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# KALUZA KLEIN UNIVERSE WITH POLYTROPIC EQUATION OF STATE

**[1] R. R. Saraogi, [2] Bhushan Birari, [3] Sant Amarjit**

[1,2,3], Assistant Professor, Department of Applied Science

ISB&M School of Technology, Pune, INDIA.

***Abstract:*** We have studied the five dimensional Kaluza Klein universe filled with perfect fluid governed by the polytropic equation of state. Exact solutions of the Einstein's field equations are obtained and physical properties of the model are discussed in details.

## I. Introduction:

Recently researches are going on for finding out the origin of accelerated expansion of the present universe. Observational evidences point towards an accelerated expansion of the universe. The astrophysical observations of the SNe Ia (Permutter et al. 1999), Cosmic Microwave Radiation (Bennett *et al.* 2003; Spergel *et al.* 2003), X-ray (Allen *et al.* 2004), are the main evidences for this cosmic acceleration. For this accelerating expansion of the universe a new energy with negative pressure is driven which is commonly known as dark energy (DE) (Peebles and Ratra 2003). Dark energy is a major component in energy field of the universe (Ade *et al.* 2013). The dark energy which is responsible for accelerated expansion of the universe has been captured a vast range of research in astrophysics. But till now the nature of dark energy is a challenging problem in theoretical physics. The earliest and simplest candidate of dark energy is the cosmological constant with time-independent equation of state $\omega=-1$.

Kumar and Yadav (2011) studied power-law and exponential-law cosmologies within the framework of Bianchi-V models with non-interacting matter fluid and DE components. Also Kumar (2013) investigated Bianchi-V space time by considering hybrid expansion law (HEL) for the average scale factor that yields power-law and exponential-law cosmologies. This motivates us in this paper to study the LRS Bianchi type II space time using hybrid expansion law (HEL) filled with dark matter and anisotropic modified holographic Ricci dark energy. Mishra and Tripathy (2015) have constructed an anisotropic dark energy cosmological model in the framework of General Relativity at the backdrop of spatially homogeneous and anisotropic Bianchi V metric by considering hybrid expansion law which establishes a cosmic transition from early deceleration to late time acceleration.

The theory of polytropes have a significant role in studying the inner structure of astrophysical compact objects (CO) very precisely. In Newtonian gravity, various useful physical phenomenon has been addressed with polytropic equations of state (EoS). Chandrasekhar (1939) provided the basic theory of Newtonian polytropes emerged through laws of thermodynamics for polytropic sphere. Kovetz (1968) redefined some anomalies in the theory of slowly rotating polytropes presented by Chandrasekhar (1939). Abramowicz (1983) extended the general form of Lane-Emden equation (LEe) for spherical, planar and cylindrical polytropes for higher dimensional spaces. In general relativity (GR), polytropes have been discussed by many researchers by means of LEe which can be derived from the hydrostatic equilibrium configuration of CO. Tooper (1964) provided the basic formulism of polytropes for compressible fluid under the assumption of quasi-static equilibrium. He extended his work for adiabatic fluid sphere and provides the fundamental framework to derive LEe for relativistic polytropes (Tooper 1965). Herrera and Barreto (2004) investigated relativistic polytropes under the assumption of post-quasi-static regime and presented a new way to describe various physical variables like pressure, mass and energy density by means of effective variables. They considered two possible configuration of polytropes in the frame of GR and found that only one was physically viable. Anisotropy plays a very vital role in the theory of GR to discuss spherical CO. Cosenza *et al.* (1981) presented a heuristic procedure to obtined anisotropic models in GR. Herrera and Barreto (2013a, 2013b) used the Tolman mass (measure of active gravitational mass) to find out stability of both Newtonian and relativistic polytropes with anisotropic inner matter configuration. Herrera *et al.* (2014) analyzed in detail anisotropic polytropes with conformally flat condition which was useful in reducing the parameters involve in relativistic modified LEe.

