>3.6244</td>
<td>0.2024</td>
<td>-0.0016</td>
</tr>
</tbody>
</table>

Most interesting feature of NM is it is non-stochastic and excellent local search solver and hence ends up consistently to the same final value of reduced error even when algorithm is repeatedly run with an initial estimate from another non stochastic gradient based approach LSE. More over time of computation is more important while estimating parameters on line. The other approaches with which comparison is made are...
VI. CONCLUSION

This paper has successfully proved the global optimality by using traditional approach LSE followed by local search Nelder-Mead (NM) to reliably estimate parameters of continuous polynomial cost function coefficients for Economic Load dispatch problem. The approach of this paper needs to be tested by considering other practical constraints such as valve-point loading effects and transmission losses.

REFERENCES