# **Boundary Layer Flow of a Casson Fluid Over an Elastic Permiable Sheet with Magnetic Field**

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#### **Abstract:**

This paper presents the study of boundary layer flow due to an exponentially extending surface within the sight of a connected attractive field. Casson liquid model is utilized to describe the non-Newtonian liquid conduct. The stream is exposed to suction/blowing at the surface. Investigation is completed within the sight of warm radiation and Synthetic response. The administering fractional differential conditions are right off the bat changed over into nonlinear standard differential conditions by utilizing fitting changes and afterward tackled numerically by utilizing Runge Kutta Shooting strategy. The impact of Casson parameter, Attractive field, Permeable parameter, Radiation parameter, Warmth age parameter, Synthetic response parameter, Prandtl number, Suction parameter and Eckert number on the stream factors is dissected.

Key Words: Casson fluid, stretching surface, permiable medium, Casson fluid

#### 1 INTRODUCTION

The investigation of limit layer stream over an extending sheet is of impressive consideration on account of its consistently expanding mechanical applications and critical bearing on a few innovative procedures. Precedents are materials fabricated by expulsion, cooling of metallic plates in cooling shower, glass fiber and paper generation, metal turning, drawing plastic movies, glass blowing and so forth. The vast majority of the accessible writing focused on the investigation of limit layer stream over an extending sheet where the speed of the extending surface is thought to be directly corresponding to the separation from the settled inception. Rajagopal et al. (1984) talked about the stream of second-request liquid over an extended sheet. Anderson et al. (1992) considered the impact of attractive field on the stream of a viscoelastic liquid past an extending sheet. Singe (1994) contemplated warmth and mass move in a hydromagnetic stream of viscoelastic liquid over an extending sheet. Abel et al. (2005) investigated MHD limit layer stream over consistently moving extending surface inserted in a permeable medium by considering the Lightness power and warm radiation impacts. Mukhopadyaya et al. (2008) talked about the free convective limit layer stream with variable thickness over an extending surface with warm radiation. Dulal buddy (2009) explored the blended convection stream of an incompressible liquid over an extending sheet within the sight of radiation.

In any case, sensibly extending of plastic sheet may not really be straight. Stream and warmth exchange qualities past an exponentially extending sheet have numerous applications in innovation, for example, strengthening and diminishing of copper wires, the last item relying upon the rate of warmth exchange at the extending consistent surface with exponentially extending speed and temperature circulation. Amid such procedures, both the kinematics of extending and the concurrent warming or cooling impacts the nature of the last items. Magyari and Keller (1999) examined the similitude arrangement of stream and warm limit layers on an exponentially extending surface. These arrangements include an exponential reliance of the temperature dispersion toward the path parallel to that of the extending. Elbashbeshy (2001) researched the stream brought about by exponentially nonstop extending surface. The impacts of thick dispersal on blended convection stream and warmth exchange over an exponentially extending surface was concentrated by Partha et al. (2005). Bidin and Nazar (2009) examined the limit layer stream over an exponential extending sheet with warm radiation, utilizing Keller-box strategy. El-Aziz (2009), Ishak (2011) depicted the stream and warmth exchange past an exponentially extending sheet. Bhattacharyya (2012) talked about the unfaltering limit layer stream and receptive mass exchange past an exponentially extending surface in an exponentially moving free stream.

Convective warmth exchange assumes an indispensable job amid the taking care of and preparing of non-Newtonian liquid streams. Mechanics of non-Newtonian liquid streams shows an exceptional test to Specialists, Physicists, and Mathematicians. On account of the multifaceted nature of these liquids, there is certifiably not a solitary constitutive condition which shows all properties of such non-Newtonian liquids. In writing, by far most of non-Newtonian liquid models is worried about basic models like the power law and grade a few. The power-

law demonstrate has a huge utilization in displaying liquids with shear-subordinate consistency yet it can't anticipate the impacts of flexibility. The liquids of evaluation a few can figure the impacts of versatility yet the consistency in these models isn't shear subordinate, and furthermore they can't anticipate the impacts of pressure unwinding. Constrained convection heat exchange on a level plate inserted in permeable media for power-law liquids was examined by Hady and Ibrahim (1997). Free convection warmth and mass exchange of non-Newtonian power-Law liquids with yield worry from a vertical level plate which is immersed in permeable media was investigated by Jumah and Mujumdar (2000). The joined impact of attractive field and gooey dispersal on a power-law liquid over plate with variable surface warmth motion installed in a permeable medium was concentrated by El-Amin (2003). Limited component technique for the impact of different infusion parameters on warmth exchange for a power-law non-Newtonian liquid over a consistently extended surface with warm radiation is introduced by Seddeek (2006). MHD control law liquid stream and warmth exchange over a non-isothermal extending sheet is examined by Prasad et al. (2009).