Higher dimensional cosmology is important because it has physical relevance to the early stages of evolution of the universe before it has undergone compactification transitions. Hence several authors (Witten 1984; Chodos and Detweller 1980; Appelquist, et al. 1987; Marchiano,1986) were attracted to the study of higher dimensional cosmology. Also, in the context of Kaluza – Klein and super string theories higher dimensions have recently acquired much significance. Several investigations have been made in higher dimensional cosmology in the frame work of different scalar – tensor theories. In particular, Reddy *et al.* (2012) have discussed a five dimensional Kaluza – Klein cosmological model in the presence of perfect fluid in f(R,T) gravity.

In the following paper, we have investigated five dimensional Kaluza-Klein Universe with polytropic EoS and hybrid expansion law and deduced various physical and geometrical properties describing the models.

## II. Metric and Field Equations:

We consider the five dimensional Kaluza-Klein line element as

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2 + dz^2\right) - B^2 d\psi^2 \quad , \tag{1}$$

where $A$ and $B$ are functions of cosmic time $t$ and the fifth coordinate $\psi$ is taken to be space-like.

The Einstein field equations in natural limits ($8\pi G = 1,\ c = 1$) are

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \quad , \tag{2}$$

where $R_{ij}$ is the Ricci tensor, $R$ is the Ricci scalar and $T_{ij}$ is the energy momentum tensor.

The energy momentum tensor $T_{ij}$ for the perfect fluid is given by

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij} \quad , \tag{3}$$

where $\rho$ is the energy density, $p$ is the pressure and $u^i$ is the four velocity vector satisfying $g_{ij}u^i u^j = 1$. We have considered the universe to be filled with perfect fluid governed by the polytropic equation of state. The perfect fluid has a general form of equation of state (EoS) $p = p(\rho)$ which in this case we consider as

$$p = K\rho^n - \rho \quad , \tag{4}$$

where $K$ and $n$ are constants known as polytropic constant and polytropic index respectively. In a co-moving co-ordinate system, the Einstein field equations (2) for the metric (1) with the help of equations (3) reduce to the following set of equations:

$$3\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\dot{A}\dot{B}}{AB} = \rho \quad , \tag{5}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = -p \quad , \tag{6}$$

$$3\frac{\ddot{A}}{A} + 3\left(\frac{\dot{A}}{A}\right)^2 = -p \quad , \tag{7}$$

where overhead dot denotes differentiation with respect to time $t$.
The directional Hubble parameters in the directions of $x, y, z$ and $\psi$ respectively are

$$H_x = H_y = H_z = \frac{\dot{A}}{A}, \quad H_\psi = \frac{\dot{B}}{B} \quad . \tag{8}$$

The mean Hubble parameter $H$ and expansion scalar $\theta$ are given by

$$H = \frac{1}{4}\theta = \frac{1}{4}\frac{\dot{V}}{V} = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \quad , \tag{9}$$

where $V = A^3 B$ is the spatial volume of the universe.
The anisotropy parameter and shear scalar are defined as

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \quad , \tag{10}$$

$$\sigma^2 = 2\Delta H^2 \quad . \tag{11}$$

Also, the conservation equation $T^{ij}_{;j} = 0$ takes the simpler form as

$$\dot{\rho} + \frac{\dot{V}}{V}(p+\rho) = 0 \quad ,$$

which on integration and using equation (4) gives

$$\rho = \gamma(\log V)^{1/(1-n)} \quad , \tag{12}$$

where $\gamma = [K(n-1)]^{1/(1-n)}$ .

**III. Solutions of the field equations:**

Solving field equations (5)-(7), we obtain

$$A = D_1 V^{1/4} \exp\left(X_1 \int \frac{dt}{V}\right) \quad , \tag{13}$$

$$B = D_2 V^{1/4} \exp\left(X_2 \int \frac{dt}{V}\right) \quad , \tag{14}$$

where $D_1, D_2, X_1, X_2$ are constants of integration satisfying the relations $D_1^3 D_2 = 1$, $3X_1 + X_2 = 1$. Also, using equations (5)-(7), we have

$$\frac{\ddot{V}}{V} = \frac{4}{3}(\rho - p) \ . \tag{15}$$

Integrating equation (15), we get

$$\dot{V} = \sqrt{3\rho V^2 + c_1} \quad ,$$

where $c_1$ is the constant of integration which can be ignored.

