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EDGE COLORING OF GRAPH USING EDGE ADJACENCY MATRIX

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ABSTRACT:

Graph coloring is the most popular area of research in graph theory. Graph coloring includes, vertex coloring ,edge coloring, region coloring etc in a graph. Proper edge coloring of a graph means, coloring of edges of a graph such that no two adjacent edges have the same color. In this paper a new method is proposed for proper edge coloring of a graph with the minimum number of colors using edge adjacency matrix. This method is explained with example and it helps to find the edge chromatic number of a graph.

Keywords:

Adjacency matrix, Edge coloring, Edge Chromatic number, Matrix representation

1. Introduction

Graph theory is one of the most rapidly developing field in Mathematics. Graphs in its applications are generally used to represent discrete objects and relationships between these objects. The visual representation of a graph is to declare an object as a vertex, while the relationship between objects is expressed as an edge [1]. The concept of graph theory initiated during the year 1735, by the Swiss Mathematician Leonhard Euler in solving Konigsberg bridge problem. Since then the field of graph theory expanded in various dimension using so many parameters, of which graph coloring is one among. Graph coloring has been studied as an algorithmic problem since early 1970s. Graph coloring problem belongs to the class of combinatorial optimization problem and studied due to its lot of applications in the area of data science, networking, register allocation and many more [2].

Graph coloring is classified as vertex coloring, edge coloring, total coloring, area coloring etc. The problem of the vertex coloring is to determine the minimum number of colors to color the vertices, so that the adjacent vertices have different colors. The problem of edge coloring is to determine the minimum number of colors to color the edges, so that the adjacent edges have different colors. The problem of total coloring is that no adjacent vertices, no adjacent edges and no edge and its end vertices are assigned the same color. The problem of area coloring is to determine the minimum number of colors to color the area, so that the adjacent areas have different colors [1]. Previously there have been several vertex and edge coloring methods were defined. In this paper a new method is proposed for proper edge coloring of graph using edge adjacency matrix.

2. Review of graph coloring

In the recent past, Tabu search techniques provide the optimal coloring of a graph [3]. David S. Johnson et al presented the simulated annealing schemes for graph coloring [4]. Daniel Brelaz presented the new methods to color the vertices of a graph [5]. One of the algorithms uses the machine based learning for graph coloring problem and used 78 identified features for that problem [6]. Amit Mittal et al described a method for graph coloring with minimum number of colors and it takes less time as compared to other techniques [7]. K A Santosa et al, presented the vertex coloring using adjacency matrix [8]. T. Ramachandran and N.Deepika proposed an algorithm for proper coloring of a vertices in a graph [2].

3. Preliminaries

For clear and better understanding of the new method some basic definitions and their remarks are presented below.

3.1 Definition

Coloring all the edges of a graph with colors such that no two adjacent edges have the same color is called the *proper* edge *coloring* .

A graph in which every edge has been assigned a color according to a proper edge coloring is called a *proper edge colored graph*. A graph G requires k different colors for its proper coloring, and no less, is called a *k- chromatic graph*, and the number k is called the *chromatic number* of G.

Remarks

- A graph consisting of only isolated edges is 1-chromatic.
- A graph with one or more edges (not a self – loop) is at least 2 – chromatic.
- A complete graph of n edges is n-chromatic.
- A graph consisting of simply one circuit with n≥3 vertices is 2 – chromatic if n is even and 3-chromatic if n is odd.

3.2 Definition

Let G be a graph with n vertices, e edges, and no self – loops. *Edge Adjacency Matrix* of G is defined by an e by e matrix denoted by A= $[a_{ij}]$, whose e rows and e columns are corresponding to the e edges. The matrix elements are

$[a_{ij}]$ = 1, if i^{th} edge is adjacent to j^{th} edge,

= 0, otherwise

Remarks

- Edge Adjacency matrix is also known as [0,1] matrix
- Edge Adjacency matrix is symmetric,i.e $[a_{ij}] = [a_{ji}]$
- Simple graph doesn't have a loop, so the diagonal elements of the edge adjacency matrix are always zero.
- The number of 1's in each row and column equals the degree of the corresponding edge.

3.3 Definition

Degree of an edge is defined as the number of edges adjacent to that edge. Degree of a vertex is defined as the number of edges connecting it.

4. Procedure for edge coloring

For proper edge coloring of graph using edge adjacency matrix, a step by step procedure is given below, which help us to color the edges such that no two adjacent edges receive the same color.

Step by Step Procedure

Step 1: Construct an edge adjacency matrix for the given graph.

Step 2: Find the sum of the elements in each row of the matrix constructed in step 1. Select the row that has the maximum value.