There is another liquid model for non-Newtonian liquid known as Casson liquid. Casson liquid displays yield pressure. It is outstanding that Casson liquid is a shear diminishing fluid which is expected to have an unbounded consistency at zero rate of shear, a yield worry beneath which no stream happens, and a zero thickness at a boundless rate of shear, i.e., if the connected shear pressure is not exactly the yield pressure, it carries on like a strong, though if the connected shear pressure is more noteworthy than yield pressure, it begins to move. The instances of Casson liquid are jam, tomato sauce, nectar, soup, concentrated organic product juices, and so on. In addition, human blood can likewise be treated as Casson liquid. Eldabe (1995) considered the warmth exchange of Casson liquid stream between two turning chambers. The stream of Casson liquid in a cylinder was concentrated by Dash et al. (2000) and Nagarani et al. (2004). The insecure limit layer stream of a Casson liquid over a moving level plate was concentrated by Mustafa et al. (2011). Shehzad (2013) talked about the impacts of mass exchange on the MHD limit layer stream of a Casson liquid with substance response. Sarojamma et al. (2014) examined the impact of substance response on MHD limit layer stream of a Casson liquid over an extending sheet.

Suction or infusion (blowing) of a liquid through the bouncing surface can essentially change the stream field. Infusion or withdrawal of liquid through a permeable jumping divider is of general enthusiasm for viable issues including limit layer control applications, for example, film cooling, polymer fiber covering, and covering of wires. The procedure of suction and blowing has likewise its significance in many building exercises, for example, in the plan of push bearing and outspread diffusers and warm oil recuperation. Suction is connected to compound procedures to expel reactants. Blowing is utilized to include reactants, cool the surface, avoid consumption or scaling and decrease the drag.

The warmth and mass exchange of thick liquids over an isothermal extending sheet with suction or passing up Gupta and Gupta (1997). Chen and Burn (1998) explored the warmth exchange on constant extending surfaces with suction or blowing. Afify (2009) contemplated the impacts of warm dissemination and dispersion thermo on free convective warmth and mass exchange over an extending surface thinking about suction or infusion. Chamkha et al. (2010) contemplated the closeness answer for precarious warmth and mass exchange from an extending surface installed in a permeable medium with suction/infusion and concoction response impacts. The impact of thick dispersal and radiation on shaky MHD free-convection stream past an unending warmed vertical plate in a permeable medium with time subordinate suction was concentrated by Israel-Cookey et al. (2003). Abo-Eldahab et al. (2004) examined the blowing/suction impact on hydromagnetic heat exchange by blended convection from a slanted extending surface with inside warmth age/ingestion.

#### MATHEMATICAL FORMULATION

Consider laminar limit layer, two-dimensional relentless stream of an incompressible, non-Newtonian Casson liquid over an exponentially extending sheet inserted in a permeable medium. Expect that the plate has a surface temperature (Tw) and focus (Cw) and is set in a qiscent liquid of uniform surrounding temperature (T $\Box$ ) and fixation (C $\Box$ ). Pick the organize framework with the end goal that x-pivot is parallel to the surface and y-hub typical to the surface . Considered the warmth and mass move forms within the sight of compound response and suction.

$$\tau_{ij} = \begin{cases}
2\left(\mu_{B} + \frac{P_{y}}{\sqrt{2\pi}}\right) e_{ij}, & \pi > \pi_{c} \\
2\left(\mu_{B} + \frac{P_{y}}{\sqrt{2\pi_{c}}}\right) e_{ij}, & \pi < \pi_{c}
\end{cases}$$
(1)

where  $\Box$ B is the plastic powerful thickness of the non-Newtonian liquid, Py is the yield worry of the liquid, e ij means the (I, j) - th part of the misshapening rate,  $\pi = e$  ij e ij is the result of the segment of disfigurement rate with itself,  $\pi c$  is the basic estimation of  $\pi$  dependent on the non-Newtonian model.

