IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Five-Dimensional Plane Symmetric String Cosmological Model with Bulk Viscosity in General Relativity

V. G. Mete[1], V.S. Deshmukh[2]

[1]Department of Mathematics, R.D.I.K. & K.D. College, Badnera– Amravati (M.S.), India
[2]Department of Mathematics, P.R.M.I.T.&R., Badnera– Amravati (M.S.), India

Abstract: In this paper, we considered five-dimensional plane symmetric Bianchi type-I cosmological model generated by a cloud of strings with particles attached to them with bulk viscosity in general relativity. To obtain solutions of field equations, we considered that shear scalar of the model is proportional to expansion scalar which leads to the anisotropic relation between the metric potentials. Also the physical and geometrical properties of the model are discussed in detail.

Keywords: Five dimensional plane symmetric Bianchi -I space-time; bulk viscosity; cloud string; general relativity.

1. Introduction:

In cosmology, the rate at which the phase transition proceeds is given by the expansion rate of the universe which is very fast in the early universe. Hence, topological defects will inevitably be produced in a cosmological phase transition [1]. String theory is one of the most important theory in cosmology that study about the unknown facts of the universe. It was developed to describe events at the early stages of the evolution of the universe. In the recent past years, in the framework of string theory several models in cosmology has been proposed by different authors in order to explain the hidden reasons of expansion of the existing universe with the acceleration. Cosmic strings are topologically stable defects, which are probably formed at some stage of the phase transition or earlier the introduction of particles in the early universe.

Furthermore, at extremely early times before the universe underwent the compactification transitions, solutions of Einstein field equations in higher dimensional space times are believed to be of physical relevance. As a consequence, now the higher dimensional theory is receiving great attention in both cosmology and particle physics. Particle physicists and cosmologists predicted the existence of GUT (Grand Unified Theory). Using a appropriate scalar field it was shown that the phase transitions on the early universe can give rise to such objects which are nothing but the topological knots in the vacuum expectation value of the scalar field and most of their energy is concentrated in a small region. As the necessity to study higher dimensional space-time in this field aiming to unify gravity with other interactions the concept of extra dimension is pertinent in cosmology [2].

Several researchers have studied Bianchi models with bulk viscosity in various frame work. Misner [3] explored the consequences of bulk viscosity in the cosmological evolution of the universe. Wang [4] have investigated Bianchi type-III string cosmological model with bulk viscosity in general relativity. Mohanty et.al. [5] have studied five dimensional axially symmetric cosmological model generated by a cloud of strings with particles attached to them in Lyra manifold. Bali and Pradhan [6], Tripathy et al. [7, 8], Rao et al. [9], Kandalkar et al. [10] have investigated different cosmological models in the presence of bulk viscosity and cosmic string. Reddy et al. [11] have studied Kaluza-Klein and Bianchi type-II bulk viscous cosmic string models in f(R,T) gravity. Naidu et.al. [12] have constructed five dimensional Kaluza-Klein space-time in the frame work of Brans-Dicke scalar-tensor theory of gravitation. Mete et.al. [13] have studied higher dimensional plane symmetric cosmological model with two fluid source in general relativity. Mahanta et.al. [14] have investigated plane symmetric bulk viscous string cosmological models with strange quark matter. Katore et.al.[15] have obtained plane symmetric cosmological model within the framework of a scalar tensor theory of Gravitation when the source for energy momentum tensor was a bulk viscous fluid containing one dimensional cosmic strings. Kandalkar et.al. [16] have constructed LRS Bianchi type-I cosmological model with bulk viscous fluid in Lyra geometry with the help of displacement vector depending upon time. Humad et.al. [17] have studied Bianchi type-I string cosmological models with bulk viscosity. Chaturvedi et.al. [18] have investigated cosmological model based on quadratic and higher order equation of state under different circumstances. Biswal [19] have studied higher dimensional Bianchi type-III cosmological models with strange quark matter coupled to the string cloud in general relativity. Singh [20] explored a five dimensional LRS Bianchi type- I metric in presence of perfect fluid in the context of Lyra's geometry by using cubic equation of state. Samanta et. al .[21] have studied cosmological singularities in Kaluza-Klein space-time. Singh et.al. [22] have investigated bulk viscous fluid Bianchi type-I string cosmological model with negative constant deceleration parameter. Baro et.al. [23] have studied five dimensional Bianchi type-III string cosmological model with bulk viscous fluid and negative constant deceleration parameter in general relativity.

Inspired by above discussion, in this paper, we have constructed five dimensional plane symmetric Bianchi type-I string cosmological model with bulk viscosity in general relativity. Physical behaviours of the model are also discussed.