Case (a): If the maximum value is unique, then find the maximal null matrix formed by the zeros in the selected row and go to step 3.

Case (b): If there is a tie in the maximum value , select all those rows and find all maximal null matrices formed by the zeros in the corresponding selected rows then select the largest null matrix among all maximal null matrices and then go to step 3.

Step 3: Check the uniqueness of the null matrix.

Case (a): If the maximal null matrix selected in step 2 is unique then go to step 4.

Case (b): If there is a tie in the largest null matrix selected in step 2, find the degree sum of all the edges associated with the rows of each largest null matrix, then choose the null matrix corresponding to the maximum degree sum.

- If it is unique, then go to step 4.
- If there is a tie in the maximum degree sum, then select any one largest null matrix arbitrarily among the tie and go to step 4.

Step 4: Assign a color to the edges corresponding to the rows of the identified maximal null matrix obtained in step 3 and go to step 5.

Step 5: Remove all the rows and columns associated with the colored edges, then go to step 2 and repeat the process until all the edges have been colored.

To understand the above procedure for proper coloring of edges of any graph, an example is discussed here under.

4.1 Example

Consider a graph with 11 vertices and 19 edges as shown in the figure 1. Find the proper edge coloring of this graph using the above procedure.

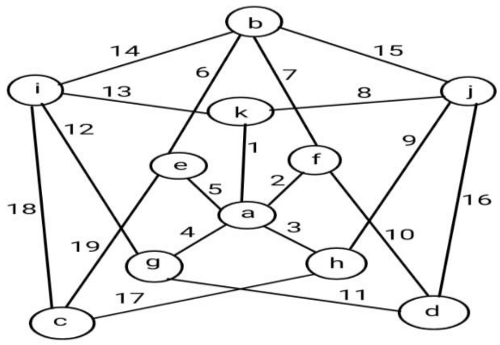

Figure:1 (11, 19) graph

Solution:

As per the first and second steps of the procedure, construct an edge adjacency matrix and compute the sum of the elements in each row of the corresponding matrix that is nothing but the degree of a edge associated with each row of the matrix and is shown in the table 1.

Table 1

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**deg**
1	0	1	1	1	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	**6**
2	1	0	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	**6**
3	1	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	**6**
4	1	1	1	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	**6**
5	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	**6**
6	0	0	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	1	**5**
7	0	1	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	0	**5**
8	1	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	0	0	0	**5**
9	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	**5**
10	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	**4**
11	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	**4**
12	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0	1	0	**5**
13	1	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0	**5**
14	0	0	0	0	0	1	1	0	0	0	0	1	1	0	1	0	0	1	0	**6**
15	0	0	0	0	0	1	1	1	1	0	0	0	0	1	0	1	0	0	0	**6**
16	0	0	0	0	0	0	0	1	1	1	1	0	0	0	1	0	0	0	0	**5**
17	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	**4**
18	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0	1	**5**
19	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	**4**

From the table 1, the maximum value is 6, and by case b) of step 2, there is tie, select the first, second, third, fourth, fifth, fourteenth, fifteenth rows and its corresponding edges are 1,2,3,4,5,14 and 15 respectively.

From the 1st row, we find that the edges 1,6,9,10,12 form the largest null matrix of order 5. From the 2nd row, the edges 2,11,13,15,17 form the largest null matrices of order 5. From the 3rd row the edges 3,6,8,10,12 form the largest null matrices of order 5. From the 4th row the edges 4,9,10,13,19 form the largest null matrices of order 5. From the 5th row the edges 5,7,8,11,17 form the largest null matrices of order 5. From the 14th row the edges 8,10,14,17 form the largest null matrices of order 4. From the 15th row the edges 10,13,15,17 form the largest null matrices of order 4.

By case (b) of step 3, there is a tie in the largest null matrices of order 5. Degree sum of each largest null matrices formed by the edges 1,6,9,10,12 is 25, 2,11,13,15,17 is 25, 3,6,8,10,12 is 25, 4,9,10,13,19 is 24 and 5,7,8,11,17 is 24.

There is a tie in the largest null matrices formed by the edges 1,6,9,10,12 and 2,11,13,15,17 and 3,6,8,10,12 with degree sum 25. By case (b) of step 3, Select any one largest null matrix arbitrarily formed by the edges 3,6,8,10 and 12.

By step 4, assign a first color(say Green) to the edges 3,6,8,10 and 12 of the identified largest null matrix.

By step 5, remove all the rows and columns associated with the colored edges of a given graph and the reduced matrix is given in the table 2.