Under the above presumptions and with the standard limit layer approximations, the administering conditions are:

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{v}} = 0 \tag{2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u - \frac{v}{K'} u$$
 (3)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\sigma B^2}{\rho c_p} u^2 + \frac{v}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{Q(x)}{\rho c_p} \left( T - T_{\infty} \right)$$
(4)

$$u\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_1(C - C_{\infty})$$
(5)

where u and v are the velocity components along the x and y directions, v is the kinematic viscosity,  $\sigma$  is the electric conductivity of the fluid,  $\rho$  is the density,  $\beta = \frac{\mu_B \sqrt{2\pi_c}}{p_y}$  is the Casson fluid parameter,  $K' = \frac{1}{2} \frac{1}{$ 

 $K_0 \, e^{-x/L}$  is the permeability of the porous medium, T is the temperature, C is the concentration,  $T_\infty$  is the temperature for away from the plate,  $C_\infty$  Species concentration of the ambient fluid,  $\alpha$  is the thermal diffusivity,  $Q(x) = Q_0 \, e^{x/L}$  is the heat generation parameter, D is the mass diffusivity,  $q_r$  is the radiative heat flux,  $K_1 = \gamma_1 \, e^{x/L}$  is the chemical reaction rate constant. The second, third, fourth and fifth terms on the right hand side of the energy equation (4) represent the radiative heat flux, joule dissipation, viscous dissipation and heat generation effects respectively. The second term on the right hand side of the concentration Eq. (5) represents chemical reaction effects.

Assume that the exponentially stretching surface is maintained at the stretching velocity  $u_w(x)$ , exponential temperature distribution  $T_w(x)$  and exponential concentration distribution  $C_w(x)$ , which are defined by

$$u_{w} = u_{0} e^{x/L}$$
,  $T_{w} = T_{\infty} + T_{0} e^{x/2L}$   $C_{w} = C_{\infty} + C_{0} e^{x/2L}$  (6)

where the subscripts w,  $\infty$  refers to the surface and ambient conditions,  $u_0$  is the characteristic velocity,  $T_0$  is the reference temperature,  $C_0$  is the reference concentration, L is the reference length.

Hence, the boundary conditions of the flow are

$$u = u_w, v = -V(x), T = T_w, C = C_w \text{ at } y = 0$$
 (7a)

$$u \to 0, T \to T_{\infty}, C \to C_{\infty}$$
 as  $y \to \infty$  (7b)

where  $V(x) = V_0 e^{\frac{x}{2L}}$ , a special type of velocity at the wall, V(x) > 0 is the velocity of suction and V(x) < 0 is the velocity of blowing,  $V_0$  is the initial strength of suction.

By using the Rosseland approximation (Brewster 1992), we can write the radiative heat flux  $q_r$  as

$$q_{r} = -\frac{4\sigma^{*}}{3s^{*}} \frac{\partial T^{4}}{\partial y} \tag{8}$$

where  $\sigma^*$  is the Stephen Boltzmann constant,  $s^*$  is the mean absorption coefficient. We assume that the temperature differences within the flow are sufficiently small so that  $T^4$  can be expanded in a Taylor series about  $T_{\infty}$  and neglecting higher order terms result in

$$T^{4} = 4T_{m}^{3}T - 3T_{m}^{4} \tag{9}$$

Substituting equations (8) and (9) into (4), we get

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_{\infty}^3}{3s^* \rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B^2}{\rho c_p} u^2$$
(10)

#### **4 DISCUSSION OF THE RESULTS**

The set non-linear differential equations 12 (a)-12(c) cannot be solved in closed-form, so it is required to solve this problem numerically in order to describe the physics of the problem well. The resulting nonlinear ordinary differential equations are solved by fourth-order Runge-Kutta method with shooting technique. In order to analyze the results, numerical computations have been carried out for various values of Casson parameter ( $\beta$ ), Magnetic parameter ( $\beta$ ), porosity parameter ( $\beta$ ), Radiation parameter ( $\beta$ ), Prandtl number ( $\beta$ ), Eckert number (Ec), heat generation parameter ( $\beta$ ), Schmidt number ( $\beta$ ), Chemical reaction parameter ( $\beta$ ), Suction parameter ( $\beta$ ). For numerical results, we considered the non dimensional parameter values as  $\beta$ 0.1,  $\beta$ 1,  $\beta$ 2,  $\beta$ 2,  $\beta$ 3,  $\beta$ 4,  $\beta$ 5,  $\beta$ 6,  $\beta$ 6,  $\beta$ 8,  $\beta$ 9.1,  $\beta$ 9,  $\delta$ 9,

