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VAGUE ĝ OPEN SETS & ITS APPLICATIONS IN VAGUE TOPOLOGICAL SPACES

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ABSTRACT

A new class of vague ĝ - open sets in vague topological spaces is introduced and also studied some of its basic properties. Further we obtained $T_V\hat{g}_{\frac{1}{2}}$, $T_V\hat{g}_{\frac{1}{2}}^{\ *}$, $T_{vs}\hat{g}_g$ spaces and also discussed some of its characterizations.

KEYWORDS

Vague ĝ open sets, $T_V\hat{g}_{\frac{1}{2}}$ space, $T_V\hat{g}_{\frac{1}{2}}^{\ *}$space, $T_{vs}\hat{g}_g$ space.

1. INTRODUCTION

In 1965 the fuzzy sets was introduced by Zadeh[14]. The theory of fuzzy topology was introduced in 1967 by C.L. Chang[4]. The concept of vague sets was first initiated by Gau and Buehrer[5]. Norman Levine[6] initiated generalized closed (briefly g-closed) sets in 1970. M.K Veera Kumar [13] introduced ĝ –Closed sets in topological spaces in 2000.

Here we introduced the concepts of vague ĝ - open sets also studied the applications of the new class and its basic properties.

2. PRELIMINARIES

Definition 2.1: [2] A vague set $\mathcal{A}$ in the universe of discourse X is characterized by two membership functions given by:

1. A true membership function $T_{\mathcal{A}} : X \to [0,1]$ and

2. A false membership function $F_{\mathcal{A}} : X \to [0,1]$,

where $T_{\mathcal{A}}(x)$ is lower bound on the grade of membership of x derived from the "evidence for x",$F_{\mathcal{A}}(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $T_{\mathcal{A}}(x) + F_{\mathcal{A}}(x) \leq 1$. Thus the grade of membership of x in the vague set $\mathcal{A}$ is bounded by a subinterval $[T_{\mathcal{A}}(x), 1 - F_{\mathcal{A}}(x)]$ of $[0, 1]$.

Definition 2.2: [2] Let $\mathcal{A}$ ad $\mathcal{B}$ be two vague sets of the form $\mathcal{A} = \{\langle x, [T_{\mathcal{A}}(x), 1 - F_{\mathcal{A}}(x)]\rangle / x \in X\}$ and $\mathcal{B} = \{\langle x, [T_B(x), 1 - F_{\mathcal{B}}(x)]\rangle / x \in X\}$ Then

a) $\mathcal{A} \subseteq \mathcal{B}$ if and only if $\mathcal{A}(x) \leq T_{\mathcal{B}}(x)$ and $1 - F_{\mathcal{A}}(x) \leq 1 - F_{\mathcal{B}}(x)$forall $x \in X$

b) $\mathcal{A} = \mathcal{B}$ if and only if $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A}$

c) $\mathcal{A}^C = \{\langle x, F_{\mathcal{A}}(x), 1 - T_{\mathcal{A}}(x) / x \in X\rangle\}$

d) $\mathcal{A} \cap \mathcal{B} = \{\langle x, \min(T_{\mathcal{A}}(x), T_{\mathcal{B}}(x)), \min(1 - F_{\mathcal{A}}(x), 1 - F_{\mathcal{B}}(x))\rangle / x \in X\}$

e) $\mathcal{A} \cup \mathcal{B} = \{\langle x, \max(T_{\mathcal{A}}(x), T_{\mathcal{B}}(x)), \max(1 - F_{\mathcal{A}}(x), 1 - \mathcal{B}(x))\rangle / x \in X\}$

VAGUE TOPOLOGICAL SPACE

Definition 2.3:[8] A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.

a) $0,1 \in \tau$

b) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

c) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a vague topological space (VTS in short) and any VS in 'τ' is known as a vague open set (VOS in short) in X.

Definition 2.4:[8] Let (X, τ) be a VTS and $\mathcal{A} = \langle x, T_{\mathcal{A}}, 1 - F_{\mathcal{A}}\rangle$ be a VS in X. Then the vague interior and a vague closure are defined by

$V\,int\,(\mathcal{A}) = \cup\{G / G \text{ is an VOS in X and } G \subseteq \mathcal{A}\}$, $V\,cl\,(\mathcal{A}) = \cap\{K / K \text{ is an VCS in X and } \mathcal{A} \subseteq K\}$

Note: $Vcl(\mathcal{A}^c) = (V\,int\,(\mathcal{A}))^C$ and $V\,int(\mathcal{A}^c) = (V\,cl\,(\mathcal{A}))^C$.

