ESTIMATION OF FUEL COST PARAMETERS FOR THERMAL POWER PLANTS USING LSE-NELDER-MEAD LOCAL SEARCH (LSE-NM) OPTIMIZATION

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Abstract: This paper presents an efficient approach to estimate the thermal power plant fuel cost parameter which are usually modeled as a polynomial of desired order. The accuracy of parameters of fuel cost is in turn influences the economic load dispatch (ELD) where cost function of fuel is to be minimized. The fuel cost polynomial parameters in this paper are obtained first by popularly used and industry adopted Least square estimation method. Upon observing the local optimality of error by LSE this paper proposes a simple geometry based, non-stochastic and derivative free optimization, known as Nelder-Mead (NM) to further reduce the absolute error by LSE. This approach LSE-NM is tested on different fuel costs such as coal, oil and gas for obtaining parameters of polynomial which results in global minimum error between practical data points and mathematical model. The global optimality for situation considered is confirmed upon comparison with global optimizers such as Particle swarm optimization(PSO) and Ant Bee colony(ABC). The prime requirements of reliability of estimated parameters and time of computation along with global optimality is possible by LSE-NM for ELD parameter estimation.

Index Terms: **Nelder-Mead, parameter estimation.**

I. INTRODUCTION

The economic load dispatch or optimal power flow or compromised fuel cost and environmental dispatch of fossil fired power plant is an optimization problem. In these problems the cost functions of fossil power plants can be mathematically modeled as linear, quadratic or cubic or non-convex fuel cost curves. The objective function accurate value says that fuel cost depends on parameters of the cost function. Due to the ageing of electric power units, ambient operating temperatures of the machines, ever fluctuating fuel cost, labour cost and transportation cost, necessitates the parameter estimation during electrical power generation from fossil fuels. so, the frequent updation of parameter of cost curve that reflect fuel cost curve becomes necessary and vital. Hence researchers from industry/academics try to explore ways to reduce error between estimated and actual values [1].

In conventional approach the parameter estimation problem is optimization technique. In optimization technique an error function is to be minimized [1]. The error is difference between the actual and estimated fuel cost. One of the widely used traditional optimization technique to this type of parameter estimation is least square estimation (LSE) [2].For an nth order polynomial LSE calculates [3],first order partial derivatives of squared error w.r.t parameters to be estimated which results in (n+1) linear algebraic equations .The (n+1) linear algebraic equation can be solved by standard gauss elimination method. This method of ELD parameter estimation was shown to end up with large error [4]. To overcome this researches focused on applying Genetic Algorithms, Particle swarm optimization PSO ,Ant Bee colony (ABC) non-derivative ,nature inspired algorithms to ELD parameter estimation problem .In recent works on this topic of parameter estimation ,it was shown [5,6] that ABC could reduce the error to a least value compared with GA,PSO and LSE. It is worth to observe that the authors who proposed Evolutionary optimization methods to polynomial curve fitting of ELD with smooth curves of quadratic and cubic polynomials did not mention number of function evolution per iteration and hence population size(which is key to the success of any EA[7]) and consequently time of computation, reliability and search range were not reported. Based only final minimum error one can't conclude the applicability of EA to smooth parameter estimation problem.

Motivated by above facts ,authors of this paper aim to bring global minimum using LSE followed by Nelder-Mead NM[8].The NM approach forms a simplex of (n+1) dimensions and evaluates the objective function and using the concept of reflection ,expansion ,and shrinking of points of simplex .The improved simplex is obtained on geometric basis rather than stochastic. Hence it was contemplated that such non-stochastic local search derivative free optimizer should definitely reduce the error and objective of reaching global optimality can be achieved. This approach referred in this paper as LSE-NM is found suitable for parameter estimation problem which is not that much non-convex. EA's are advantageous for highly non-smooth ,non-convex and mixed integer problems of power systems[9].

II. MATHEMATICAL MODEL OF FUEL COST:

In practical power system the heat rate curve or fuel cost function can be mathematically modeled as suitable polynomial by estimating the parameters of mathematical model that exactly suits the desired polynomial fit. The general mathematical model of polynomial is

$$F = a_o + \sum_{k=1}^{j} a_k P^k \qquad (1)$$

In equation a_o is minimum cost operation .The coefficients a_k reflects the combined steam boiler process. P is the power output of the unit.'j' is the order of the polynomial or desired mathematical model. During the course of operation the re estimation of coefficients are required due to fluctuations in various costs and ageing of the power unit. The above formulation of fuel cost is for power units without load point effects. The least square estimation of above polynomial is explained below.

