IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

ANALYSIS OF SINGLE AND MULTI OBJECTIVE INVENTORY MODEL WITH EXPONENTIAL DEMAND BY FUZZY NON LINEAR PROGRAMMING AND GENERAL FUZZY PROGRAMMING TECHNIQUES

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Abstract: A single objective stochastic inventory model is formulated and introducing the impreciseness to the budget, it is considered as a fuzzy single objective stochastic inventory model. Then we reformulate the model as a multi objective inventory problem and it is solved by General Fuzzy Non Linear Programming technique. The solutions are illustrated by numerical examples when the demand follows exponential distribution and the results are compared, also.

Keywords - Fuzzy stochastic model, General fuzzy non-linear programming, Fuzzy Programming, stochastic inventory.

Introduction

In 1965, the first publication in fuzzy set theory by Zadeh (1965) showed the intention to accommodate uncertainty in the non-stochastic sense rather than the presence of random variables. Bellman and Zadeh (1970) first introduced fuzzy set theory in decision-making processes. Later, Zimmermann (1976) showed that the classical algorithms could be used to solve a fuzzy linear programming problem. Fuzzy mathematical programming has been applied to several fields like project network, reliability optimization, transportation, media selection for advertising; air pollution regulation etc. problems formulated in fuzzy environments. We can see from real markets that many products such as clothes, shoes and vegetables, whose backorder rate (or equivalently, lost sales rate) may be influenced by substitute, brand loyalty, customers' preference and waiting patience, etc. In other words, the lost sales rate may change slightly due to these potential factors, and is difficult to measure an exact value for lost sales rate. In most of the existing inventory models, it is assumed that the inventory parameters, objective goals and constraint goals are deterministic and fixed. But, if we think of their practical meaning, they are uncertain, either random or imprecise. The budget constraint can easily be converted to a storage constraint. Islam and Roy (2007) considered Fuzzy multi-item economic production quantity model under space constraint. Al-Fawzan and Hariga (2002) analyzed an integrated inventory-targeting problem with time-dependent process mean. Halim and Chaudhuri (2011) analyzed Fuzzy production planning models for an unreliable production system with fuzzy production rate and stochastic and fuzzy demand. Jaggi and Arneja (2011) discussed Stochastic integrated vendor-buyer model with unstable lead-time and set up cost. Chou (2009) developed a

fuzzy backorder inventory model and application to determining the optimal empty-container quantity at a port. Gani and Maheswari (2010) discussed economic order quantity for items with imperfect quality where shortages are backordered in fuzzy environment.
In this paper, we formulate a single objective stochastic inventory model and introducing the impreciseness to the budget, it is considered as a fuzzy single objective stochastic inventory model. Then we consider the problem as a multi objective problem and it is solved by General Fuzzy Non Linear Programming technique.

1. Mathematical model

We consider a multi item single period inventory problem with budgetary and floor or shelf- space constraints. We assume that the demand of the items follows probability distribution. So n products are stocked to satisfy a random external demand during a single period. For each item, an order quantity Q_i can be made for delivery prior to the beginning of the period. No subsequent orders can be made during the period. Excess demand is disposed of at a lower price. We consider also inventory-carrying cost for items sold during the period and those remaining at the end. This cost is based on the average inventory rather than simply ending inventory as is generally assumed, under the following assumptions:
For i-th item (i=1, 2, ….., n),
p_i = purchasing price of each product,
s_i = selling price of each product,
C_{1i} = inventory carrying cost per quantity per unit time,
C_{3i} = shortage cost for unsatisfied demand,
L_i = salvage value per unit,
f_i = floor space available per unit,
D_i = demand of the ith item.
F = floor space available,
B = budget available for replenishment,
We assume that the demand for the period for the ith item is a random variable which follows probability distribution and then we assume $f_i(x)$ is the probability density function.
Mainly two models are considered here.
1. Single Objective Stochastic Inventory Model [SOSIM]
2. Fuzzy Single Objective Stochastic Inventory Model [FSOSIM]