Table 2

	1	**2**	**4**	**5**	**7**	**9**	**11**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**Deg**
1	0	1	1	1	0	0	0	1	0	0	0	0	0	0	**4**
2	1	0	1	1	1	0	0	0	0	0	0	0	0	0	**4**
4	1	1	0	1	0	0	1	0	0	0	0	0	0	0	**4**
5	1	1	1	0	0	0	0	0	0	0	0	0	0	1	**4**
7	0	1	0	0	0	0	0	0	1	1	0	0	0	0	**3**
9	0	0	0	0	0	0	0	0	0	1	1	1	0	0	**3**
11	0	0	1	0	0	0	0	0	0	0	1	0	0	0	**2**
13	1	0	0	0	0	0	0	0	1	0	0	0	1	0	**3**
14	0	0	0	0	1	0	0	1	0	1	0	0	1	0	**4**
15	0	0	0	0	1	1	0	0	1	0	1	0	0	0	**4**
16	0	0	0	0	0	1	1	0	0	1	0	0	0	0	**3**
17	0	0	0	0	0	1	0	0	0	0	0	0	1	1	**3**
18	0	0	0	0	0	0	0	1	1	0	0	1	0	1	**4**
19	0	0	0	1	0	0	0	0	0	0	0	1	1	0	**3**

Table 2 shows that, the maximum value is 4, and starts with step 2, repeating the process we get a set of edges 2,11,13,15 and 17 are need to be colored with second color (say Red) . Remove all the rows and columns associated with the colored edges of a given graph.

Again by the first and second steps, the edge adjacency matrix and the sum of the elements in each row of the matrix for the uncolored edges of a graph is given in table 3.

Table 3

	1	**4**	**5**	**7**	**9**	**14**	**16**	**18**	**19**	**Deg**
1	0	1	1	0	0	0	0	0	0	**2**
4	1	0	1	0	0	0	0	0	0	**2**
5	1	1	0	0	0	0	0	0	1	**3**
7	0	0	0	0	0	1	0	0	0	**1**
9	0	0	0	0	0	0	1	0	0	**1**
14	0	0	0	1	0	0	0	1	0	**2**
16	0	0	0	0	1	0	0	0	0	**1**
18	0	0	0	0	0	1	0	0	1	**2**
19	0	0	1	0	0	0	0	1	0	**2**

Table 3 shows that, the third row has the maximum value 3. By case a) of step 2, it is unique and its corresponding edge is 5 and the matrix formed by the zeros in the third row is given in the table 3(a).

Table 3(a)

	5	**7**	**9**	**14**	**16**	**18**
5	0	0	0	0	0	0
7	0	0	0	1	0	0
9	0	0	0	0	1	0
14	0	1	0	0	0	1
16	0	0	1	0	0	0
18	0	0	0	1	0	0

Table 3(a) shows that, the edges 5,7,9 and 18 form the maximal null matrix. By case a) of step 3, it is unique. By step 4, assign a third color (say Pink) to the edges 5,7,9 and 18 associated with the rows of the identified maximal null matrix. By step 5, remove all the rows and columns associated with the colored edges of a given graph. Again by the first and second steps the edge adjacency matrix and the sum of the elements in each row of the matrix for the uncolored edges of a graph is given in table 4.

Table 4

	1	**4**	**14**	**16**	**19**	**Deg**
1	0	1	0	0	0	**1**
4	1	0	0	0	0	**1**
14	0	0	0	0	0	**0**
16	0	0	0	0	0	**0**
19	0	0	0	0	0	**0**

Table 4 shows that, the maximum value is 1, and starts with step 2, repeating the process we get a set of edges 1,14,16 and 19 are need to be colored with second color (say Blue). Remove all the rows and columns associated with the colored edges of a given graph.

The remaining edge is 4. Assign fifth color (say Yellow) to this edge. The resulting graph is shown in figure 2.

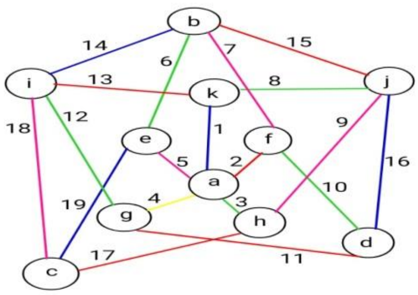

Figure:2

By the proposed method, the edges of a given graph is colored with minimum five colors and its chromatic number is 5.

Conclusion

The proposed method to find edge coloring of a graph using edge adjacency matrix with illustration clearly explain the procedure and final solution gives edge chromatic number of graph. By developing computer algorithm in an appropriate language for the above proposed method we can make easy in finding the edge chromatic number of any large size of graph.

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