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# GRAVITATIONAL COLLAPSE OF HIGHER DIMENSIONAL MONOPOLE AND CHARGED VAIDYA SPACE-TIME

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*Abstract:* We analyze the gravitational collapse of higher dimensional monopole Vaidya space-time and also charged Vaidya space-time. We show that singularities arising in monopole and charged null fluids in higher dimension are naked, it violating Cosmic Censorship Hypothesis (CCH).

Keywords - Cosmic censorship hypothesis, naked singularity, Gravitational collapse

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#### I. INTRODUCTION

Most important and challenging problem in classical general relativity is proof of Cosmic Censorship Hypothesis (CCH). Cosmic Censorship Hypothesis proposed by R. Penrose [1] which state that singularities formed in gravitational collapse physically reasonable matter cannot be observed. There are two versions of this hypothesis. The weak Cosmic Censorship Hypothesis state that all singularities formed in gravitational collapse are hidden behind the event horizon of the gravity and are invisible to distant observer from infinity. On the other hand the strong Cosmic Censorship Hypothesis asserts that no singularities are visible. Many researchers have attempted to give precise reformulation to this hypothesis, but neither proof nor mathematical formulation for this hypothesis is available so far, on the contrary several examples of naked singularities have been found which includes collapse of dust [2-9], radiation [10-15], perfect fluid [16-22] etc. The existence of a black hole or a naked singularity may have shown all these models. One of the most important examples having naked singularities is the Vaidya solution [23]. This was shown first by Papapetrou [12], since then this solution is being used to analyze the scenario of gravitational collapse in general relativity. Recently Anzhong Wang [24] introduced a more general family of Vaidya space-times which covers monopole solutions, de- sitter and anti de-sitter solutions and charged Vaidya solutions as special cases.

Recently, there has been renewed interest in studying higher dimensional space-ties from the point of view of both cosmology [25] and gravitational collapse [25-26]. An interesting problem that arise the effect of the higher dimension can have on the formulation of naked singularity [27-32]. The present work deals with the spherically symmetric collapse of monopole Vaidya case and charged Vaidya case. It admits strong curvature naked singularities.

In this paper we have generalized the higher dimensional Vaidya space-time collapse [27, 30, 33] and investigated the nature of the singularities arising in this space-time. Also we describe the existence of strong curvature naked singularities in monopole Vaidya solution.

#### **II HIGHER DIMENSIONAL VAIDYA SPACE-TIME**

The (n+2) dimensional space time of generalized Vaidya metric is as follows [34]

$$ds^{2} = -\left(1 - \frac{m(v, r)}{r^{n-1}}\right)dv^{2} + 2dvdr + r^{2}\left[d\theta_{1}^{2} + \sin^{2}\theta_{1}d\theta_{2}^{2} + \sin^{2}\theta_{1}\sin^{2}\theta_{2}d\theta_{3}^{2} + \sin^{2}\theta_{1}\sin^{2}\theta_{2}....sin^{2}\theta_{n-1}d\theta_{n}^{2}\right]$$
(1)

where v is advanced Eddington time coordinate and r is radial coordinate with the condition  $0 < r < \infty$ where m(v, r) is gravitational mass which will present in the sphere of radius r and

$$d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_2^3 + \sin^2\theta_1 \sin^2\theta_2 \dots \sin^2\theta_{n-1} d\theta_n^2$$
(2)

is the metric of n- sphere and we denote this metric as  $d\psi^2$ . The corresponding energy momentum tensor is given by

$$T_{\gamma \lambda} = T_{\gamma \lambda} {}^{(n)} + T_{\gamma \lambda} {}^{(m)}$$
(3)

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$$T_{\gamma\lambda} \stackrel{(n)}{=} \mu l_{\gamma} l_{\lambda} \tag{4}$$

And 
$$T_{\gamma\lambda} = (P + \rho) \left( l_{\gamma} n_{\lambda} + l_{\lambda} n_{\gamma} \right) + P g_{\gamma\lambda}$$
 (5)

Here the above P and  $\rho$  represents the thermodynamic pressure and energy density, where as  $\mu$  represents energy density of Vaidya null radiation.

 $l_{\gamma}$ ,  $n_{\lambda}$  are linearly independent two eigen vectors of energy momentum tensor.