1.1 Single Objective Stochastic Inventory Model [SOSIM]

Most of the probabilistic inventory models are considered as an unconstrained probabilistic optimization model. But, in real life, problems are considered under some limited restrictions, e.g. total floor space for inventory problem is not unlimited. Similarly, total budget amount is also limited. So the following model may be considered:
Maximize expected profit rate under limited storage space and budgetary constraints. It is a Single Objective Stochastic Inventory Model [SOSIM]
The model can be stated as:
Maximize expected profit rate = total revenue earn – total inventory related cost
= [Revenue from sales + salvage value]
- [Inventory carrying cost + purchase price + shortage cost]
Max PF ($Q_1, Q_2, \ldots\ldots, Q_n$) = (RS ($Q_1, Q_2, \ldots\ldots, Q_n$) - IC ($Q_1, Q_2, \ldots\ldots, Q_n$))

$$= \sum_{i=1}^{n}((s_i\int_0^{Q_i} x f_i(x)dx + s_i Q_i \int_{Q_i}^{\infty} f_i(x)dx + L_i\int_0^{Q_i}(Q_i - x) f_i(x)dx) - (p_i Q_i + \frac{C_{1i}Q_i}{2}$$
$$+ \frac{C_{1i}}{2}\int_0^{Q_i}(Q_i - x) f_i(x)dx + C_{3i}\int_{Q_i}^{\infty}(x - Q_i) f_i(x)dx)) \quad \ldots.(1)$$

subject to the constraints

$$\sum_{i=1}^{n} f_i Q_i \le F$$
$$\sum_{i=1}^{n} p_i Q_i \le B$$
$$Q_i \ge 0 \ (i = 1, 2, \ldots., n).$$

1.2 Fuzzy Single Objective Stochastic Inventory Model [FSOSIM]

Fuzziness means imprecision of meaning of an object. We know that in practical information (specially, in a system of communication) is intrinsically statistical in nature and is dealt with probability theory. But in many applications meaning of information is more important. Meaning of information is a fuzzy concept, which in some respect quite different from the concept of probability. Fuzziness may occur not only because certain phenomena or relationships are vague but also because of abundance or lacking of information. Thus, when the above budget B for replenishment becomes fuzzy, the model (1) is transformed to a Fuzzy Single Objective Stochastic Inventory Model [FSOSIM].

The model can be stated as:

$$M\tilde{ax}\ PF(Q_1, Q_2, \ldots\ldots, Q_n) = (RS\ (Q_1, Q_2, \ldots\ldots, Q_n) - IC\ (Q_1, Q_2, \ldots\ldots, Q_n))$$

$$= \sum_{i=1}^{n}((s_i \int_0^{Q_i} x f_i(x)dx + s_i Q_i \int_{Q_i}^{\infty} f_i(x)dx + L_i \int_0^{Q_i}(Q_i - x) f_i(x)dx) - (p_i Q_i + \frac{C_{1i}Q_i}{2}$$

$$\frac{C_{1i}}{2}\int_0^{Q_i}(Q_i - x) f_i(x)dx + C_{3i}\int_{Q_i}^{\infty}(x - Q_i) f_i(x)dx)) \quad \ldots.(2)$$

subject to the constraints

$$\sum_{i=1}^{n} f_i Q_i \le F$$

$$\sum_{i=1}^{n} p_i Q_i \le \tilde{B}$$

$$Q_i \ge 0\ (i = 1, 2, \ldots., n).$$

(Here wavy bar '~' indicates "fuzzification" of the parameters).

1.3 Multi Objective Stochastic Inventory Model [MOSIM]

Traditional single objective linear or non-linear programming problem aims at optimization of the performance in terms of combination of resources. In reality, a managerial problem of a responsible organization involves several conflicting objectives to be achieved simultaneously subject to a system of restrictions (constraints) that refers to a situation on which the DM has no control. For this purpose a latest tool is linear or non-linear programming problem with multiple conflicting objectives. So the following model may be considered:

Maximize total revenue earn as well as minimize total inventory related cost under limited storage space and budgetary constraints. It is a Multi-Objective Stochastic Inventory Model [MOSIM].

