Mathematical Model Formulation for Investigation of Influence of Air Induction Pressure as an Operating Variable on a Stationary Compression Ignition Engine Performance

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Abstract— Operating parameters of any particular Compression Ignition engine are firstly critically reviewed. Then by using the theories of engineering experimentation design of experiments is done. Dimensional analysis technique is used for formulation of mathematical model. Internal Combustion engines subject area is especially important as per the environmental aspects are concerned. Design of each machine is having common objectives, first is to obtain highest possible efficiency and another is to obtain minimum possible environmental degradation. The Induction pressure acting on engine is most important operating parameter of engine. The present work Shows vital scope for improvement of operating performance of engines by taking wider operating range in case of each engine.

Keywords— Air Induction Pressure, Efficiency, Compression Ignition Engine, Performance

I. INTRODUCTION

Operating parameters of an internal combustion engine are discussed here, for the study of the effects of different parameters on engine performance. Engine performance is dependent on fuel consumption, which bears direct influence on efficiency and engine out emissions. So, fuel consumption rate is a basic dependent variable. It is an important parameter of an engine, varies because of any possible variable variations such as engine design (its operating conditions) & fuel (type of fuel) and after treatment system(flow resistance offered by exhaust system). Based upon the literature reviews, the following different variables are selected for reduction of variables using dimensional analysis technique. Different \( \pi \) terms are obtained, using Buckingham’s \( \pi \) – theorem. Let us begin here with a basic but essential step in the experimental investigation, different input and output variables are briefly discussed below.

The indicator diagram of a four-stroke diesel engine cycle, as shown in the figure-1, consists of two enclosed areas. The large area represents the gross work done. The smaller shaded area, formed by the suction and exhaust operations is called pumping loop and represents the loss of work due to exhausting of burnt gases and admission of new unburnt gas or charge. This work obtained from the negative area is to be deducted from the gross work to obtain the net work done. The pumping loop is shown magnified in figure for explanation purpose. The gas exchange processes affects the volumetric efficiency of the engine. The performance of the engine, to a great deal, depends on the volumetric efficiency.

During the exhaust stroke when the piston moves from bottom dead centre to top dead centre, pressure rises and gases are pushed into exhaust pipe. Thus the power required to drive the exhaust gases is called the exhaust stroke loss and increase in speed increases the exhaust stroke loss. The indicator diagram of a diesel four-stroke cycle engine shows the suction line “ea” lies below the atmospheric pressure line. This fall of pressure below the atmospheric pressure is a result of the restricted area of the inlet passages, due to the restricted area, the entering air cannot flow into the cylinder in sufficient quantity to keep the pressure with the rapidly moving piston. With the use of supercharger the air pressure of the inlet on I.C. engine can be increased which results in decrease in the negative loop of the indicator diagram of a 4- stroke diesel engine cycle. This can increase the net work done.
II. OPERATING VARIABLES FOR COMPRESSION IGNITION ENGINE

Operating parameters of an internal combustion engine are reviewed and summary of important operating variables is given below [2-6].

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Parameters</th>
<th>Type of variable</th>
<th>Causes of variations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fuel consumption rate</td>
<td>Basic dependent variable</td>
<td>It is a dependent &amp; essential Input parameter of an engine varies because of any possible variable variations such as engine design (type of engine) &amp; fuel (type of fuel) and After treatment system (type of devices used and flow resistance offered by exhaust system)</td>
</tr>
<tr>
<td>2</td>
<td>Load on engine</td>
<td>Independent variable</td>
<td>As per the output power requirements such as vehicle weight, No. of persons seating, road resistance or slope condition, drag force because of air while in motion.</td>
</tr>
<tr>
<td>3</td>
<td>Speed</td>
<td>Independent variable</td>
<td>As per the availability of time or time requirements. Speed can be kept constant as in case of electric generator applications with the help of a governor.</td>
</tr>
<tr>
<td>4</td>
<td>back pressure on the engine</td>
<td>Independent variable</td>
<td>As per the complete exhaust system designs and their maintenance aspects because of aging effects. Particulate Matter accumulations causes variations of pressure drop across each component of exhaust system.</td>
</tr>
<tr>
<td>5</td>
<td>Air induction pressure</td>
<td>Independent variable</td>
<td>As per the design of air intake system and its maintenance aspects because of aging effects. Dust particles accumulation in air filter causes variations of pressure drop across each component of induction system. Turbocharger or Supercharger like air induction system components also affects on air induction pressure at the inlet to C.I. engine.</td>
</tr>
</tbody>
</table>