These Eigen vectors are having the components

$$l_{\gamma} = \lambda_{\gamma}^{0}, \qquad (6)$$

$$n_{\gamma} = \frac{1}{2} \left[ 1 - \frac{m(v, r)}{r} \right] \lambda_{\gamma}^{0} - \lambda_{\gamma}^{1} , \qquad (7)$$

$$l_{\gamma} n^{\nu} = -1 \qquad l_{\gamma} l^{\nu} = n_{\gamma} n^{\nu} = 0$$

$$l_{\nu}n^{\nu} = -1, \qquad l_{\nu}l^{\nu} = n_{\nu}n^{\nu} = 0$$
(8)

Especially for the above equation (5) when  $P = \rho = 0$  then it reduces to Vaidya solution of higher dimensional space time with m= m (v)

Now we consider energy momentum tensor of equation (7) as the general case.

The energy conditions for the above will be as follows:

1. The dominant energy conditions are

|    | $\mu \ge 0, \ \rho \ge P \ge 0$           | (9)  |
|----|-------------------------------------------|------|
| 2. | The weak and strong energy conditions are |      |
|    | $\mu \ge 0, \ P \ge 0 \ , \ \rho \ge 0$   | (10) |

Einstein field equations is given by

$$G_{\gamma\lambda} = k T_{\gamma\lambda} \tag{11}$$

Where  $G_{\gamma\lambda}$  is Einstein tensor, k is Gravitational constant

From equations (1), (3) and (4) which is having Stress Energy tensor is given by

$$\rho = \frac{nm}{k(n-1)r^{n}}$$

$$P = \frac{-m}{k(n-1)r^{n-1}}$$

$$\mu = \frac{nm}{k(n-1)r^{n}}$$
(12)
(13)
(14)

Here dash and dot represent differentiation with respect to 'r' and 'v' respectively. From the above equations, the limitations on 'm' should be

(*i*)  $m' \ge 0$ ,  $m'' \ge 0$  (*ii*) m > 0 to satisfy the energy conditions (i) indicates the mass function either increases with 'r' or is constant

(ii) indicates the matter within radius 'r' increases with time.

## III NAKED SINGULARITIES IN MONOPOLE VAIDYA SPACE-TIME

Monopole solutions in Vaidya background are given by [35-36]

$$m(v, r) = \alpha v^{n-1} + \beta v^{n-1}$$
  
where  $\alpha$  and  $\beta$  are arbitrary constants.

With this mass function, the metric (1) becomes

$$ds^{2} = -\left[1 - \frac{\alpha v^{n-1}}{r^{n-1}} - \beta\right] dv^{2} + 2dv dr + r^{2} \left(d\theta_{1}^{2} + \sin^{2} \theta_{1} d\theta_{2}^{2} + \sin^{2} \theta_{1} \sin^{2} \theta_{2} d\theta_{3}^{2} + \dots + d\theta_{n}^{2}\right)$$
(16)

To investigate the nature of singularity that may form in the gravitational collapse we need to consider the radial null geodesics defined by  $ds^2 = 0$ , taking  $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0$  into account.

(15)

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The equation for the radial null geodesics for the metric (16) is given by

$$\frac{dr}{dv} = \frac{1}{2} \left[ 1 - \frac{m(v, r)}{r^{n-1}} \right] \tag{17}$$

To investigate the nature of the singularity, we need to consider the radial null geodesics defined by  $ds^2 = 0$ . Equation for the null geodesics for the metric (17) is given by

$$\frac{dr}{dv} = \frac{1}{2} \left[ 1 - \frac{\alpha v^{n-1}}{r^{n-1}} - \beta \right]$$
(18)

It can be easily checked that the above differential equation has singularity at v = 0, r = 0.

For the geodesic tangent to be uniquely defined and exist at this point we must have [37]

$$X_{0} = \lim_{v \to 0 \atop r \to 0} \frac{v}{r} = \lim_{v \to 0 \atop r \to 0} \frac{dv}{dr} = \frac{2}{1 - \alpha X_{0}^{n-1} - \beta}$$
(19)

i.e. 
$$\alpha X_0^n + (\beta - 1)X_0 + 2 = 0$$
 (20)

The variable X can be interpreted as the tangent to the outgoing geodesics, hence if equation (20) has at least one positive and real root then the singularity could be naked. If the equation (20) has no real and positive root then the collapse ends into a black hole.

It can be checked from the Theory of equations that above equation has at least three positive roots.

To investigate whether the naked singularities will arise or not, we take some different values of  $\alpha$ ,  $\beta$  and n.