Integrating above equation with the help of equation (12), we obtain

$$V = \exp\left[(\alpha t)^{2(1-n)/(1-2n)}\right] \quad , \tag{16}$$

where $\alpha = \dfrac{\sqrt{3\gamma}(1-2n)}{2(1-n)}$ .

Using equation (16), equations (13) and (14) become

$$A = D_1 \exp\left[\frac{1}{4}(\alpha t)^{2(1-n)/(1-2n)} + X_1 \frac{(1-2n)t}{2(1-n)} E_{\frac{1}{2(1-n)}}(\alpha t)^{2n/(1-2n)}\right] , \tag{17}$$

$$B = D_2 \exp\left[\frac{1}{4}(\alpha t)^{2(1-n)/(1-2n)} + X_2 \frac{(1-2n)t}{2(1-n)} E_{\frac{1}{2(1-n)}}(\alpha t)^{2n/(1-2n)}\right] , \tag{18}$$

where $E_n(z)$ is the exponential integral function.

Thus, the directional Hubble parameters are given by

$$\left.\begin{aligned} H_x = H_y = H_z = \beta(\alpha t)^{(1+2n)/(1-2n)} + X_1 \exp\left[-(\alpha t)^{2(1-n)/(1-2n)}\right] , \\ H_\psi = \beta(\alpha t)^{(1+2n)/(1-2n)} + X_2 \exp\left[-(\alpha t)^{2(1-n)/(1-2n)}\right] , \end{aligned}\right\} \tag{19}$$

where $\beta = \dfrac{\alpha(1-n)}{2(1-2n)}$.

This gives the mean Hubble parameter and expansion scalar as

$$H = \frac{1}{4}\theta = \beta(\alpha t)^{(1+2n)/(1-2n)} \quad . \tag{20}$$

The anisotropy parameter and the shear scalar take the form as

$$\Delta = \frac{X^2}{4\beta^2}(\alpha t)^{-2(1+2n)/(1-2n)} \exp\left[-2(\alpha t)^{2(1-n)/(1-2n)}\right] \quad , \tag{21}$$

$$\sigma^2 = \frac{X^2}{2}\exp\left[-2(\alpha t)^{2(1-n)/(1-2n)}\right] \quad , \tag{22}$$

where $X^2 = 3X_1^2 + X_2^2$.

Using equation (16) in equation (12) and (4), the density and pressure of the given model becomes

$$\rho = \gamma(\alpha t)^{2/(1-2n)} \quad , \tag{23}$$

$$p = \gamma(\alpha t)^{2/(1-2n)}\left[K\gamma^{(n-1)}(\alpha t)^{2(n-1)/(1-2n)} - 1\right] \quad . \tag{24}$$

The EoS parameter $\omega$ is given by

$$\omega = \frac{p}{\rho} = K\gamma^{(n-1)}(\alpha t)^{2(n-1)/(1-2n)} - 1 \quad . \tag{25}$$

The energy density parameter $\Omega$ of the model is given by

$$\Omega = \frac{\rho}{4H^2} = \frac{\gamma}{4\beta^2}(\alpha t)^{4n/(2n-1)} \quad . \tag{26}$$

The deceleration parameter can be written as

$$q = -\frac{\alpha(1+2n)}{\beta(1-2n)(\alpha t)^{2/(1-2n)}} - 1 \quad . \tag{27}$$