2. Metric and field equation:

Here, plane symmetric Bianchi type -I metric in five dimension is considered as:

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2) - B^2(t)dz^2 - C^2(t)d\psi^2, \quad (1)$$

where A, B, C are functions of cosmic time t only.

The energy momentum tensor for cosmic strings with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j - g_{ij}) \quad . \quad (2)$$

Here ρ as the energy density, λ is string tension density, are related by $\rho = \rho_p + \lambda$, ξ is the bulk coefficient of viscosity, θ is the expansion scalar, u^i the five velocity cloud particles, x^i the unit space like vector represents the direction of string.

$$u^i = (0,0,0,0,1) \text{ and } x^i = (A^{-1},0,0,0,0) \quad (3)$$

Here u^i and x^i satisfy the orthogonal relationship

$$u^i u_i = -x^i x_i = 1 \text{ and } u_i x^i = 0 \quad (4)$$

The Einstein field equation in general relativity is

$$R_{ij} - \frac{1}{2}R g_{ij} = T_{ij} \quad (5)$$

From equations (2) - (5), line element (1) leads the following system of equations:

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = \xi\theta \quad (6)$$

$$2\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{C}}{AC}=\lambda+\xi\theta \tag{7}$$

$$2\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{B}}{AB}=\xi\theta \tag{8}$$

$$\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+2\frac{\dot{A}\dot{C}}{AC}=\rho\ , \tag{9}$$

where overhead dot (**.**) denote derivative with respect to time *t.*

The spatial volume (V) of the metric (1) are defined by

$$V=A^2BC \tag{10}$$

The physical quantity of dynamical interest in cosmology are given below:
The expansion scalar (θ) :

$$\theta=2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{11}$$

The shear scalar (σ^2) :

$$\sigma^2=\frac{1}{2}\left[2\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2\right]-\frac{\theta^2}{8} \tag{12}$$

The Hubble parameter (H) :

$$H=\frac{1}{4}\sum_{i=1}^{4}H_i \tag{13}$$

The deceleration parameter q which is defined as

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \tag{14}$$

3. Cosmological Solutions:

The set of field equations (6) – (9) are the system of four independent equations with six unknowns so in order to obtain deterministic solution of above system, we consider the analytic relation between scale factors.

$$C=A^n\quad , \tag{15}$$

where n is constant.

From equations (6) and (8) , we obtain

$$\frac{\dot{A}}{A}\left[\frac{\ddot{A}}{\dot{A}}+(1+n)\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right]=0 \tag{16}$$

which yields following two cases:

Case I: $\frac{\ddot{A}}{\dot{A}}+(1+n)\frac{\dot{A}}{A}+\frac{\dot{B}}{B}=0$

Case II: $\dot{A}=0$

In the following subsections we intend to determine cosmic string models for the above mentioned cases.

3.1 Case I: $\frac{\ddot{A}}{\dot{A}}+(1+n)\frac{\dot{A}}{A}+\frac{\dot{B}}{B}=0$

In this case we obtain

$$A(t)=\left[(n+2)\left(\int\frac{k}{B}dt+k_1\right)\right]^{\frac{1}{(n+2)}} \quad ,n\neq -2 \tag{17}$$

from which it is clear that given any function $B(t)$,we can find $A(t)$. Therefore the solutions are not unique. However for further studies here we consider

$$\frac{\ddot{A}}{\dot{A}}+(1+n)\frac{\dot{A}}{A}=-\frac{\dot{B}}{B}=k \quad \text{(Constant)} \tag{18}$$

Solving equation (18), we get

$$A(t)=\left[(n+2)\left(k_1\frac{e^{kt}}{k}+k_2\right)\right]^{\frac{1}{(n+2)}}, \quad n\neq -2 \tag{19}$$

and

$$B=k_3e^{-kt} \tag{20}$$

$k_1(\neq 0), k_2,$ and $k_3(\neq 0)$ are constants of integrations.

From equations (15) and (19), we get

$$C=\left[(n+2)\left(\frac{k_1}{k}+k_2\right)\right]^{\frac{n}{n+2}} \tag{21}$$

From equations (7)-(9) using equations (19) to (21), we obtain

The string tension density is

$$\lambda=\frac{kk_1e^{kt}}{\left(\frac{k_1}{k}e^{kt}+k_2\right)}-k^2 \tag{22}$$

From equation (8) we get,

$$\xi\theta=k^2-\frac{(2n+1)k_1^2e^{2kt}}{\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^2} \tag{23}$$

The energy density ρ is

$$\rho=\frac{(2n+1)k_1^2e^{2kt}}{\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^2}-\frac{kk_1e^{kt}}{\left(\frac{k_1}{k}e^{kt}+k_2\right)} \tag{24}$$