Now if we take n = 4 then the equation (20) reduces to

$$\alpha X_0^4 + (\beta - 1)X_0 + 2 = 0 \tag{21}$$

If we take  $\alpha = 0.001$  then the roots of equation (21) obtained for different values of  $\beta$  in monopole space-time are shown in the following table.

| Table 1 Values of $X_0$ for different values of $\beta$ |                                            | _              |              |
|---------------------------------------------------------|--------------------------------------------|----------------|--------------|
|                                                         | β                                          | X <sub>0</sub> |              |
| 0.01<br>0.02<br>0.03                                    |                                            | 2.03761        |              |
|                                                         |                                            | 2.05916        |              |
|                                                         |                                            | 2.08119        |              |
|                                                         | 0.04                                       | 2.1037         |              |
|                                                         | 0.05                                       | 2.1268         |              |
|                                                         | 0.06                                       | 2.1504         |              |
|                                                         | 0.07                                       | 2.1745         |              |
|                                                         | 0.08                                       | 2.1993         |              |
|                                                         | 0.09                                       | 2.2247         |              |
|                                                         |                                            |                |              |
|                                                         | 2.3 –                                      | α=0.001        |              |
|                                                         | 2.3<br>2.2                                 | α=0.001        |              |
| X,                                                      | 2.3<br>2.2<br>2.1                          | α=0.001        |              |
| Xo                                                      | 2.3<br>2.2 -<br>2.1 -<br>2 -               | α=0.001        |              |
| X <sub>o</sub>                                          | 2.3<br>2.2<br>2.1<br>2<br>1.9              | α=0.001        |              |
| X <sub>o</sub>                                          | 2.3<br>2.2<br>2.1<br>2<br>1.9<br>0.01 0.02 | α=0.001        | 07 0.08 0.09 |

Figure.1 Graph of the Values of  $X_0$  against the values of  $\beta$  for fixed value of  $\alpha$ 

If we observe above graph we see that the values of  $X_0$  increases as we increase the values of  $\beta$ . If we take  $\beta$ = 0.001 then the roots of equation (21) obtained for different values of  $\alpha$  in monopole space-time are shown in the following table. Table 2.Values of  $X_0$  for different values of  $\alpha$ 

| α    | $\mathbf{X}_0$ |
|------|----------------|
| 0.01 | 2.03761        |
| 0.02 | 2.0591         |
| 0.03 | 2.0811         |
| 0.04 | 2.1037         |
| 0.05 | 2.1268         |
| 0.06 | 2.1504         |
| 0.07 | 2.1745         |
| 0.08 | 2.1999         |
| 0.09 | 2.2247         |
| 0.1  | 2.2507         |



Figure 2. Graph of the Values of  $X_0$  against the values of  $\alpha$  for fixed value of  $\beta$ 

If we observe the above graph then we see that the values of  $X_0$  increases as we increase the values of  $\alpha$ .

Now if we take n = 5 then the equation (20) reduces to

$$\alpha X_0^5 + (\beta - 1)X_0 + 2 = 0 \tag{22}$$

If we take  $\alpha = 0.001$  then the roots of equation (22) obtained for different values of  $\beta$  in monopole space-time are shown in the following table.

| β    | $X_0$  |
|------|--------|
| 0.01 | 2.0574 |
| 0.02 | 2.0806 |
| 0.03 | 2.1044 |
| 0.04 | 2.1288 |
| 0.05 | 2.154  |
| 0.06 | 2.18   |
| 0.07 | 2.2068 |
| 0.08 | 2.2344 |
| 0.09 | 2.263  |
| 0.1  | 2.2925 |

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|---------------------------------------|
|---------------------------------------|



Figure 3. Graph of the Values of  $X_0$  against the values of  $\beta$  for fixed value of  $\alpha$ 

If we observe above graph we see that the values of  $X_0$  increases as we increase the values of  $\beta$ , which ensures that the singularity is naked.

If we take  $\beta$ = 0.001 then the roots of equation (22) obtained for different values of  $\alpha$  in monopole space-time are shown in the following table.

Table 4. Values of  $X_0$  for different values of  $\alpha$ 

| α     | X0                   |
|-------|----------------------|
| 0.001 | 2.0371               |
| 0.002 | 2.0799               |
| 0.003 | 2.1353               |
| 0.004 | 3.0377               |
| 0.005 | 2.4 <mark>031</mark> |



Figure.4. Graph of the Values of  $X_0$  against the values of  $\alpha$  for fixed value of  $\beta$ 

We observe from the above graph we see that the values of  $X_0$  increases as we increase the values of  $\beta$  up to  $\alpha = 0.004$ . Afterwards  $X_0$  decreases and beyond  $\alpha = 0.006$  we get imaginary roots which gives black holes.

