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A Study Of Laplace Transform Method For Solving Ordinary Differential Equations

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***Abstract*:** The Laplace transform is very efficient method for solving ordinary differential equations. There are many mathematical models arising in engineering in the form of ordinary differential equations. The motive of this paper is that a scientific review on solving some ordinary differential equations using Laplace transform method.

Keywords: Laplace Transform, Linear Ordinary Differential Equations, Initial Value Problem.

I. INTRODUCTION

The Laplace transform was primary used and named after by Pierre Simon Laplace. Pierre Simon Laplace was a French Mathematician an Astronomer. The Laplace transformation is applicable in so many fields such as engineering, science and technology. It is an effective tool for solving linear differential equation either ordinary differential equations or partial. It reduces ordinary differential equation into algebraic equation with constant coefficient and variable coefficient without finding the overall solution and the arbitrary constant.

II. DEFINITION: LAPLACE TRANSFORM

Let $f(t)$ be a function of t defined for all positive values of t. Then the Laplace transforms of $f(t)$, denoted by $L\{f(t)\}$ is defined as

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt = \bar{f}(s), \qquad (1)$$

Provided that the integral exists, s is a parameter which may be a real or complex number.

From (1), we get

$$L\{f(t)\} = \bar{f}(s),$$

∀ Values of s for which the improper integral converges,i.e.

$$\lim_{k\to\infty} \int_0^k e^{-st} f(t)dt < \infty \qquad (2)$$

If the equation (2) doesn't exist then the improper integral diverges and $f(t)$ has no transform.

$$L\{f(t)\} = \lim_{k\to\infty} \int_0^k e^{-st} f(t)dt < \infty$$

$$= F(s)$$

In other words,

$$L\{f(t)\} = F(s)$$

Where L is the operator of the Laplace Transform.

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III. SOLVE ORDINARY DIFFERENTIAL EQUATION BY USING LAPLACE TRANSFORM

In this section we solve some Ordinary Differential equation using Laplace transform method.

Example 1: Consider the following ordinary differential equation

$$y'' + y' = t \qquad y(0) = 0, \qquad y'(0) = -2$$

Taking Laplace on both sides we get

$$L\{y"\} + L\{y\} = L\{t\}$$

We know that

$$L\{y"\} = s^2 y(s) - sy(0) - y'(0)$$
$$L\{y\} = y(s)$$
$$L\{t\} = \frac{1}{s^2}$$

Therefore,

$$\{s^2 y(s) - sy(0) - y'(0)\} + y(s) = \frac{1}{s^2}$$
$$s^2 y(s) - s + 2 + y(s) = \frac{1}{s^2}$$
$$(s^2+1) y(s) = \frac{1}{s^2} + s - 2$$
$$y(s) = \frac{1}{s^2(s^2+1)} + \frac{s}{(s^2+1)} - \frac{2}{(s^2+1)}$$

Partial fractions of $\frac{1}{s^2(s^2+1)} = \frac{1}{s^2} - \frac{1}{(s^2+1)}$

$$y(s) = \frac{1}{s^2} - \frac{1}{(s^2+1)} + \frac{s}{(s^2+1)} - \frac{2}{(s^2+1)}$$
$$y(s) = \frac{1}{s^2} - \frac{3}{(s^2+1)} + \frac{s}{(s^2+1)}$$
$$y = L^{-1}\left\{\frac{1}{s^2} - \frac{3}{(s^2+1)} + \frac{s}{(s^2+1)}\right\}$$
$$y = L^{-1}\left\{\frac{1}{s^2}\right\} - 3L^{-1}\left\{\frac{3}{(s^2+1)}\right\} + L^{-1}\left\{\frac{s}{(s^2+1)}\right\}$$

The solution is

$$y = t - 3sint + cost$$

Example 2: Consider the following ordinary differential equation

$$y'' + 4y' + 3y = e^{-t}, \qquad y(0) = y'(0) = 1$$

Taking Laplace on both sides we get

$$L\{y"\} + 4L\{y'\} + 3L\{y\} = L\{e^{-t}\}$$

We know that

$$L\{y"\} = s^2 y(s) - sy(0) - y'(0)$$
$$L\{y'\} = sy(0) - y(0)$$
$$L\{y\} = y(s)$$
$$L\{e^{-t}\} = \frac{1}{s+1}$$

Therefore,

$$\{s^2 y(s) - sy(0) - y'(0)\} + 4\{sy(s) - y(0)\} + 3y(s) = \frac{1}{s+1}$$
$$s^2 y(s) - sy(0) - y'(0) + 4sy(s) - 4y(0) + 3y(s) = \frac{1}{s+1}$$

$$s^2y(s) - s - 1 + 4sy(s) - 4 + 3y(s) = \frac{1}{s+1}$$

$$\{s^2 + 4s + 3\}y(s) = \frac{1}{s+1} + s + 5$$

$$y(s) = \frac{1}{(s+1)\{s^2+4s+3\}} + \frac{s}{\{s^2+4s+3\}} + \frac{5}{\{s^2+4s+3\}}$$

Partial fractions

$$\frac{1}{(s+1)\{s^2+4s+3\}} = -\frac{1}{4(s+1)} + \frac{1}{2(s+1)^2} + \frac{1}{4(s+3)}$$

$$\frac{s}{\{s^2+4s+3\}} = -\frac{1}{2(s+1)} + \frac{3}{2(s+3)}$$

$$\frac{5}{\{s^2+4s+3\}} = \frac{5}{2(s+1)} - \frac{5}{2(s+3)}$$

$$y(s) = -\frac{1}{4(s+1)} + \frac{1}{2(s+1)^2} + \frac{1}{4(s+3)} - \frac{1}{2(s+1)} + \frac{3}{2(s+3)} + \frac{5}{2(s+1)} - \frac{5}{2(s+3)}$$

$$y(s) = -L^{-1}\left\{\frac{1}{4(s+1)}\right\} + L^{-1}\left\{\frac{1}{2(s+1)^2}\right\} + L^{-1}\left\{\frac{1}{4(s+3)}\right\} - L^{-1}\left\{\frac{1}{2(s+1)}\right\} + L^{-1}\left\{\frac{3}{2(s+3)}\right\} + L^{-1}\left\{\frac{5}{2(s+1)}\right\} - L^{-1}\left\{\frac{5}{2(s+3)}\right\}$$

$$y = -\frac{1}{4}e^{-t} + \frac{1}{2}e^{-t}t + \frac{1}{4}e^{-3t} - \frac{1}{2}e^{-t} + \frac{3}{2}e^{-3t} + \frac{5}{2}e^{-t} - \frac{5}{2}e^{-3t}$$

$$y = \frac{7}{4}e^{-t}$$

IV. CONCULSION

Through this paper we solve some ordinary differential equations by using Laplace transform. Laplace transform is very effective tool to simplify very complex problems in the field of engineering, science and technology.

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