III. LEAST SQUARE ESTIMATION(LSE):

Assuming the availability of practical data of fuel cost $F_i$power units as (P_i, F_i) at various ith power outputs the squared error can be calculated using equation (2)

$$error = [F_i - F]^2 \tag{2}$$

In equation 2 , F is shown in equation.

The error minimization is done by finding partial derivatives of $error$ with respect to each a_k coefficient to be determined. Thus for j th coefficient the general formula is [2]

$$\frac{\partial error}{\partial a_j} = -2\sum_{i=1}^{n}[F_i - F]P_i^j = 0 \tag{3}$$

The above partial derivatives cab put in standard algebraic form as follows.

$$AX = B \tag{4}$$

In above equation

$$A = \begin{bmatrix} n & \cdots & \sum P_i^j \\ \vdots & \ddots & \vdots \\ \sum P_i^j & \cdots & \sum P_i^{J+J} \end{bmatrix}, \ B = \begin{bmatrix} \sum F_i \\ \sum(P_i F_i) \\ \vdots \\ \vdots \\ \sum(P_i^j F_i) \end{bmatrix}, \quad \mathrm{X} = \begin{bmatrix} a_o \\ a_1 \\ \vdots \\ \vdots \\ a_j \end{bmatrix} \tag{5}$$

Using Gauss elimination method can be used to solve the above equation.The LSE method can be invoked in MATLAB[10] environment using polyfit(),command.This general command is as follows. The usage form of polyfit command is as follows.

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x=polyfit(P,F,j);%
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in the above command P is a row vector of real power outputs.F is a row vector of actual cost while j is power of polynomial to be fit for P and F.The function returns x which are the coefficients of the polynomial.

IV.NELDER-MEAD (NM) OPTIMIZATION METHOD:

The NM is a geometry based non-derivative, simplex local search function extrapolation approach developed by John Nelder et.al [1].The prime requirement of this method is to have an initial simplex with $(N+1)$ vertices.The N is number of search directions. The method [8] in brief sorts the vertices objective function values from best to worst. Then reflects the worst point towards centroid of simplex excluding the worst point. The reflected point x_r may be better than x_n or better than current best value x_1 or worse than second best point (x_n).In case of option one x_r, replaces x_{n+1}.The option two i.e x_r better than x_1 the NM explores along the line centroid and the move is known as expansion and computes the point x_e,The function value at x_e decides replacement of x_{n+1} withx_e or x_r . And the third option of NM is contraction move, in this move contraction point x_c replaces the worst value upon comparison of worst vertex with x_c .Every time when worst vertex is replaced.

The new simplex is reformed and procedure is repeated till termination conditions of optimization which may include shrink of simplex to down- hill search.

Assume (N+1) simplex points with function values arranged from best to worst.

Itr=0 (initialize the iteration counter)

step-A : $f(x_1) \leq f(x_2) \leq \cdots \ldots \ldots \leq f(x_{N+1})$

Till stopping criteria of the ALGORITHN repeat the following steps.

Compute the centroid of $x_o = \frac{1}{N}\sum_{i=1}^{N} x_i$

Find reflection point x_r using $x_r = x_o + \alpha(x_o - (x_{N+1})$.

Compute objective value at x_r i.e $f(x_r)$

If $f(x_r) < f(x_n) \& f(x_r) > f(x_1)$ replace x_{N+1} with x_r go to step A.

If $f(x_r) < f(x_1)$, obtain an expansion point x_e using $x_e = x_o + \gamma(x_r - x_o)$

If $f(x_e) < f(x_r)$, replace x_{N+1} with x_e go to step A.else replace with x_r go to step A.

If $(x_r) \geq f(x_n)$, calculate contraction point $x_c, x_c = x_o + \rho(x_{N+1} - x_o)$

Replace x_c with x_{N+1} with x_c go to step 1 if $(x_c) < f(x_{N+1})$.

Shrink the simplex i.e replace all points except the best x_1 , using

$x_i = x_1 + \sigma(x_i - x_1)$ go to step A

Itr=Itr+1.

go to step A.

Typical values for . γ, ρ, σ and α are respectively 2,(1/2),(1/2) and 1.

The stopping criteria of the NM method may be number of iterations or number of function evaluations or distance norm of initial best point and current best point. Step A is called as initialization of the simplex. During Step A, function values are arranged from best to worst (descending) order. This algorithm can be simply implemented in MATLAB[10] by invoking a function known as " fminsearch ()".Typically fminsearch () accepts address of the function to be minimized and initial search value and returns possibly an improved value and objective function value at current iterate. For detailed usage of this function is available [10].