III. REDUCTION OF VARIABLES

The theory of experimentation as suggested by Hilbert (1961) is a good approach of representing the response of any phenomenon in terms of proper interaction of various inputs of the phenomenon. In fact, it is felt that such an approach is not yet seen towards research on engines. This approach finally establishes an experimental data based model for any phenomenon. It is felt in this investigation to adopt this approach for formulating the model correlating various inputs to a phenomenon of engine operation. The response variable of the phenomenon is fuel consumption during experimentation. A quantitative relationship is decided to be established amongst the response and the Inputs. The inputs are varied experimentally and the corresponding responses are measured. Such relationships are known as models. The data of the independent and dependent parameters of the system has been gathered during the experimentation. The optimum values of the independent pi terms can be decided by optimization of these models for maximum efficiency and minimum fuel energy. Future modifications must be done in such a way that each alternation should not cause air induction fall.
Finally in this analysis the fuel consumption \((F_c)\) of four stroke, single cylinder C.I. engine during a test run can be considered as dependent upon load \(I_d\), speed \(N\), back pressure \(P_b\) and suction pressure of intake air \(P_1\) of the engine. Hence \(F_c\) is a function of \(I_d\), \(N\) and \(P_b\). Using Buckingham’s Pi theorem, Mathematically,

\[
F_c = f(I_d, N, P_b, P_1) \quad \text{…….(I)}
\]

Or it can be written as

\[
f_1(F_c, I_d, N, P_b, P_1) = 0 \quad \text{……..(II)}
\]

Therefore, total number of variables, \(n = 5\)

No of fundamentals dimensions, \(m = 3\)

\(\text{(m is obtained by writing dimensions of each variables)}\)

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Parameters</th>
<th>Symbols</th>
<th>MLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fuel consumption rate</td>
<td>(F_c)</td>
<td>(MT^{-1})</td>
</tr>
<tr>
<td>2</td>
<td>Load on engine</td>
<td>(I_d)</td>
<td>(MLT^{-2})</td>
</tr>
<tr>
<td>3</td>
<td>Speed</td>
<td>(N)</td>
<td>(T^{-1})</td>
</tr>
<tr>
<td>4</td>
<td>Back pressure on the engine</td>
<td>(P_b)</td>
<td>(ML^{-1}T^{-2})</td>
</tr>
<tr>
<td>5</td>
<td>Air induction pressure</td>
<td>(P_1)</td>
<td>(ML^{-1}T^{-2})</td>
</tr>
</tbody>
</table>

Thus, the fundamental dimensions in the problems are \(M, L, T\) and hence \(m = 3\).

Therefore the number of dimensionless \(\pi\) terms = \(n - m\) 

\[= 5 - 3 = 2\]

Thus, two \(\pi\) terms say \(\pi_1, \pi_2\) are formed.

Hence equation (II) is written as,

\[
f_1(\pi_1, \pi_2) = 0 \quad \text{………..(III)}
\]

Each \(\pi\) terms = \(m + 1\) variables

Where \(m\) is equal to 3 and also called repeating variables. Out of five variables \(F_c, I_d, N, P_b,\) and \(P_1\), three variables are to be selected as repeating variables; \(F_c\) is dependent variable and should not be selected as repeating variables. Out of 4 remaining variables (One variable should have geometric property; the second variable should have fluid property and third one should have fluid property. These requirements must be fulfilled.) the variables \(I_d, N\) and \(P_b\) are selected as repeating variables. The variables themselves should not form a dimensionless term and should have themselves fundamental dimensions equal to \(m\), i.e. 3 here. Dimensions of \(I_d, N\) and \(P_b\) are \(MLT^{-2}, T^{-1}\), \(ML^{-1}T^{-2}\) and hence the 3 fundamental dimensions exists in \(I_d, N\) and \(P_b\) and they themselves do not form dimensionless group.

Each \(\pi\) terms is written as according to the equation

\[
\pi_1 = I_d^{a_1} \cdot N^{b_1} \cdot P_b^{c_1} \cdot F_c \quad \text{………..(IV)}
\]

\[
\pi_2 = I_d^{a_2} \cdot N^{b_2} \cdot P_b^{c_2} \cdot P_1 \quad \text{………..(V)}
\]

\(\pi\) terms are determined by the principle of dimensional homogeneity.