The skin-friction ( $C_f$ ), Nusselt number (Nu) and Sherwood numbers (Sh) are evaluated for variations in the governing parameters and are presented in Tables 3-6. Table 3 shows the skin friction coefficient for the different values of  $\beta$  and S. By increasing the values of  $\beta$ , the value of the skin friction coefficient, Nusselt number, Sherwood number decreases, but it increases upon increasing suction parameter. Table 4 represents the variation of the skin-friction, Nusselt number and Sherwood numbers for various values of K and M. It is observed that the skin friction coefficient increases with the increase in magnetic parameter M and Porous parameter K while Nusselt number and Sherwood numbers decreases.

We first focus on the impacts of Casson parameter  $\square$  on speed and temperature profiles. Figure 2 represents the impact of Casson parameter  $\square$  on speed profile within the sight of suction/blowing. Speed is found to diminish with expanding the estimations of Casson parameter  $\square$  as a result of increment in  $\square$  plastic powerful thickness increments and it makes opposition smooth movement. It is likewise seen that force limit layer thickness diminishes with expanding  $\square$ . Liquid speed is substantially more smothered if there should be an occurrence of suction (S = 0.5) than that of blowing (S = -0.5).

The impact of Casson parameter on temperature profiles within the sight of suction/blowing is appeared in figure 3. Warm limit layer thickness increments with expanding  $\Box$ . The thickness of the limit layer happens because of the expansion in versatility stress parameter. It is unmistakably appeared with expanding Casson liquid parameter  $\Box$ , the temperature is found to diminish for both the instances of suction and blowing.

Figure 4 demonstrates the variety of speed profile against the attractive parameter within the sight of suction/blowing. We see that the impact of the attractive parameter is to diminish the speed of the liquid in the limit layer district. This plainly uncovers the transverse attractive field restricts the liquid transport because of expanding Lorentz compel related with expanding attractive parameter. Figure 5 displays the impact of attractive parameter on temperature profiles within the sight of suction/blowing. It is seen that the temperature is observed to be expanded. The warm limit layer thickness increments, as attractive parameter increments. Impacts of permeable parameter on speed and temperature field within the sight of suction/blowing is displayed in figure 6 and 7. It is seen that ascent in the estimation of permeable parameter decreases the speed profiles and improves the temperature. It is because of the way that expansion in porosity augments the permeable layer and builds the energy limit layer thickness.

Figure 8 speaks with the impact of warmth age parameter on temperature profiles within the sight of suction/blowing. Physically, the nearness of warmth age impacts tends to build the liquid temperature. Figure 9 delineates the impact of radiation on temperature profiles within the sight of suction/blowing. It shows that warm radiation improves the temperature in the limit layer district. Along these lines radiation should keep at its base so as to encourage better cooling condition.

The impact of Eckert number on warmth move within the sight of suction/blowing is appeared in Figure 10. It is discovered that temperature in the limit layer area increments with increment in the gooey dispersal parameter. Figure 11 portrays the impacts of Prandtl number Pr on temperature profiles. It demonstrates that the temperature diminishes with expanding Pr. Additionally, the warm limit layer thickness diminishes with expanding Prandtl number.