The general command in MATLAB environment is [x,fvals]=fminsearch(@objfunction,x);

Where fminsearch is NM solver in MATLAB environment and @objfunction is another matlab script file where objective function to be minimized is written.

The function returns x which are further improved quantities of x computed by polyfit() command.

The objective function for fminsearch is as follows

$$err = |F_i - F| \tag{5}$$

In the above equation F_i are data points and $= x_o + \sum_{k=1}^{j} x_k P_i^j$. As stated earlier x are coefficients of polynomial to be estimated. As both solvers LSE and NM are used for parameter estimation the method is named as LSE-NM.

V.IMPLEMENTATION USING IN MATLAB:

Read the data such as j, F_i and P_i

Call polyfit command as explained above. This returns x the coefficients for polynomial order j.

Use fminsearch() command using x returned by polyfit.

Output the x and absolute error.

5.1 Test results and discussion:

In this paper the data of fuel cost at real power output available for coal, oil and gas [6] are taken for comparison with PSO and Ant Bee colony (ABC) algorithms.

Test case-1 pertains to j=2, i.e quadratic cost modeling. The comparative results are shown in Table 1.

Table 1: comparative table of error estimated for quadratic parameter estimation.

Unit	**P(MW)**	**F_{actual}(GJ/h) F_i**	**$F_{estimated}$ (GJ/h)**				**Error($F_{estimated}$-F_{actual})**			
			Optimization Methods							
			LSE-NM	**ABC(6)**	**PSO(5)**	**LSE(5)**	**Unit**	**LSE-NM**	**ABC**	**PSO**	**LSE**
1	10	176.62	176.62	176.619	176.358	174.252	**1**	**0**	-0.001	0.262	2.368
coal	20	256.4	264.92	264.913	264.765	261.968	**coal**	**8.52**	8.513	-8.365	-5.568
	30	361.5	361.5	361.487	361.5	359.004		**0**	-0.013	0	2.496
	40	467.6	466.36	466.341	466.562	465.36		**-1.24**	-1.259	1.038	2.24
	50	579.5	579.5	579.475	579.952	581.036		**0**	-0.025	-0.452	-1.536
							|∑Error|	**9.76**	9.81	10.117	14.208
2	10	184.75	184.75	184.735	183.6	182.346	**2**	**0**	-0.015	-1.15	-2.404

oil	20	268.2	276.806	276.774	275.4	273.862	**oil**	8.6062	8.574	7.2	5.662
	30	377.7	377.7	377.653	376.4	375.258		**0**	-0.047	-1.3	-2.442
	40	488.8	487.431	487.372	486.6	486.534		-1.3688	-1.428	-2.2	-2.266
	50	606	606	605.931	606	607.69		**0**	-0.069	0	1.69
							|∑Error|	9.975	10.133	11.85	14.464
3	10	187.2	187.2	187.799	185.78	184.824	**3**	**0**	0.599	-1.42	-2.376
gas	20	272.8	281.363	281.36	279.121	278.368	**gas**	8.5625	8.56	6.321	5.568
	30	384.3	384.3	384.301	381.732	381.732		**0**	0.001	-2.568	-2.568
	40	497.2	496.013	496.022	494.484	494.916		-1.1875	-1.178	-2.716	-2.284
	50	616.5	616.5	616.523	616.507	617.92		**0**	0.023	0.007	1.42
							|∑Error|	9.75	10.361	12.742	14.216

The following are the observation on quadratic estimation

1) Absolute error by LSE-NM is less than ABC and PSO for all power plants.

2) The error at 10,30 and 50 MW is zero by the LSE-NM while the error by other approaches are not exactly zeros.

3) The percentage reduction in total error compared with LSE is shown in table 2 below.

Table 2:% reduction in total error for quadratic polynomial parameter estimation

unit	LSE-NM	ABC	PSO
coal	31.3063	30.9544	28.7936
oil	31.0357	29.9433	18.0725
gas	31.4153	27.1173	10.3686

It is very clear from the above table that NM has reduced the error compared to ABC and PSO with reference to LSE.

4)The total iteration by NM is only 132.

5)The time of parameter estimation is 0.312s.