To solve the problem (1) as a MOSIM, it can be reformulated as:

$$\text{Max RS } (Q_1, Q_2, \ldots\ldots, Q_n) = \sum_{i=1}^{n}((s_i \int_0^{Q_i} x f_i(x)dx + s_i Q_i \int_{Q_i}^{\infty} f_i(x)dx + L_i \int_0^{Q_i}(Q_i - x) f_i(x)dx)$$

$$\text{Min IC } (Q_1, Q_2, \ldots, Q_n) = \sum_{i=1}^{n}((p_i Q_i + \frac{C_{1i}Q_i}{2} +$$

$$\frac{C_{1i}}{2}\int_0^{Q_i}(Q_i - x) f_i(x)dx + C_{3i}\int_{Q_i}^{\infty}(x - Q_i) f_i(x)dx)) \quad \ldots.(3)$$

subject to the constraints,

$$\sum_{i=1}^{n} f_i Q_i \le F$$

$$\sum_{i=1}^{n} p_i Q_i \le B$$

$$Q_i \ge 0\ (i = 1, 2, \ldots., n)$$

2.Stochastic Models with Exponential Demand

We assume that demand for the period for the i^{th} item is a random variable that follows exponential distribution. Then the probability density function $f_i(x)$ are given by:

$$f_i(x) = \lambda_i e^{(-\lambda_i x)} \qquad x>0 \qquad \textit{for } i = 1, 2, \dots, n.$$
$$= 0 \qquad \text{otherwise}$$

So, RS ($Q_1, Q_2, \dots, Q_n$) $= \sum_{i=1}^{n}((s_i\int_0^{Q_i} x f_i(x)dx + s_iQ_i\int_{Q_i}^{\infty} f_i(x)dx + L_i\int_0^{Q_i}(Q_i-x)f_i(x)dx))$

$$= \sum_{i=1}^{n}(\frac{s_i}{\lambda_i}+\frac{L_i}{\lambda_i}+L_iQ_i - e^{-\lambda_iQ_i}(\frac{s_i}{\lambda_i}+2\lambda_iQ_i+\frac{L_i}{\lambda_i}))$$

IC ($Q_1, Q_2, \dots, Q_n$) $= \sum_{i=1}^{n}((p_iQ_i + \frac{C_{1i}Q_i}{2}$

$$+(\frac{C_{1i}}{2})\int_0^{Q_i}(Q_i-x)f_i(x)dx + C_{3i}\int_{Q_i}^{\infty}(x-Q_i)f_i(x)dx))$$

$$= \sum_{i=1}^{n}(p_iQ_i + C_{1i}Q_i + \frac{C_{1i}}{2\lambda_i} + e^{-\lambda_iQ_i}(C_{3i}Q_i + \frac{C_{3i}}{\lambda_i} - C_{1i}Q_i - \frac{C_{1i}}{2\lambda_i}))$$

3.Mathematical Analysis

3.1 Single Objective Non-Linear Programming Problem [SONLP]

A Single Objective Nonlinear Programming Problem may be taken in the following form:

Min f(x) ….(4)

subject to $x \in X = \{x = (x_1, x_2, \dots, x_n) \in R^n : g_j(x) \le b_j$ for $j = 1, 2, \dots, m$

and $l_i \le x_i \le u_i$ $(i = 1, 2, \dots, n)\}$.

where, x is an n-dimensional vector of decision variables, f and g_j (j = 1, 2, …, m), are given real valued non-linear functions of n variables $x_1, x_2, \dots, x_n$ and X is the feasible set of the constraints.

3.2 Fuzzy Single Objective Non-Linear Programming Problem [FSONLP]

A general Non-Linear Programming Problem with a fuzzy objective function and fuzzy constraints may be taken in the following form:

$M\tilde{in}f(x)$ ….(5)

subject to $x \in X = \{x = (x_1, x_2, \dots, x_n) \in R^n : g_j(x) \lesssim b_j$ for $j = 1, 2, \dots, m$;

$h_r(x) \le b_r'$ $(r = 1, 2, \dots, p)$ and $l_i \le x_i \le u_i$ $(i = 1, 2, \dots, n)\}$.

where, x is an n-dimensional vector of decision variables, f , h_r (r=1, 2, …,p) and g_j (j = 1, 2, …, m), are given real valued non-linear functions of n variables $x_1, x_2, \dots, x_n$ and X is the feasible set of the constraints.