### IV Naked Singularities in Charged Vaidya space-time

Following Anzhong Wang [24], we defined the general expression for charged Vaidya space-time as

$$m(v,r) = \alpha v^{n-1} - \frac{\beta v^{2(n-1)}}{r^{n-1}}$$
(23)

where  $\alpha$  and  $\beta$  are arbitrary constants.

With this mass function the metric (1) becomes,

$$ds^{2} = -\left[1 - \frac{\alpha u^{n-1}}{r^{n-1}} + \frac{\beta u^{2(n-1)}}{r^{2(n-1)}}\right] dv^{2} + 2dv dr + r^{2} \left(d\theta_{1}^{2} + \sin^{2} \theta_{1} d\theta_{2}^{2} + \sin^{2} \theta_{1} \sin^{2} \theta_{2} d\theta_{3}^{2} + \dots + d\theta_{n}^{2}\right)$$

After some computation we get  $X_0$ , which is the tangent to the radial null geodesic at the singularity given by

$$X_{0} = \frac{2}{1 - \alpha X_{0}^{n-1} + \beta X_{0}^{2(n-1)}}$$

which implies

$$\beta X_0^{2n-1} - \alpha X_0^n + X_0 - 2 = 0 \tag{24}$$

The variable X can be interpreted as the tangent to the outgoing geodesics, hence if equation (24) has at least one positive and real root, then the singularity could be naked. If the equation (24) has no real and positive root then the collapse ends into a black hole.

It can be checked from the Theory of equations that above equation has at least three roots.

Now if we take n = 4 then the equation (24) reduces to

$$\beta X_0^7 - \alpha X_0^4 + X_0 - 2 = 0$$

If we take  $\alpha = 0.001$  then the roots of equation (25) obtained for different values of  $\alpha$  in monopole space-time are shown in the following table.

Table 5 Values of  $X_0$  for different values of  $\beta$ 

| 0     |                |        |
|-------|----------------|--------|
| β     | $\mathbf{X}_0$ |        |
| 0.001 | 1.918          |        |
| 0.002 | 1.8586         |        |
| 0.003 | 1.8156         |        |
| 0.004 | 1.7819         |        |
| 0.005 | 1.754          |        |
| 0.006 | 1.7303         |        |
| 0.007 | 1.7096         |        |
| 0.008 | 1.6913         |        |
| 0.009 | 1.6749         | $\sim$ |
|       |                |        |



Figure 5 Graph of the Values of  $X_0$  against the values of  $\beta$  for fixed value of  $\alpha$ 

If we observe above graph we see that the values of  $X_0$  decreases as we increase the values of  $\beta$ , which ensures that the singularity is naked. If we take  $\beta = 0.001$  then the roots of equation (25) obtained for different values of  $\alpha$  in monopole space-time are shown in the following table.

(25)

ofα

| Tab | able 6 Values of X <sub>0</sub> for different values |        |
|-----|------------------------------------------------------|--------|
|     | α                                                    | X0     |
|     | 0.01                                                 | 2.028  |
|     | 0.02                                                 | 2.22   |
|     | 0.03                                                 | 2.5681 |
|     | 0.04                                                 | 3.0283 |
|     | 0.05                                                 | 3.4073 |
|     | 0.06                                                 | 3.7075 |
|     | 0.07                                                 | 3.9583 |
|     | 0.08                                                 | 4.1764 |
|     | 0.09                                                 | 4.3708 |
|     | 0.1                                                  | 4 5475 |



Figure 6 Graph of the Values of  $X_0$  against the values of  $\alpha$  for fixed value of  $\beta$ 

If we observe above graph we see that the values of  $X_0$  increases as we increase the values of  $\alpha$ , which ensures that the singularity is naked.

# V CONCLUDING REMARK

Gravitational collapse is the most striking phenomenon in general relativity. The CCH has provided strong motivation for research in the field. In the absence of general proof for CCH, many examples have been proposed in which naked singularity is the outcome of gravitational collapse.

In the present work we have studied the higher dimensional monopole and charged Vaidya space-time and shown that naked singularities do occur as the end stage of gravitational collapse.

Thus one may argue that the dimension of space-time does not play any fundamental role in the formation of naked singularities. Thus the higher dimensional monopole and charged Vaidya space-time violates Cosmic Censorship Hypothesis.

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