The Particle density is obtained as

$$\rho_p=\frac{(2n+1)k_1^2e^{2kt}}{\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^2}-\frac{2kk_1e^{kt}}{\left(\frac{k_1}{k}e^{kt}+k_2\right)}+k^2 \tag{25}$$

The spatial volume (V) is

$$V = k_3 e^{-kt}\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right] \quad , k_3 > 0 \tag{26}$$

The expansion scalar is

$$\theta = \frac{k_1 e^{kt}}{\left(\frac{k_1}{k}e^{kt}+k_2\right)} - k \tag{27}$$

The Hubble parameter (H) is obtained as

$$H = \frac{k_1 e^{kt}}{4\left(\frac{k_1}{k}e^{kt}+k_2\right)} - \frac{k}{4} \tag{28}$$

From equations (23) and (27), we obtain

$$\xi = -\frac{k_1}{k_2}e^{kt} + \frac{k_1^2 e^{2kt}(2n+1)}{kk_2(n+2)^2\left(\frac{k_1}{k}e^{kt}+k_2\right)} - k \tag{29}$$

The deceleration parameter (q) is

$$q = -\left[\frac{4k_1 e^{kt}}{kk_2}+1\right] \tag{30}$$

The shear scalar of the model is

$$\sigma^2 = \frac{1}{8}\left[\frac{(3n^2-4n+4)\,k_1^2 e^{2kt}}{8(n+2)^2\left(\frac{k_1}{k}e^{kt}+k_2\right)} + 3k^2 + \frac{2kk_1 e^{kt}}{\left(\frac{k_1}{k}e^{kt}+k_2\right)}\right] \tag{31}$$

Thus the metric given in equation (1) can be written as

$$ds^2 = dt^2 - \left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^2 (dx^2+dy^2) - k_3^2 e^{-2kt} dz^2$$
$$-\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^{\frac{2n}{n+2}} d\psi^2 \tag{32}$$

3.2 Case II: $\dot{A} = 0$

In this case ,we get,

$$A = p \quad (\text{Constant}) \tag{33}$$

From equation (15) and (33),we get

$$C = p^n \tag{34}$$

Now field equations (6) – (9) together with equation (33) and (34), reduces to

$$\frac{\ddot{B}}{B} = \xi\theta \tag{35}$$

$$\rho = 0 \tag{36}$$

$$\lambda + \xi\theta = 0 \tag{37}$$

Here there are three equations (35)-(37) involving four unknowns $\lambda, \xi, \theta, B, \rho$.

In order to obtain a determinate solution we have to assume a physical or mathematical condition. In the literature [24] ,we find different equations of state for string model as

$$\rho = \lambda \text{ (geometric string or Nambu string)} \tag{38}$$

and

$$\rho = (1+\omega)\lambda \quad \text{(p-string)}, \tag{39}$$

where ω is a constant such that $\omega > 0$.

Therefore using (38) and (39), we derive the solutions of equations (35)–(37) in the following subsections.

3.2.1 Geometric string $\rho = \lambda$

In this case (35)-(37) together with (38) we get,

$$\rho = \lambda = 0 \tag{40}$$

$$\xi = 0 \tag{41}$$

$$B = lt + m \,, \tag{42}$$

where $l\,(\neq 0)$ and m are integrating constants.

Thus this case leads to five dimensional vacuum model in Einstein theory and metric (1) becomes

$$ds^2 = dt^2 - p^2(dx^2 + dy^2) - (lt+m)^2 dz^2 + p^{2n} d\psi^2 \tag{43}$$

3.2.2 p-string $\rho = (1+\omega)\lambda$

From equation (36),we get

$$(1+\omega)\lambda = 0 \tag{44}$$

which yields either $(1+\omega) = 0$ or $\lambda = 0$

But $\omega = -1$ is not acceptable as $\omega > 0$.

Since if $\lambda = 0$ the model reduces to the model already obtained above in equation (43).

4. Conclusion :

In this paper, we have presented five dimensional plane symmetric Bianchi type-I string cosmological model with bulk viscosity. The model shown in equation (32) is inflationary. At an initial epoch $t = 0$, the metric in equation (32) becomes flat. Expansion scalar (θ) is finite at the initial epoch $t = 0$ and is tends to zero when t tends to infinity. Thus expansion of the model is finite. Also since $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$,the universe remains anisotropic throughout the evolution. The spatial volume for this model increases with time. The deceleration parameter obtained here is negative for $kk_2 > 0$ and $k_1 > 0$ thus indicates inflation in the model.

References:

[1] Kibble, T.W.B. (1976) , Topology of cosmic domains and strings, J. Phys. A , 9, 1387.

[2] Singh, K.P., Mollah, M.R. (2016), Higher Dimensional LRS Bianchi Type-I Cosmological Model Universe Interacting with Perfect Fluid in Lyra Geometry, The African Review of Physics 11, 33-38.