Here the symbol ‘ $Mi\tilde{n}$ ’ denotes a relaxed or fuzzy version of ‘ Min ’ with the interpretation that the objective function should be minimize as much as possible under the given constraints. Again the symbol ‘$\lesssim$’ denotes a relaxed or fuzzy version of the order inequality ‘ $\le$ ’.

Here fuzzy objective function $M\tilde{in}f(x)$ is a fuzzy set (f(x), μ_0(f(x))) with membership function:

$$\mu_0 f(x) = \begin{cases} 1 & \text{if } f(x) \le b_0 \\ \dfrac{b_0 + p_0 - f(x)}{p_0} & \text{if } b_0 \le f(x) \le b_0 + p_0 \\ 0 & \text{if } f(x) > b_0 + p_0 \end{cases}$$

and fuzzy constraints $g_j(x) \lesssim b_j$ (j=1, 2, ……, m) are fuzzy sets ($g_j(x)$, $\mu_j(g_j(x))$) with membership function:

$$\mu_j(g_j(x)) = \begin{cases} 1 & \text{if } g_j(x) \le b_j \\ \dfrac{b_j + p_j - g_j(x)}{p_j} & \text{if } b_j \le g_j(x) \le b_j + p_j \\ 0 & \text{if } g_j(x) > b_j + p_j \end{cases}$$

[j=1, 2, …., m]

According to Bellman and Zadeh's (1970) fuzzy decision making process and following Zimmermann (1976), we have the crisp NLP problem as:

$$Max\ \mu_0 f(x)$$

subject to $\mu_j(g_j(x)) \ge \mu_0(f(x))$ for j=1, 2, …..,m

$h_r(x) \le b_r'$ for r =1, 2, …, p

$x \ge 0$.

3.3 Multi-Objective Non-Linear Programming Problem [MONLP]

A Multi-Objective Non-Linear Programming (MONLP) or Vector Minimization Problem (VMP) may be taken in the following form:

$$\text{Min } f(x) = [f_1(\mathbf{x}), f_2(x), f_3(x), \ldots, f_k(x)]^T$$

Subject to $x \in X = \{x \in R^n : g_j(x) \le \text{ or } = \text{ or } \ge b_j$ for $j = 1, \ldots, m$ ….(6)

and $l_i \le x_i \le u_i$ $(i = 1, 2, \ldots, n)\}$.

Zimmermann (1978) showed that fuzzy programming technique could be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used:

STEP 1: Solve the MONLP problem (6) as a single objective non-linear programming problem using only one objective at a time and ignoring the others; these solutions are known as ideal solution.

STEP 2: From the result of step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c|cccc} & f_1(x) & f_2(x) & \cdots\cdots & f_k(x) \\ \hline x^1 & f_1^*(x^1) & f_2(x^1) & \cdots & f_k(x^1) \\ x^2 & f_1(x^2) & f_2^*(x^2) & \cdots & f_k(x^2) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x^k & f_1(x^k) & f_2(x^k) & \cdots & f_k^*(x^k) \end{array}$$

Here $x^1, x^2, \ldots, x^k$ are the ideal solutions of the objective functions $f_1(x), f_2(x), \ldots, f_k(x)$ respectively.

So $U_r = \max\{f_r(x^1), f_r(x^2), \ldots, f_r(x^k)\}$

and $L_r = f_r^*(x^r)$.

[L_r and U_r be lower and upper bounds of the r^{th} objective functions $f_r(x)$ for r = 1, 2, ……,k].

STEP 3: Using aspiration level of each objective of the MONLP problem (6) may be written as follows:

Find x so as to satisfy

$f_r(x) \ \tilde{\lessgtr}\ L_r$ (for r = 1, 2,…., k)

$x \in X$

Here objective functions of (6) are considered as fuzzy constraints. These types of fuzzy constraints can be quantified by eliciting a corresponding membership function:

$\mu_r[f_r(x)] = 0 \text{ or } \to 0$ if $f_r(x) \ge U_r$

$= d_r(x)$ if $L_r \le f_r(x) \le U_r$ $(r = 1, 2, \ldots, k)$ ….(7)

$= 1 \text{ or } \to 1$ if $f_r(x) \le L_r$

Here $d_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$ (r=1, 2, …, k).