For the \(\pi\) term, we have

\[
\pi_1 = M^0 \cdot L^0 \cdot T^0
\]

\[
\pi_1 = (MLT^{-2})^{a_1} \cdot (T^{-1})^{b_1} \cdot (ML^{-1}T^{-2})^{c_1} \cdot (MT^{-1})
\]

Equating the powers of \(M, L\), and \(T\) on both sides, we get

Power of \(M\),

\[
0 = a_1 + b_1 + 1
\]

\[
a_1 + b_1 = -1 \quad \text{…..(i)}
\]

Power of \(L\),

\[
0 = a_1 - c_1 \quad \text{…..(ii)}
\]

Subtracting equation (ii) from (i),

\[
c_1 = \frac{-1}{2}\]

and by putting this value in equation (i), \(a_1 = \frac{-1}{2}\)

Power of \(T\),

\[
0 = -2a_1 - b_1 - 2c_1 - 1
\]

\[
-1 = 2a_1 + b_1 + 2c_1 \quad \text{…..(iii)}
\]

Putting values of \(a_1\) and \(c_1\), we get
\[
2 \left( \frac{-1}{2} \right) + b_1 + 2 \left( \frac{-1}{2} \right) = -1
\]
\[
-1 + b_1 - 1 = -1
\]

Substituting the values of \(a_1, b_1\) and \(c_1\) in equation (IV),
\[
\pi_1 = l_d^{-1/2} N^1 \cdot P_b^{-1/2} \cdot F_c
\]
\[
\pi_1 = \frac{N \cdot F_c}{\sqrt{P_b l_d}}
\]
\[
\pi_2 = l_d^{a_2} N^{b_2} \cdot P_b^{c_2} \cdot P_1
\]
\[
\pi_2 = \frac{M^0 \cdot L^0 \cdot T^0}{P_1}
\]
\[
\pi_2 = (MLT^{-2})^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-1}T^{-2})^{c_2} \cdot (ML^{-1}T^{-2})
\]

Equating the powers of \(M, L,\) and \(T\) on both sides, we get

- **Power of \(M,\)**
  \[
  0 = a_2 + c_2 + 1
\]
  \[
  a_2 + c_2 = -1 \quad \text{(iv)}
\]

- **Power of \(L,\)**
  \[
  0 = a_2 - c_2 - 1
\]
  \[
  a_2 - c_2 = 1 \quad \text{(v)}
\]

Subtracting equation (iv) from (v),
\[
   c_2 = -1\quad \text{and by putting this value in equation (iv),} \\
   a_2 = 0
\]

- **Power of \(T,\)**
  \[
  0 = -2a_2 - b_2 - 2c_2 - 2
\]

Putting values of \(a_2\) and \(c_2\), we get
\[
-2 = 2a_2 + b_2 + 2c_2 \quad \text{... (vi)}
\]
\[
2(0) + b_2 + 2(-1) = -2
\]
\[
0 + b_2 - 2 = -2
\]
\[
b_2 = -2 + 2 = 0
\]

Substituting the values of \(a_2, b_2\) and \(c_2\) in equation (V),
\[
\pi_2 = l_d^{0} N^{0} \cdot P_b^{-1} \cdot P_1
\]
\[
\pi_2 = \frac{P_1}{P_b}
\]

Substituting the value of \(\pi\)-terms in equation (III),
\[
f_1(\pi_1, \pi_2) = 0
\]
\[
f_1 \left( \frac{N \cdot F_c}{\sqrt{P_b l_d}} \cdot \frac{P_1}{P_b} \right) = 0
\]

or
\[
\frac{N \cdot F_c}{\sqrt{P_b l_d}} = f_1 \left( \frac{P_1}{P_b} \right)
\]

or
\[
F_c = \frac{\sqrt{P_b l_d}}{N} \cdot f_1 \left( \frac{P_1}{P_b} \right)
\]

### IV. EXPERIMENTATION

In the present work, throughout the complete trials conducted, engine jacket cooling water is kept constant at 0.1666 liters/sec, so as to provide ease in comparison of different parameters. Perforated circular copper plates arrangement in Diesel Particulate Filter is used as a test piece for back pressure variations, in some cases by varying the load and speed of the engine also. Further during the trials on DPF, each times the fresh perforated plates and rings are used. To reduce the backpressure on the engine, maximum no. of plates (i.e. 20 plates) provided with extra 8 no. of holes, of 5 mm diameter are used for determining the effect, that is of back pressure
reduction mainly. The different parameters are compared with the values of same parameters without using DPF, for the same engine output conditions [9,11,12].

During the trials, 10 test runs are conducted and distinct observations are obtained. These deviations may be a result of the lack of control in holding the variables at their planned levels, or simple lack of precision in the measurements [7]. Each observation is taken when the engine setup reaches at steady state condition to minimize error (See Appendix calculation table). [8-14].