[3] Misner, C.W. (1968), The isotropy of the Universe, J. Astrophys, 151, 431-457.

[4] Wang,X.X. (2005), Bianchi Type-III string cosmological model with bulk viscosity in general relativity, Chin. Phys. Lett., 22, 29-32.

[5] Mohanty ,G., Mahanta, K.L. (2007), Five – dimensional axially symmetric string cosmological model in Lyra manifold, Astrophys Space Sci., 312, 301-304.

[6] Bali, R., Pradhan, A. (2007), Bianchi Type-III string cosmological model with time dependent bulk viscosity, Chinese Phys. Lett., 24, 585-588.

[7] Tripathy, S.K. ,Nayak, S.K., Sahu, S.K., Routray, T.R. (2009), Bulk viscous barotropic magnetised string cosmological models, Astrophys. Space Sci. 323, 281-287.

[8] Tripathy, S.K., Behera, D., Routray, T.R. (2010), Anisotropic universe with cosmic strings and bulk viscosity ,Astrophys. Space Sci. 325, 93-97.

[9] Rao, V.U.M, Kumari, G.S.D., Sireesha, K.V.S. (2011), Anisotropic universe with cosmic strings and bulk viscosity in a scalar tensor theory of gravitation, Astrophys. Space Sci. 335, No.2,635-638.

[10] Kandalkar, S.P, Khade, P.P. Gawande, S.P. (2011), Bianchi Type-VI Bulk Viscous Fluid String Cosmological Model in General Relativity, Bulg. J. Phys. 38, 145-154.

[11] Reddy, D.R.K., Naidu, R.L., Naidu, K.D., Prasad, T.R. (2013), Kaluza-Klein universe with cosmic strings and bulk viscosity in f (R,T) gravity, Astrophys. Space Sci. 346 (1), 261-265.

[12] Naidu, R.L., Naidu · K. D., Babu, K.S., Reddy, D.R.K. (2013), A five dimensional Kaluza-Klein bulk viscous string cosmological model in Brans-Dicke scalar-tensor theory of gravitation, Astrophys. Space Sci., 347(1), 197-201.

[13] Mete, V.G., Umarkar, U.M., Pund, A.M. (2013), Higher Dimensional Plane Symmetric Cosmological Models with Two-Fluid Source in General Relativity, Int. J. Theor. Phys., 52, 4439-4444.

[14] Mahanta, K.L., Tripathy,S.K. (2014),Cosmic Strings and Anisotropic Universe, Romanian Journal of Physics, 60,7.

[15] Katore, S.D., Shaikh, A.Y. (2014), Plane Symmetric Universe with Cosmic String and Bulk Viscosity in Scalar Tensor Theory of Gravitation, Rom. Journ. Phys., 59, Nos.7-8, 715-723.

[16] Kandalkar, S.P., Samdurkar, S. (2015), LRS Bianchi Type I Cosmological Model with Bulk Viscosity in Lyra Geometry, Bulg J Phys., 42, 42-52.

[17] Humad, V., Nagar, H., Shrimali, S. (2016), Bulk viscous fluid Bianchi type-I string cosmological model in general relativity, IOSR-JM, 12(2), 11-15.

[18] Chaturvedi, B.B., Gupta, B.K. (2017), Six-Dimensional Bulk Viscous Fluid Cosmological Model in f(R, T) Gravity Theory, Bulg. Journal of Phys., 44, 288 -298.

[19] Biswal,S.K. (2018), Strange Quark Matter Coupled to String Cloud in Five Dimensional Bianchi Type-III Space Time in General Relativity Research & Reviews: Journal of Physics, 7(2),54-61.

[20] Singh, A.K. (2018), LRS Bianchi type- I Higher Dimensional Cosmological Model Universe with Equation of State, International journal of basic and applied research, 8(9), 318.

[21] Samanta, G.C., Goel, M. , Myrzakulov, R. (2018), Strength of the singularities, equation of state and asymptotic expansion in Kaluza–Klein space time, New Astronomy, Elsevier, 60, 74-79.

[22] Singh, P.K., Baro, J. (2020),Bulk Viscous Fluid Bianchi Type-I String Cosmological Model with Negative Constant Deceleration Parameter, Adv. Math. Sci. J., 9(7), 4907-4916.

[23] Baro, J., Singh, K.P. (2020) , Higher Dimensional Bianchi Type-III String Universe With Bulk Viscous Fluid and Constant Deceleration Parameter (DP), Adv. Math. Sci.J.,9 no.10, 8779–8787.

[24] Letelier, P.S. (1983), String cosmologies, Phys. Rev. D, 28 (10), 2414.