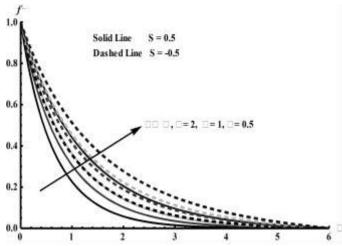


Figure 2: Variation of velocity profile for different values of  $\boldsymbol{\beta}$ 

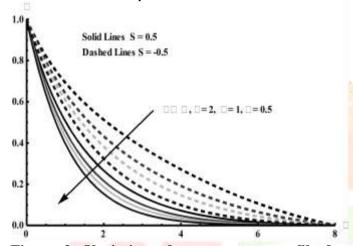


Figure 3: Variation of temperature profile for different values of  $\beta$ 

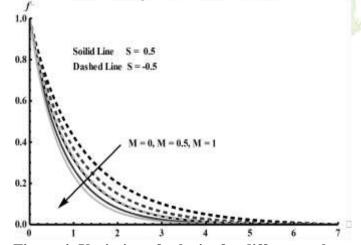


Figure 4: Variation of velocity for different values of M

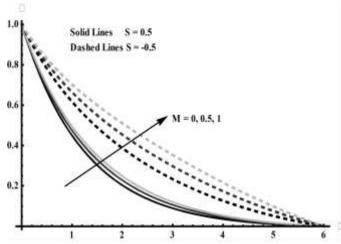


Figure 5: Variation of temperature profile for different values of M

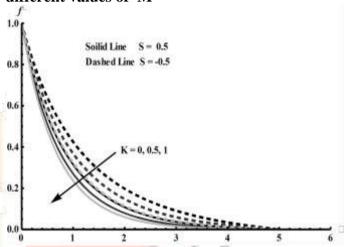


Figure 6: Variation of velocity for different values of K

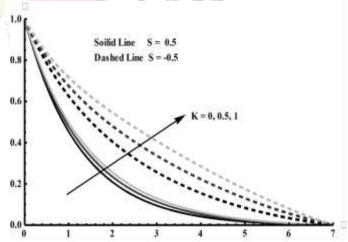


Figure 7: Variation of temperature profile for different values of K

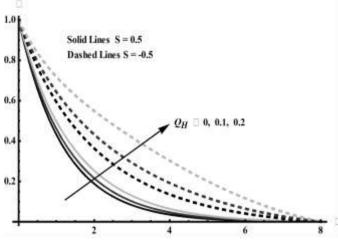


Figure 8: Variation of temperature profile for different values of  $\,Q_{\rm H}\,$ 

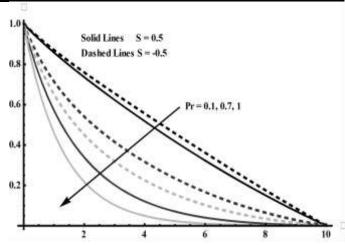


Figure 11: Variation of temperature profile for different values of Ec

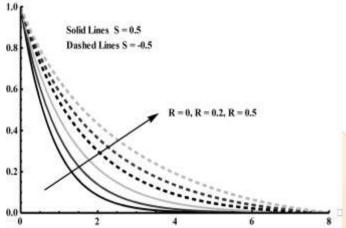


Figure 9: Variation of temperature profile for different values of R

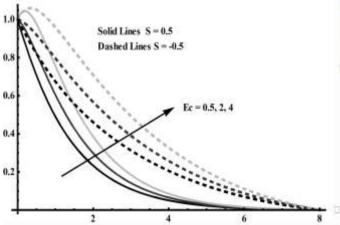


Figure 10: Variation of temperature profile for different values of Ec



## CONCLUSIONS

The present paper gives the numerical arrangement of Warmth and mass exchange issue of Radiative, dissipative MHD stream in a Casson liquid over an exponentially extending sheet implanted in a permeable medium with warmth age, Joule dissemination and suction/blowing. By utilizing comparable change, the overseeing conditions are decreased in to customary differential conditions which are explained by Runge Kutta fourth Request Shooting strategy. The principle discoveries can be abridged as pursues

□ layer t	Increase in Casson parameter causes decrease in energy limit layer thickness however the warm limit thickness increments for this situation.
	Magnetic parameter decreases the rate of transport.
	Prandtl number decreases the warm limit layer thickness.
	Due to substance response the centralization of the liquid abatements.
□ grindi	Effect of suction parameter on a liquid is to smother the speed field which thus causes the upgrade of skin ng coefficient.
	Skin grinding coefficient is higher for suction than for blowing.