Having elicited the membership functions (as in (7)) $\mu_r[f_r(x)]$ for r = 1, 2, …… , k, introduce a general aggregation function

$$\mu_{\tilde{D}}(x) = \mu_{\tilde{D}}(\mu_1(f_1(x)), \mu_2(f_2(x)), \ldots\ldots, \mu_k(f_k(x))).$$

So a fuzzy multi-objective decision making problem can be defined as

Max $\mu_{\tilde{D}}(x)$ ….(8)

subject to
$x \epsilon X$.

Here we adopt the fuzzy decision as:

Fuzzy decision based on minimum operator (like Zimmermann's approach (1976)). In this case (6) is known as $FNLP_M$.

STEP 4: Solve (8) to get optimal solution.

3.4 General Fuzzy Non-linear Programming [GFNLP] technique to solve Multi-Objective Non-Linear Programming Problem [MONLP]

A Multi-Objective Non-Linear Programming (MONLP) problem or Vector Minimization problem (VMP) may be taken in the following form:

$$\text{Min } f(x) = [f_1(\mathbf{x}), f_2(x), f_3(x), \ldots, f_k(x)]^T \quad \ldots.(9)$$

Subject to $x \in X = \{x \in R^n : g_j(x) \leq \text{ or } = \text{ or } \geq b_j \text{ for } j = 1, \ldots\ldots, m$
and $l_i \leq x_i \leq u_i \quad (i = 1, 2, \ldots\ldots, n)\}$.

Zimmermann (1978) showed that fuzzy programming technique could be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used:

STEP 1: Solve the MONLP problem (9) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

STEP 2: From the result of step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \cdots \\ x^k \end{array} \begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & \ldots\ldots & f_k(x) \end{array} \\ \begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \ldots & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & \ldots & f_k(x^2) \\ \ldots & \ldots & \ldots & \ldots \\ f_1(x^k) & f_2(x^k) & \ldots & f_k^*(x^k) \end{bmatrix} \end{array}$$

Here $x^1, x^2, \ldots\ldots, x^k$ are the ideal solutions of the objective functions $f_1(x), f_2(x), \ldots, f_k(x)$ respectively.

$$\text{So } U_r = \max \{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$$
$$\text{and } L_r = \min \{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$$

[L_r and U_r be lower and upper bounds of the r^{th} objective functions $f_r(x)$ for $r = 1, 2, \ldots\ldots, k$].

STEP 3: Using aspiration level of each objective of the MONLP problem (9) may be written as follows:

Find x so as to satisfy

$f_r(x) \ \tilde{\leq} \ L_r$ (for $r = 1, 2, \ldots, k$)

$x \varepsilon X$

Here objective functions of (9) are considered as fuzzy constraints. These type of fuzzy constraints can be quantified by eliciting a corresponding membership function:

$$\mu_r^{w_r}[f_r(x)] = \begin{cases} 0 \text{ or } \to 0 & \text{if } f_r(x) \geq U_r \\ w_r\,\mu_r(f_r(x)) & \text{if } L_r \leq f_r(x) \leq U_r \ (r = 1, 2, \ldots\ldots, k) \\ w_r & \text{if } f_r(x) \leq L_r \end{cases} \quad \ldots.(10)$$

Here w_r are the weights $\mu_r(f_r(x))$ is a strictly monotonic decreasing function with respect to $f_r(x)$ (r=1, 2, …, k).

Having elicited the membership functions (as in (10)) $\mu_r^{w_r}$ $[f_r(x)]$ for $r = 1, 2, \ldots\ldots, k$, introduce a general aggregation function

$$\mu_{\tilde{D}}^{w}(x) = G(\mu_1^{w_1}(f_1(x)), \mu_2^{w2}(f_2(x)), \ldots\ldots\ldots, \mu_k^{w_k}(f_k(x))).$$

So a fuzzy multi-objective decision making problem can be defined as

Max $\mu_{\tilde{D}}^{w}(x)$

subject to $x \in X$ ….(11)

Here we adopt Fuzzy decision based on minimum operator (like Zimmermann's approach (1978). In this case (11) is known as $FNLP_M$.