Definition 2.5:[8] A Vague set $\mathcal{A}$ *of* (X, τ) is said to be a (VSCS if $Vint(Vcl(\mathcal{A})) \subseteq \mathcal{A}$), (VSOS if $\mathcal{A} \subseteq Vcl(Vint(\mathcal{A}))$), (VPCS if $Vcl(Vint(\mathcal{A})) \subseteq \mathcal{A}$), (VPOS if $\mathcal{A} \subseteq Vint(Vcl(\mathcal{A}))$), (VαCS if $Vcl(Vint(Vcl(\mathcal{A}))) \subseteq \mathcal{A}$), (VαOS if $\mathcal{A} \subseteq Vint(Vcl(Vint(\mathcal{A})))$), (VROS if $\mathcal{A} = Vint(Vcl(A))$), (VRCS if $\mathcal{A} = Vcl(Vint(\mathcal{A}))$),

Definition 2.6:[9] A vague set $\mathcal{A}$ of (X, τ) is said to be a **vague ĝ-closed** sets (VĜCS in short) if $Vcl(\mathcal{A}) \subseteq U$ whenever $\mathcal{A} \subseteq U$ and U is vague semi open set in X.

4. VAGUE ĝ - OPEN SETS

We introduced the notion of vague ĝ open sets and studied some of their properties.

Definition 3.1: A vague set $\mathcal{A}$ of (X, τ) is said to be a vague ĝ - open sets (VĜOS in short) if $Vint(\mathcal{A}) \supseteq U$ whenever $\mathcal{A} \supseteq U$ and U is vague semi closed set in X.

The family of all V ĝ OSs of an VTS (X,τ) is denoted by V ĝ O(X).

Note that the complement of $\mathcal{A}$ (i.e.,$\mathcal{A}^C$) is V ĝ CS, $\mathcal{A}$ in an VTS (X,τ) is a V ĝ OS in X.

Example 3.2: Let $X = \{a, b\}, \tau = \{0, G, 1\}$ be a VT on X, $G = \{< x, [0.2,0.5], [0.1,0.4] >\}$, then the vague set $\mathcal{A} = \{< x, [0.2,0.7], [0.3,0.6] >\}$ is a V ĝ OS.

Theorem 3.3:

i. Every VOS is a VĜOS.
ii. Every VĜOS is a VSGOS.
iii. Every VĜOS is a VGOS.
iv. Every VĜOS is a VRGOS.

v. Every $V\hat{G}OS$ is a VGPROS.

vi. Every $V\hat{G}OS$ is a $V\alpha GOS$.

Proof: Straight forward.

The converse of the above statements need not be true, which can be verified by the following examples.

Example 3.4: Let $X = \{a, b\}, \tau = \{0, G, 1\}$ be a VT on X, $G = \{< x, [0.1,0.4], [0.2,0.3] >\}$, then the vague set $\mathcal{A} = \{< x, [0.2,0.8], [0.3,0.6] >\}$ is a $V\hat{g}OS$ but not a VOS.

Example 3.5: Let $X = \{a, b\}, \tau = \{0, G, 1\}$ be a VT on X, $G = \{< x, [0.6,0.9], [0.7,0.8] >\}$ then the vague set $\mathcal{A} = \{< x, [0.9,0.9], [0.8,0.8] >\}$ is a VSGOS but not $V\hat{g}OS$.

Example 3.6: Let $X = \{a, b\}, \tau = \{0, G_1, G_2, G_3, G_4 1\}$ be a VT on X, $G_1 = \{< x, [0.2,0.5], [0.5,0.5] >\}, G_2 = \{< x, [0.5,0.9], [0.3,0.4] >\}, G_3 = \{< x, [0.5, 0.9], [0.4,0.5] >\}, G_4 = \{< x, [0.2,0.5], [0.3,0.4] >\}$ then the vague set $\mathcal{A} = \{< x, [0.4,0.5], [0.5, 0.6] >\}$ is a VGOS but not $V\hat{g}OS$.

Example 3.7: Let $X = \{a, b\}, \tau = \{0, G, 1\}$ be a VT on X, $G = \{< x, [0.2,0.7], [0.5,0.8] >\}$ then the vague set $\mathcal{A} = \{< x, [0.6,0.8], [0.6,0.9] >\}$ is a VRGOS but not $V\hat{g}OS$.

Example 3.8: Let $X = \{a, b\}, \tau = \{0, G, 1\}$ be a VT on X, $G = \{< x, [0.5,0.7], [0.2,0.7] >\}$ then the vague set $\mathcal{A} = \{< x, [0.4,0.5], [0.5,0.8] >\}$ is a VGPROS but not $V\hat{g}OS$.

Example 3.9: Let $X = \{a, b\}, \tau = \{0, G, 1\}$ is a VT on X, $G = \{< x, [0.5,0.9], [0.4,0.8] >\}$ then the vague set $\mathcal{A} = \{< x, [0.2,0.5], [0.3,0.6] >\}$ is a $V\alpha GOS$ but not $V\hat{g}OS$.