### References

- [1]. Abel S., Prasad K. V., Mahaboob A. (2005), Buoyancy force and thermal radiation effects in MHD boundary layer visco-elastic fluid flow over continuously moving stretching surface, International Journal of Thermal Sciences 44, 465–476
- [2]. Abo-Eldahab E.M., El-Aziz M.A. (2004), Blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined stretching surface with internal heat generation/ absorption, Int J Thermal Sciences 43, 709-719.
- [3]. Afify A.A. (2009), Similarity solution in MHD: Effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection, Commun Nonlinear Sci Numer Simulat, Vol. 14, 2202-2214.
- [4]. Andersson H. I.(1992), MHD flow of a viscoelastic fluid past a stretching surface, Acta Mechanica 95, 227–230
- [5]. Bhattacharyya K.( 2012), Steady boundary layer flow and reactive mass transfer past an exponentially stretching surface in an exponentially moving free stream, Journal of the Egyptian Mathematical Society 20, 223–228
- [6]. Bidin B., Nazar R. (2009), Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation, Euro J. Sci. Res. 33, 710-717
- [7]. Brewster M. Q, Thermal radiative transfer and properties, John Wiley & Sons, Inc., Newyork, 1992.
- [8]. Chamkha A.J., Aly A.M., Mansour M.A.(2010), Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/ injection and chemical reaction effects, Chem Engng Comm 197, 846-858.
- [9]. Char M.I. (1994), Heat and mass transfer in a hydromagnetic flow of viscoelastic fluid over a stretching sheet, J. of Math. Anal. Appl 186, 674-689.
- [10]. Chen C.K., Char M. (1988), Heat transfer on a continuous stretching surfaces with suction or blowing, J Math Anal Appl. 135, 568-580.
- [11]. Dash R. K, Jayaraman G., Mehta K. N. (2000), Shear-augmented dispersion of a solute in a Casson fluid flowing in a conduit, Annals of Biomedical Engineering 28, 373-385..

- [12]. Dulal Pal (2009), Heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation, Meccanica 44, 145–158.
- [13]. El-Amin M.F. (2003), Combined effect of magnetic field and viscous dissipation on a power-law fluid over plate with variable surface heat flux embedded in a porous medium, Journal of Magnetism and Magnetic Materials 261,228-237.
- [14]. El-Aziz M.A. (2009), Viscous dissipation effect on mixed convection flow of a micropolar fluid over an exponentially stretching sheet, Canadian Journal of Physics. 87, 359-368.
- [15]. Elbashbeshy E.M.A. (2001), Heat transfer over an exponentially stretching continuous surface with suction, Archive of Mechanics 53, 643-651.
- [16]. Eldabe N.T.M., Salwa M.G.E. (1995), Heat transfer of MHD non-Newtonian Casson fluid flow between two rotating cylinders, J. Phys. Soc. Japan 64, 4164- 1995.
- [17]. Gupta P.S., Gupta A.S. (1977), Heat and mass transfer on a stretching sheet with suction or blowing, Canadian Journal of Chemical Engineering 55, 744-746.
- [18]. Hady F.M., Ibrahim F.S. (1997), Forced convection heat transfer on a flat plate embedded in porous media for power-law fluids, J. Transport in Porous Media 28, 125-34.
- [19]. Ishak A. (2011), MHD boundary layer flow due to an exponentially stretching sheet with radiation effect, Sains Malaysiana 40,391–395.
- [20]. Israel-Cookey C., Ogulu A., Omubo-Pepple V.B. (2003), Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction, Int J Heat Mass Transfer 46, 2305-2311.
- [21]. Jumah R.Y., Mujumdar A.S. (2000), Free convection heat and mass transfer of non-Newtonian power-law fluids with yield stress from a vertical flat plate in saturated porous media, International Communications in Heat and Mass Transfer 27, 485-494.
- [22]. Magyari E., Keller B. (1999), Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface, J. Phys. D: Appl. Phys. 32, 577-585.
- [23]. Mukhopadhyaya S., Layek G. C. (2008), Effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface, International Journal of Heat and Mass Transfer, 51, 2167–2178.
- [24]. Mustafa M., Hayat T., Pop I., Aziz A (2011), Unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate, Heat Transfer- Asian Research 40, 563–576.
- [25]. Nagarani P., Sarojamma G, Jayaraman G (2004), Effect of boundary absorption in dispersion in Casson fluid flow in a tube, Ann Biomed Eng., 706-719.
- [26]. Partha M. K., Murthy P. V. S. N., Rajasekhar G. P. (2005), Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface, Heat Mass Transfer 41, 360–366