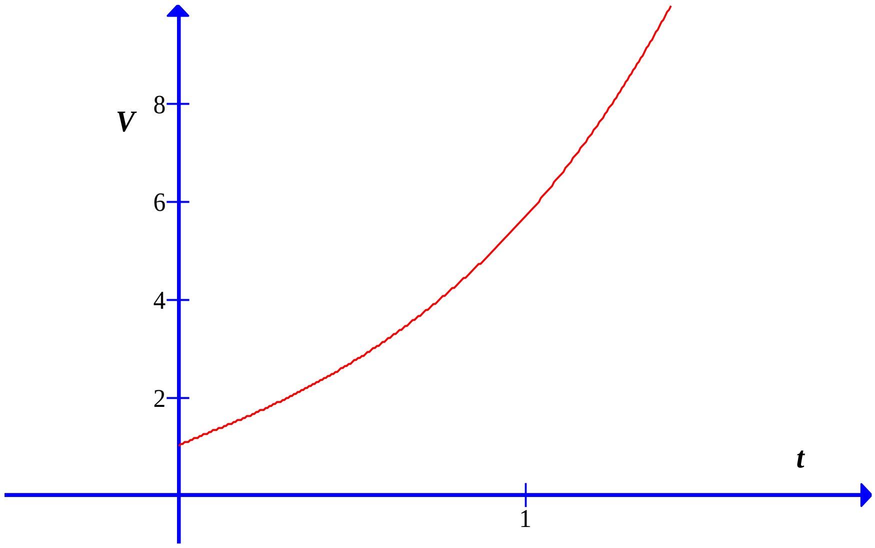


Fig. 1: Volume vs. Time for $\alpha = 2$, $n = 3$.

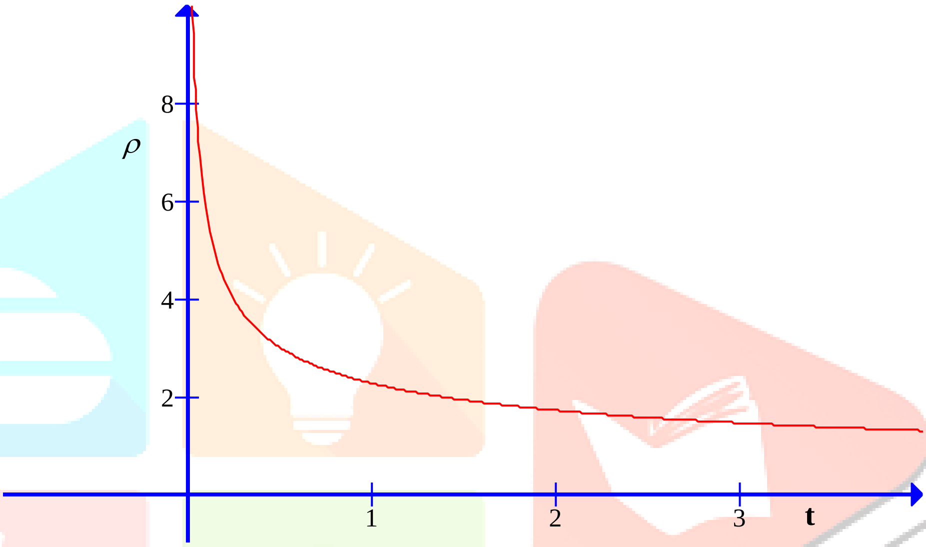


Fig. 2: Energy Density vs. Time for $\gamma = 3$, $\alpha = 2$, $n = 3$.

## IV. Discussion:

It can be verified from equation (16) and Fig. 1 that the spatial volume of the universe which was finite at the beginning of time increases exponentially with time and becomes infinite after some finite time. Fig. 2 shows the variation of the energy density of the universe with time. It is noteworthy that the energy density had an infinite value at $t = 0$ but it decreases as time increases.

## V. Result:

Kaluza Klein cosmological model with a polytropic equation of state has been investigated. The physical and cosmological properties of the model have been obtained. Exponential expansion of volume with time and decrease of the energy density of the universe with time can be observed. Obtained value of the deceleration parameter show an agreement with the present day situation, i.e. from the value of the deceleration parameter, the early inflation era and the present accelerated expansion phase can be satisfactorily explained. So our results are in compliance with the results obtained by Riess *et al.* (1998) and Perlmutter *et al.* (1999).

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