Theorem 3.10: The union and the intersection of any two $V\hat{g}OS$ in (X, τ) may be a $V\hat{g}OS$ in (X, τ). This can be seen from the below example.

Example 3.11: Let $X = \{a, b\}, \tau = \{0, G_1, G_2, G_3, G_4 1\}$ be a VT on X, $G_1 = \{< x, [0.1,0.4], [0.2,0.3] >\}, G_2 = \{< x, [0.2,0.5], [0.1,0.4] >\}, G_3 = \{< x, [0.2, 0.5], [0.2,0.4] >\}, G_4 = \{< x, [0.1,0.4], [0.1,0.3] >\}$ then the vague set $\mathcal{A} \cup \mathcal{B} = \{< x, [0.2,0.8], [0.3, 0.6] >\}$ & $\mathcal{A} \cap \mathcal{B} = \{< x, [0.2,0.7], [0.3, 0.6] >\}$ is a $V\hat{g}OS$, but depending upon the vague points it may vary.

Theorem 3.12: A vague set $\mathcal{A}$ of a VTS (X, τ) is a $V\hat{G}OS$ in (X, τ) if and only if $U \subseteq Vint(\mathcal{A})$, whenever U is a vague semi closed set and $U \subseteq \mathcal{A}$.

Proof: Necessity: Suppose $\mathcal{A}$ is $V\hat{G}OS$ in (X, τ). Let U be a vague semi closed set in (X, τ) such that $U \subseteq \mathcal{A}$. Then U^C is a VSOS in X such that $\mathcal{A}^C \subseteq U^C$, By hypothesis, $\mathcal{A}^C$ is a $V\hat{G}CS$ in (X, τ).

We have $Vcl(\mathcal{A}^C) \subseteq U^C$, hence $U \subseteq Vint(\mathcal{A})$.

Sufficiency: Let $\mathcal{A}$ be a vague set of X and let $U \subseteq Vint(\mathcal{A})$, whenever U is a vague semi closed set and $U \subseteq \mathcal{A}$. Then $\mathcal{A}^C \subseteq U^C$ and U^C is a vague semi open set. By hypothesis, $Vcl(\mathcal{A}^C) \subseteq U^C \Rightarrow \mathcal{A}^C$ is a $V\hat{G}CS$ in (X, τ). Hence $\mathcal{A}$ is a $V\hat{G}OS(X)$.

Theorem 3.13: Let (X, τ) be a VTS, then for every $\mathcal{A} \in V\hat{G}OS$ and for every $V\hat{G}OS$ in (X, τ) $\mathcal{B} \in VS(X), Vint\ (\mathcal{A}) \subseteq \mathcal{B} \subseteq \mathcal{A} \Rightarrow \mathcal{B} \in V\hat{G}OS$ in (X, τ).

Proof: Suppose $\mathcal{A}$ is a $V\hat{G}OS$ in (X, τ) and $Vint\ (\mathcal{A}) \subseteq \mathcal{B} \subseteq \mathcal{A} \Rightarrow \mathcal{A}^C \subseteq \mathcal{B}^C \subseteq (Vint\ (\mathcal{A}))^C$.

Let $\mathcal{B}^C \subseteq U$ and U be VSOS, Since $\mathcal{A}^C \subseteq \mathcal{B}^C \subseteq U$, Hence $\mathcal{A}^C \subseteq U$. But $\mathcal{A}^C$ is a $V\hat{G}CS(X) \Rightarrow Vcl(\mathcal{A}^C) \subseteq U$. Also $\mathcal{B}^C \subseteq (Vint\ (\mathcal{A}))^C = Vcl(\mathcal{A}^C)$. $\therefore\ Vcl(\mathcal{B}^C) \subseteq Vcl(Vcl(\mathcal{A}^C) \subseteq U$.

Hence $\mathcal{B}^C$is a $V\hat{G}FCS \Rightarrow \mathcal{B}$ is a $V\hat{G}FOS$ in (X,τ). Hence $\mathcal{B} \in V\hat{G}FOS$ (X).

4. Properties of $T_V\,\hat{g}_{\frac{1}{2}}$ **spaces**

We introduce the following definition:

Definition 4.1: A space (X,τ) is called a $T_V\,\hat{g}_{\frac{1}{2}}$ space if every V $\hat{g}$ - closed set in it is vague closed.

Example 4.2: " $X = \{a,b\}, \tau = \{0, G, 1\}$ is a VT on X, $G = \{< x, [0.5,0.5], [0.5,0.5] >\}$ then the vague set $\mathcal{A} = \{< x, [0.5,0.5], [0.5,0.5] >\}$". Thus (X,τ) is a $T_V\,\hat{g}_{\frac{1}{2}}$space.