Then the problem (11), using the membership function as in (10), according to min-operator is reduced to:

$$\begin{aligned} &\text{Max } \alpha \\ &\text{Subject to } \mu_i^w [f_i(x)] \geq \alpha \quad \text{for } i = 1, 2, \ldots\ldots, k \\ &x \in X, \quad \alpha \varepsilon [0, w], w \varepsilon (0,1] \end{aligned}$$

$w = \min(w_1, w_2, \ldots, w_k)$ ….(12)

STEP 4: Solve (12) to get optimal solution.

4.Solution of Proposed Model

4.1 Single Objective Stochastic Inventory Model [SOSIM]

To solve this model in (1), in this paper we use computer software.

4.2 Fuzzy Single Objective Stochastic Inventory Model [FSOIM]

In fuzzy set theory, the fuzzy constraint is defined by its membership function, which may be linear or non-linear. Here we assume μ_B ($Q_1, Q_2, \ldots, Q_n$) and μ_{PF} ($Q_1, Q_2, \ldots, Q_n$) to be the linear membership functions for the second constraint and for the objective function of (2) respectively. Then,

$$\mu_B(Q_1, Q_2, \ldots\ldots, Q_n) = \begin{cases} 0 & \text{for } \sum_{i=1}^{n} p_i Q_i > B + \delta_B \\ 1 - \dfrac{\sum_{i=1}^{n} p_i Q_i - B}{\delta_B} & \text{for } B \le \sum_{i=1}^{n} p_i Q_i \le B + \delta_B \\ 1 & \text{for } \sum_{i=1}^{n} p_i Q_i < B \end{cases}$$

Here δ_B is the maximum acceptable violation of the aspiration level B.

$$\mu_{PF}(Q_1, Q_2, \ldots\ldots, Q_n) = \begin{cases} 0 & \text{for } PF(Q_1, Q_2, \ldots\ldots, Q_n) < PF' \\ \dfrac{PF(Q_1, Q_2, \ldots\ldots, Q_n) - PF'}{PF^0 - PF'} & \text{for } PF' \le PF(Q_1, Q_2, \ldots\ldots, Q_n) \le PF^0 \\ 1 & \text{for } PF(Q_1, Q_2, \ldots\ldots, Q_n) > PF^0 \end{cases}$$

Where PF^0 and PF' are the objective values when the problem (1) is solved by taking the budget B and B+δ_B respectively. Thus the following crisp NLP problems are obtained:

Max$\mu_B(Q_1, Q_2, \ldots, Q_n)$

subject to

$\mu_{PF}(Q_1, Q_2, \ldots, Q_n) \ge \mu_B(Q_1, Q_2, \ldots, Q_n)$ ….(13)

$\sum_{i=1}^{n} f_i Q_i \le F$

$Q_i \ge 0$ for i = 1, 2, ……., n.

OR

Max$\mu_{PF}(Q_1, Q_2, \ldots, Q_n)$ ….(14)

subject to

$\mu_B(Q_1, Q_2, \ldots, Q_n) \ge \mu_{PF}(Q_1, Q_2, \ldots, Q_n)$

$\sum_{i=1}^{n} f_i Q_i \le F$

$Q_i \ge 0$ for i = 1, 2, …., n.