6)The polynomial coefficients by proposed approach is shown in table 3

Table 3 :Parameters for quadratic polynomial fitting

unit	ao	a1	a2
Coal	96.6000	7.5880	0.0414
oil	101.5312	7.8800	0.0442
gas	101.8125	8.1000	0.0439

Test case 2: In this case for power outputs and cost given in table1 is used to estimate parameters with j=3 i.e cubic polynomial cost function. The error of individual units and total error are shown in table 4 below.

Table 4 :A comparative table for cubic polynomial fuel cost of thermal plants

Unit	**P(MW)**	**F$_{actual}$(GJ/h) F$_i$**	**F$_{estimated}$ (GJ/h)**				**Error(F$_{estimated}$-F$_{actual}$)**				
			Optimization Methods								
			LSE-NM	**ABC(6)**	**PSO(5)**	**LSE(5)**	**Unit**	**LSE-NM**	**ABC**	**PSO**	**LSE**
1	10	176.62	176.62	176.615	176.806	176.227		**0**	-0.0048	0.186	-0.393
coal	20	256.4	256.413	257.134	260.557	258.274	**coal**	**0.0132**	0.7342	4.157	1.874
	30	361.5	356.656	357.093	361.951	359.721		**-4.8445**	-4.4068	0.451	-1.779
	40	467.6	467.6	467.492	471.446	470.968		**0**	-0.1078	3.846	3.368

	50	579.5	579.5	579.331	579.5	582.415		**0**	-0.1688	0	2.915
							|∑Error|	**4.8578**	5.4224	8.641	10.329
2	10	184.75	184.75	184.739	184.076	184.301	**2**	0	-0.0109	-0.674	-0.449
oil	20	268.2	268.706	269.163	268.2	269.562	**oil**	0.5059	0.9631	0	1.362
	30	377.7	373.212	373.507	373.01	374.223		-4.4877	-4.1929	-4.69	-3.477
	40	488.8	488.8	488.771	488.863	488.863		0	-0.0289	0.063	0.063
	50	606	606	605.955	606.119	600.945		0	-0.0449	0.119	-5.055
							|∑Error|	4.9936	5.2407	5.547	11.059
3	10	187.2	187.2	187.188	187.101	186.804	**3**	0	0.0117	-0.099	-0.396
gas	20	272.8	272.8	274.632	274.326	274.688	**gas**	0	-1.8323	1.526	1.888
	30	384.3	379.385	380.561	381	382.452		-4.9154	3.7387	-3.3	-1.848
	40	497.2	497.202	497.17	498.074	500.496		0.002	0.0297	0.874	3.296
	50	616.5	616.5	616.659	616.5	619.22		0	-0.1593	0	2.72
							|∑Error|	4.9174	5.7767	5.799	10.148

The following are the observations from the above table 4 :

1)This case also the total error and individual errors by proposed LSENM is less compared to other approaches.

2) At certain power outputs error is exactly driven to zero by NM while other approaches could not achieve this performance.

3) The percentage reduction in total error compared with LSE is shown in table below.

Table 5 :%reduction in total error for cubic polynomial parameter estimation.

unit	**LSE-NM**	**ABC**	**PSO**
COAL	52.9693	47.5031	16.3423
oil	54.8458	52.6114	49.8418
Gas	51.5432	43.0755	42.8557

A significant percentage reduction is definite by LSE-NM.

4) In this case total iteration after which no further reduction in error is 154.

5) The time of parameter estimation is 0.45s

6) The polynomial coefficients by proposed approach is shown in table 6

Table 6: parameters for cubic polynomial fitting

unit	**ao**	**a1**	**a2**	**a3**
Coal	7.0226	3.1251	0.1997	-0.0016
oil	0.8137	3.5770	0.1974	-0.0016
Gas	2.3367	3.6244	0.2024	-0.0016

Most interesting feature of NM is it is non-stochastic and excellent local search solver and hence ends up consistently to the same final value of reduced error even when algorithm is repeatedly run with an initial estimate from another non stochastic gradient based approach LSE. More over time of computation is more important while estimating parameters on line. The other approaches with which comparison is made in this paper definitely can’t arrive at same final parameters as the mechanism involved in updating the function value depends on current state of pseudo random number generation .The intelligent approaches are known for global optimality and may be useful for planning purpose but definitely not for on line parameter estimation where time and reliability matters more.

VI.CONCLUSION

This paper has successfully proved the global optimality by using traditional approach LSE followed by local search Nelder-Mead (NM) to reliably estimate parameters of continuous polynomial cost function coefficients for Economic Load dispatch problem. The approach of this paper needs to be tested by considering other practical constraints such as valve-point loading effects and transmission losses.

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