**Engine specifications:**
1. Make: Kirloskar, single cylinder, four stroke Compression Ignition engine
2. Rated power output: 5 H.P
3. Stroke length: 110 mm
4. Bore diameter: 80mm
5. Loading type: Water resistance type load, with copper element and load changing arrangement
6. Moment arm: 0.2 meter
7. Orifice diameter (for air box): 25mm
8. Co-efficient of discharge of orifice: 0.64

V. DEVELOPMENT OF EXPERIMENTAL DATA BASED MODEL

One dependent π term (viz. π₁) and one independent π-term (viz. π₂) and have been recognized in the design of experimentation. These π-terms are accessible for the model formulation. Dependent π-term is assumed to be the function of the available independent π-term.

**Model Development for dependent π-term (π₁):**
For the dependent π-term, π₁ we have,

\[ π₁ = f(π₂) \]

Where f stands for “function of”

A possible accurate numerical form for this phenomenon could be

\[ π₁ = K₁ * (π₂)^{a₁} \] ...1.1

Equation (1.1) is one of the mainly significant and universal relations assumed in technical work and general functional relationship obtained as a result after applying dimensional analysis method.

In the equation 1.1, Constant of proportionality \( K_1 \) and indices \( a_1 \) are the two unknown terms. To obtain the values of these unknowns we require at least two sets of values of π₂. As per the experimental plan in design of experimentation we have n sets of these values. If any random one set from this table is selected and the values of unknowns, \( K_1 \) and \( a_1 \) are computed, then it may not result in one best unique solution representing a best-fit unique curve for the remaining set of the values.

To be very specific we can find out \( ^nC_r \) combinations of r sets taken together out of the available n sets of the values. The value \( ^nC_r \), in this case will be \( ^nC_2 \). Solving these many sets and finding their solutions will be a extraordinary task. Hence, it was decided to solve this problem by curve fitting technique (Spiegel 1980). To follow this method it is necessary to have the equations in the form as under.

\[ Z = a + b * x + c * y + d * z \] ………..1.2

The equation 1.1 can be brought in the form of equation 1.2 by taking the log of both sides. By taking the log of both the sides of this equation we get,

\[ \log π₁ = \log K₁ + a₁ * \log(π₂) \] ………..1.3

Let, \( \log π₁ = Z₁, \log K₁ = K₁, \log(π₂) = A \)

Then the equation 1.3 can be written as

\[ Z₁ = K₁ + a₁ * A \] ...1.4

Equation 1.4 is a regression equation of Z on A. In an n-dimensional co-ordinate system this represents a regression hyper-plane.

To find out the regression hyper-plane we determine \( a_1 \) in equation 1.4 so that

\[ \sum Z₁ = n K₁ + a₁ * \sum A \]

\[ \sum Z₁ * A = K₁ \sum A + a₁ * \sum A * A \] ………..1.5

Where, n is the number of runs or the number of sets of the values.

These equations are called usual equations equivalent to the equation 1.4 and are obtained as per the definition. In the above sets of equation the values of the multipliers of \( K₁ \) and \( a₁ \) are substituted to calculate the values of the unknowns (viz. \( K₁ \) and \( a₁ \)). The values of the terms on Left Hand Side and the multipliers of \( K₁ \) and \( a₁ \) in the sets of equation 1.5 are calculated. After substituting these values in the equation 1.5 we will get one set of equation which is to be solved simultaneously to get the values of \( K₁ \) and \( a₁ \).

The matrix method of solving these equations using MATLAB is given below.

Let, \( A = 2x2 \) matrix of the multipliers of \( K₁ \) and \( a₁ \).
\( B = 2x1 \) matrix of the terms on Left Hand Side and
\( C = 2x1 \) matrix of solutions or values of \( K₁ \) and \( a₁ \).

Then, \( C = inv(A) * B \) ……1.6
gives the unique values of $K_1$ and $\alpha_1$. In our case –

Engine performance Model

$$\pi_1 = x (\pi_2)^y$$

VI. CONCLUSIONS

The data of the independent and dependent parameters of the system has been gathered during the experimentation. In this case there are dependent and independent pi terms. It is necessary to correlate quantitatively various independent and dependent pi terms involved in this man-machine-animal system. This correlation is nothing but a mathematical model as a design tool for such work station. The optimum values of the independent pi terms can be decided by optimization of these models for maximum efficiency and minimum fuel energy. Since induction pressure is one of the operating parameters of a particular engine, which must be maximized for overall engine performance improvement. It would be a challenge to continue to do this with future emission limits. In this paper an important correlation for C.I. engine operating variables, for limited test run region, is investigated & procedure of experimental data based modeling is shown for a C.I. engine. Further scope is, to check that the applicability of model and feasible operating regime, for any particular internal combustion engine and its utility for performance analysis & optimization.

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