4.3 Fuzzy Programming Technique to solve MOSIM

To solve MOSIM (3) problem, step 1 of section 3.3 is used. After that according to step 2 Pay-off matrix is formulated as follows:

$$\begin{array}{c} \\ Q^1 \\ Q^2 \end{array} \begin{array}{cc} RS(Q_1, Q_2, \ldots, Q_n) & IC(Q_1, Q_2, \ldots, Q_n) \\ \left[\begin{array}{c} RS^*(Q_1^{\,1}, Q_2^{\,1}, \ldots, Q_n^{\,1}) \\ RS(Q_1^{\,2}, Q_2^{\,2}, \ldots, Q_n^{\,2}) \end{array}\right. & \left.\begin{array}{c} IC(Q_1^{\,1}, Q_2^{\,1}, \ldots, Q_n^{\,1}) \\ IC^*(Q_1^{\,2}, Q_2^{\,2}, \ldots, Q_n^{\,2}) \end{array}\right] \end{array}$$

Now, U_1, L_1, U_2, L_2 (where $L_1 \le RS(Q_1, Q_2, \ldots\ldots, Q_n) \le U_1$ and $L_2 \le IC(Q_1, Q_2, \ldots\ldots, Q_n) \le U_2$) are identified and $Q^1 = (Q_1^{\,1}, Q_2^{\,1}, \ldots, Q_n^{\,1})$, $Q^2 = (Q_1^{\,2}, Q_2^{\,2}, \ldots, Q_n^{\,2})$ are the ideal solutions of the objective functions $RS(Q_1, Q_2, \ldots\ldots, Q_n)$ and $IC(Q_1, Q_2, \ldots\ldots, Q_n)$.

Here, for simplicity, fuzzy linear membership functions $\mu_{RS}(Q_1, Q_2, \ldots\ldots, Q_n)$ and $\mu_{IC}(Q_1, Q_2, \ldots\ldots, Q_n)$ for the objective functions RS and IC respectively are identified as follows:

$$\mu_{RS}(Q_1, Q_2, \ldots\ldots, Q_n) = \begin{cases} 0 & \text{for } RS(Q_1, Q_2, \ldots\ldots, Q_n) \le L_1 \\ \dfrac{RS(Q_1, Q_2, \ldots\ldots, Q_n) - L_1}{U_1 - L_1} & \text{for } L_1 \le RS(Q_1, Q_2, \ldots\ldots, Q_n) \le U_1 \\ 1 & \text{for } RS(Q_1, Q_2, \ldots\ldots, Q_n) \ge U_1 \end{cases}$$

$$\mu_{IC}(Q_1, Q_2, \ldots\ldots, Q_n) = \begin{cases} 0 & \text{for } IC(Q_1, Q_2, \ldots\ldots, Q_n) \le L_2 \\ \dfrac{U_2 - IC(Q_1, Q_2, \ldots\ldots, Q_n)}{U_2 - L_2} & \text{for } L_2 \le IC(Q_1, Q_2, \ldots\ldots, Q_n) \le U_2 \\ 1 & \text{for } IC(Q_1, Q_2, \ldots\ldots, Q_n) \ge U_2 \end{cases}$$

According to step 3 of section 3.3 having elicited the above membership functions crisp linear programming problems are formulated as follows:

(Following to (8))

Max $\mu_{RS}(Q_1, Q_2, \ldots\ldots, Q_n)$ ….(15)

subject to

$$\mu_{IC}(Q_1, Q_2, \ldots\ldots, Q_n) \ge \mu_{RS}(Q_1, Q_2, \ldots\ldots, Q_n)$$

$$\sum_{i=1}^{n} f_i Q_i \le F$$

$$\sum_{i=1}^{n} p_i Q_i \le B$$

$$0 \le \mu_{RS}(Q_1, Q_2, \ldots\ldots, Q_n),\ \mu_{IC}(Q_1, Q_2, \ldots\ldots, Q_n) \le 1.$$

$$Q_i \ge 0 \quad \text{for} \quad i = 1, 2, \ldots, n$$

OR

Max $\mu_{IC}(Q_1, Q_2, \ldots\ldots, Q_n)$ ….(16)

subject to

$$\mu_{RS}(Q_1, Q_2, \ldots\ldots, Q_n) \ge \mu_{IC}(Q_1, Q_2, \ldots\ldots, Q_n)$$

$$\sum_{i=1}^{n} f_i Q_i \le F$$

$$\sum_{i=1}^{n} p_i Q_i \le B$$

$$0 \le \mu_{RS}(Q_1, Q_2, \ldots\ldots, Q_n),\ \mu_{IC}(Q_1, Q_2, \ldots\ldots, Q_n) \le 1.$$

$$Q_i \ge 0 \quad \text{for} \quad i = 1, 2, \ldots, n$$

Solve the crisp non-linear programming problem (15), (16) by an appropriate mathematical programming algorithm.

