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A NESTED INFINITE RADICAL FORMULA FOR ODD NUMBERS

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Abstract

I am going to provide a Nested Infinite Radical Formula for all Odd Numbers which are greater than 1. Besides that, I will also prove the Proposition mentioned by various methods.

1 Introduction

Srinivasa Ramanujan had proposed a nested infinite radical problem in JIMS (Journal of the Indian Mathematical Society) in the year 1908. The problem (Q.289) says,
Find the value:

$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + \ldots}}}$$

I am going to provide a formula that would help us find the solution of the problem proposed by Srinivasa Ramanujan and later I have generalized the formula for all odd numbers bigger than 1.

Proposition 1. *For all odd numbers (in the form 2k-1) greater than 1 we have*

$$2k - 1 = \sqrt{1 + 4\sqrt{1 + (k^2 - k - 1)\sqrt{1 + (k^2 - k)\sqrt{1 + \ldots}}}}$$

Later, In section 2, I will be showing that how the above-mentioned formula can be proved using the nested formula given by Ramanujan, along with that I will also be showing how a nested complicated radical expression can be simplified and vice versa.

2 Generalization

I begin by proving Proposition 1,

$$2k - 1 = \sqrt{1 + 4\sqrt{1 + (k^2 - k - 1)\sqrt{1 + (k^2 - k)\sqrt{1 + \ldots}}}}$$

Proof : From the generalization of (1) given by Ramanujan, i.e

$$\sqrt{ax+(n+a)^2+x\sqrt{a(x+n)+(n+a)^2+(x+n)\sqrt{\dots}}}=x+n+a$$

if we set a = 0, $x=k^2-k-1$, and n = 1 we get,

$$\sqrt{1+(k^2-k-1)\sqrt{1+(k^2-k)\sqrt{1+\dots}}}=k^2-k$$

After some algebraic manipulation we get,

$$\sqrt{1+4\sqrt{1+(k^2-k-1)\sqrt{1+(k^2-k)\sqrt{1+\dots}}}}=\sqrt{1+4(k^2-k)}$$

$$\Rightarrow\sqrt{1+4\sqrt{1+(k^2-k-1)\sqrt{1+(k^2-k)\sqrt{1+\dots}}}}=\sqrt{(2k-1)^2}$$

$$\Rightarrow\sqrt{1+4\sqrt{1+(k^2-k-1)\sqrt{1+(k^2-k)\sqrt{1+\dots}}}}=(2k-1)\ \square$$

and this completes our proof. Now I will provide some examples which would help us understand the need for the formula (2).

Example 2.1 Find a nested radical expression for : i) 49 ii) 897

Solution. i) To find the expression required we first set

$$2k-1=49\Rightarrow k=25$$

Now, using (2) we get,

$$49=\sqrt{1+4\sqrt{1+599\sqrt{1+600\sqrt{1+\dots}}}}$$

ii) Again, to find the required expression we first set

$$2k-1=897\Rightarrow k=449$$

Now, using (2) we get,

$$897=\sqrt{1+4\sqrt{1+201151\sqrt{1+201152\sqrt{1+\dots}}}}$$

Example 2.2 Find the value of :

$$\sqrt{1+4\sqrt{1+425755\sqrt{1+425756\sqrt{1+\dots}}}}$$

Solution. Now for solving this we need to compare this with (2),

$$\sqrt{1+4\sqrt{1+425755\sqrt{1+425756\sqrt{1+\ldots}}}}$$

$$So,\ k^2-k-1=425755$$

$$\Rightarrow k=-652\ (rejected\ as\ k>0)$$

or

$$\Rightarrow k=653\ (accepted)$$

Since k is positive so our value for k is 653 and thus our final solution is 1305.

3 The Famous Problem

Now I will show how we can use the above-mentioned formula to solve the problem given by Srinivasa Ramanujan, i.e.

$$\text{Let, S} = \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+\ldots}}}}}$$

On comparing with (2) we get,

$$\text{S} = \sqrt{1+2\sqrt{1+3\sqrt{\mathbf{1+4\sqrt{1+5\sqrt{1+\ldots}}}}}}$$

The bold portion is in the form of (2), we get,

$$k^2-k-1=5$$

$$\Rightarrow k=-2\ (rejected\ as\ k>0)$$

or

$$\Rightarrow k=3\ (accepted)$$

So our final value for k is 3, Now on moving back to our original equation we get,

$$\Rightarrow \text{S} = \sqrt{1+2\sqrt{1+3\,.(2(3)-1)}} = \sqrt{1+2\sqrt{16}} = \sqrt{9} = 3\ \square$$

and this completes our proof, So using my formula one can easily deduce the value of an Infinite Radical Expression.

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