Example 4.3: " $X = \{a,b\}, \tau = \{0, G, 1\}$ is a VT on X, $G = \{< x, [0.1,0.4], [0.2,0.3] >\}$ then the vague set $\mathcal{A} = \{< x, [0.2,0.8], [0.3,0.6] >\}$". Here (X,τ) is not a $T_V\,\hat{g}_{\frac{1}{2}}$space.

Definition 4.4: A space (X,τ) is called a $T_V\,\hat{g}_{\frac{1}{2}}{}^*$ space if every Vg- closed set in it is V $\hat{g}$ − closed.

Example 4.5: " $X = \{a,b\}, \tau = \{0, G, 1\}$ is a VT on X, $G = \{< x, [0.3,0.6], [0.6,0.8] >\}$ then the vague set $\mathcal{A} = \{< x, [0.2,0.4], [0.3,0.6] >\}$". Thus (X,τ) is a $T_V\,\hat{g}_{\frac{1}{2}}{}^*$space.

Definition 4.6: A space (X,τ) is called a $T_{v\,s}\,\hat{g}_g$ space if every Vsg - closed set in it is V $\hat{g}$ − closed.

Example 4.7: " $X = \{a,b\}, \tau = \{0, G, 1\}$ is a VT on X, $G = \{< x, [0.3,0.5], [0.4,0.6] >\}$ then the vague set $\mathcal{A} = \{< x, [0.5,0.8], [0.4,0.7] >\}$". Thus (X,τ) is a $T_{v\,s}\,\hat{g}_g$space.

Theorem 4.8: Every $T_{V\frac{1}{2}}$ space is a $T_V\,\hat{g}_{\frac{1}{2}}$ space but not conversely.

Proof: Follows from the result "Every vague $\hat{g}$ closed set is vague g closed but not conversely".

The converses of the above theorem need not be true as seen from the following example.

Example 4.9: "$X = \{a,b\}, \tau = \{0, G, 1\}$ is a VT on X, $G = \{< x, [0.5,0.5], [0.5,0.5] >\}$ as same in the example 4.2 then the vague set $B = \{< x, [0.2,0.8], [0.3,0.6] >\}$ is not vague closed". Hence (X,τ) is not a $T_{V\frac{1}{2}}$ space.

Theorem 4.10: Every $T_{V\frac{1}{2}}$ space is a $T_V\,\hat{g}_{\frac{1}{2}}{}^*$space but not conversely.

Proof: Follows from the result "Every vague closed set is vague $\hat{g}$ closed but not conversely".

The converses of the above theorem need not be true as seen from the following example.

Example 4.11: "$X = \{a,b\}, \tau = \{0, G, 1\}$ is a VT on X, $G = \{< x, [0.5,0.5], [0.5,0.5] >\}$ as same in the example 4.2 then the vague set $\mathcal{A} = \{< x, [0.5,0.5], [0.5,0.5] >\}$ is V $\hat{g}$ closed". Thus (X,τ) is a $T_V\,\hat{g}_{\frac{1}{2}}{}^*$ space, but the vague set $\mathcal{B} = \{< x, [0.2,0.8], [0.3,0.6] >\}$ is not vague closed". Hence (X,τ) is not a $T_{V\frac{1}{2}}$ space.

Theorem 4.12: Let (X,τ) be a VTS and (X,τ) is $T_V\,\hat{g}_{\frac{1}{2}}$ space. Then the following statements hold.

(i) Any union of $V\,\hat{g}$ CS is $V\,\hat{g}$ CS.

(ii) Any intersection of $V\,\hat{g}$ OS is $V\,\hat{g}$ OS.

Proof: (i) Let $\{U_i\}_{i\in J}$ be a collection of $V\,\hat{g}$ CS in a $T_V\,\hat{g}_{\frac{1}{2}}$ space (X,τ). Therefore every $V\,\hat{g}$ CS is VCS. But the union of VCS is VCS. Hence the union of $V\,\hat{g}$ CS is $V\,\hat{g}$ CS in X.

(ii) It can be proved by taking the complement in (i).

Theorem 4.13: A VTS (X, τ) is a $T_V\widehat{g}_{\frac{1}{2}}$ space if and only if $V\widehat{g}O(X) = VO(X)$.

Proof: Necessity: Assume $\mathcal{A}$ be $V\widehat{g}OS$ in X. Then $\mathcal{A}^C$ is $V\widehat{g}CS$ in X. By hypothesis, $\mathcal{A}^C$ is VCS in X. Therefore $\mathcal{A}$ is VOS in X. Hence $V\widehat{g}O(X) = VO(X)$.

Sufficiency: Suppose $\mathcal{A}$ is $V\widehat{g}CS$ in X. Then $\mathcal{A}^C$ is $V\widehat{g}OS$ in X. Therefore $\mathcal{A}$ is VCS in X. Hence (X, τ) is $T_V\widehat{g}_{\frac{1}{2}}$ space.

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