5. Numerical Illustration: (Demand follows exponential distribution)

Example 1.

The Dolphin fish stall sells mainly four varieties of rare fishes per kg. at the rate of $28, $15, $15 and $25, respectively. If the stall fails to sell according to requirement it pays for shortages at the rate of $28, $15, $10 and $25 per kg., respectively. The stall has to purchase per kg. of such fishes respectively by $20, $10, $10, $20. The stall has only 4000 sq.ft. of storage area and the annual budget is $7000. The respective floor spaces per unit are 10 sq.ft., 5 sq.ft., 5 sq.ft. and 15 sq.ft., whereas, the salvage values per unit are $15, $3, $3, $15, respectively. The stall uses inventory carrying charges of $4, $2, $2 and $4 per unit per year respectively. If the demand x per year per unit follows exponential distribution where the density function is:

$f_i(x) = \lambda_i e^{(-\lambda_i x)}$, $x>0$

$= 0$, otherwise (for i =1, 2, 3 and 4)

Where, $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_3 = 0.03$, $\lambda_4 = 0.04$, then determine the optimal lot size for each variety of fishes and the maximum expected profit.

[For convenience it is taken that the optimal lot sizes lie from 30 to 800]

Example 2.

If in the above problem the annual budget is about to $7000 but not more than $7500 then determine the optimal lot size for each variety of fishes and the maximum expected profit.

The optimal solutions of example 1 and example 2 corresponding to SOSIM (1) and FSOSIM (2) are shown below in Table – 1:

Optimal solutions of SOSIM (1) and FSOSIM (2) with exponential demand

MODEL	Q_1^*	Q_2^*	Q_3^*	Q_4^*	$PF^*(\$)$	ASPIRATION LEVEL
SOSIM	30.0000	314.7552	199.4229	62.91095	204.1545	---
FSOSIM	30.0000	327.3050	207.7814	63.23588	230.4945	$\mu_{PF}=0.5688$ $\mu_B=0.5684$

Table – 1

Example 3.

In the above problem, considering the Maximum Revenue earn from sales is about $11,450 with degree of acceptance 0.8 and maximum tolerance limit is $11,455 as well as Minimum Inventory related cost is about $10,850 with degree of acceptance 0.9 and maximum tolerance limit is $10,855 in the above problem, determine the Maximum Revenue earn from sales, Minimum Inventory related cost, optimal lot sizes keeping equal importance on Maximum Revenue earn from sales and Minimum Inventory related cost.

Considering the above problem determine the maximum total revenue earn, minimum total inventory related cost and the optimal lot size for each type of Dolphin fishes, keeping equal importance on total revenue earn as well as total inventory related cost.

The optimal solutions of Example 3 corresponding to MOSIM (3) with equal weights by FNLP methods and GFNLP methods are shown below in Table – 2.

Optimal solutions of MOSIM (3) with equal weights by FNLP and GFNLP methods

METHODS	Q_1^*	Q_2^*	Q_3^*	Q_4^*	$RS^*(\$)$	$IC^*(\$)$	ASPIRATION LEVEL
FNLP	106.9961	146.7374	139.2704	100	11,448.42	10854.32	$\mu_{RS}=0.8621$ $\mu_{IC}=0.8472$
GFNLP	107.2103	145.9873	139.5921	100	11454.87	10850.71	$\mu_{RS}=0.8782$ $\mu_{IC}=0.8362$

Table – 2

6. Conclusion

In this paper, our objective is to establish the better performance of the General Fuzzy Non Linear Programming Technique i.e. to prove that General Fuzzy Non Linear Programming (GFNLP) Technique optimizes the objective function more than the usual Fuzzy Non Linear Programming (FNLP) Technique. In Table 2 in case of the MOSIM the objective function RS (Revenue earn from Sales) is more maximized and IC (Inventory related Cost) is more minimized under GFNLP technique than the FNLP technique

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