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Inverse Optimization: An Application Based Survey

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Abstract

The inverse optimization problem determines the values of parameters of optimization problem i.e. cost coefficients, coefficient matrix, right hand side vector etc. that make the given feasible solution optimal. In this paper, we have presented various application of inverse optimization problems reported in the literature.

Keywords: Inverse Optimization; Inverse Problem

Introduction

A variety of real life problems can be formulated as a mathematical programming problem and solved by using suitable techniques. Whenever we model these problems mathematically, it is assumed that all the parameters associated with the problem are known exactly and we wish to find the solution which is optimal for the present values of parameter. However, in practice, there are many situations when we are not very much sure about these parameters or we only have some estimates of these parameters, but we may have a solution from the observation, experiment or experiences. The known solution may or may not be optimal for the present values of parameters, so we need to adjust these parameters to make the given solution optimal. This problem can be considered as an inverse problem, but whenever we talk about optimization, we always look for the best solution i.e. the adjustment of the parameters should be minimum or the cost associated behind it should be minimum. Thus, an inverse optimization problem is to find the values of parameters (cost, capacity, travel time, etc.) which makes the given solution optimum and which are differ from the given parameters as little as possible.

A Variety of Inverse Optimization problems have been Investigated in past few years. From the year 1992 to 2000 the most of the researchers had considered inverse optimization of various combinatorial optimization problems e.g. Inverse shortest path problem, spanning tree problem, minimum cost flow problem, minimum cut problem, maximum flow problem etc. From 2000 to 2014, mainly linear programming problem, quadratic programming problem, integer programming problem and fractional programming problems were considered for inverse optimization. From 2014 onward the inverse optimization of Multi objective optimization problem, non linear optimization problems, optimization with noisy data etc have been reported.

Mathematical Formulation

Let us consider the following optimization problem

$$\min \quad f(c,x)$$

$$\text{s.t.} \quad x \in S \qquad (1)$$

where $S \in R^n$ is the set of feasible solutions, $c \in R^n$ a given cost vector. If $\hat{x} \in S$ is the given feasible solution, $\hat{c} \in R^n$ is the perturbed value of c and $\|.\|$ be some norm then the inverse problem to (1) is

$$\min \quad \|\hat{c} - c\|$$

$$\begin{aligned} \text{s.t.} \quad & f(\hat{c},\hat{x}) = \min\{f(\hat{c},x): x \in S\} \\ & l \le \hat{c} \le u \qquad (2) \\ & \hat{c} \in R^n \end{aligned}$$

Applications of Inverse Optimization

1. Geophysical Sciences

In the problems related to geophysical sciences, the model parameters (such as the radius of Earth's metallic core) are very difficult or impossible to determine, but we may have some estimates of these parameters and the values of these parameters can be improve with the help of the values of observable parameters. An important application based on inverse shortest path problem in the area of geophysical sciences is, predicting the movement of earthquakes (see Torntola [36], Burton [35]). In the model of earthquake movements, the geographical zone is discretized in a number of square cells, where the transmission time of certain seismic waves from corresponding cells is not known accurately, but we have some estimates of it. When earthquake is observed and the arrival time of resulting seismic perturbations at various points are observed and it is assume that earthquake travel along shortest path, then the problem is to refine the estimates of the transmission times between the cells.

1. Medical Imaging

A useful application of inverse problem is arise in X-ray tomography [37, 38], where the dimension of a body part is estimated by the observation from a CT- scan of that body part together with a priori knowledge of the body.

2. Traffic Equilibrium

In a transportation network, users generally select the route (or flow) that minimize their travel cost (or time), this type of flow in a network (road or rail) is called user equilibrium flow, where no user can decrease his/her cost by changing his/her route. If a transportation planner wants to enforce the user to use a particular flow (route) then this flow is called the system optimal flow. The user equilibrium flow may or may not be equal to the system optimal flow, in the case when these are not equal, a minimum toll can be imposed on some road segment of the route, so that the user equilibrium flow become identical with the system optimal flow. The problem is an example of inverse shortest path problem and is reported by Burton [35], Dial [1,2].

Another application of inverse optimization in transportation planning is the inverse maximum capacity path problem. For example, if a transportation planner wants to make a particular path between two cities, the maximum capacity path between them, where the capacity of the path is defined as the minimum capacity of the arc on the path, then by using inverse optimization, the capacity of the arc of the network are adjusted as little as possible so that the given path becomes the maximum capacity path. The problem is an example of inverse maximum capacity path problem reported by Yang and Zhang [3].

3. Location Problem

A location problem is to find the best place to install a facility or to build a centre for the system in a given network (road network), so that the total cost, including setup cost and transportation cost is minimum. However, there are many situations when the centre has already been constructed or the facility has already been fixed at certain place, and we wish to modify the network system (improving road, upgrading travelling tools) as little as possible so that the given location becomes the centre (optimal) location. Zhang et al. [4] identified these types of problems, called inverse centre location problem. This type of problems have wide applications such as locations of fire stations, hospitals, police stations, schools, shopping centers, warehouses and many more

4. Production planning problem

Optimal production planning is an important process of supply chain management. In the present scenario, production planning is not only concern with the allocation of resources during the production task, to maximize profit or minimize cost, but also with organizing resources to fulfil the sudden market demand most efficiently. As we know that a numerous production planning problems can be formulated as a linear programming problem and we solve them for minimizing cost or maximizing profit, assuming that the resources are rigid in nature. However, the practical situations are quite different where we have to fulfil the market demand, which is not possible due to rigid treatment of resources. In these situations the theory of inverse optimization for linear programming problems comes into picture.

The inverse linear programming problem (Ahuja and Orlin [37], Zhang and Liu [5]) overcome the deficiency of the existing linear programming problem and provide the flexibility over the allocation of resources during the production task to match the production with the present market demand.

5. Data Envelopment Analysis (DEA)

In the present scenario, DEA become the main tool to management scientists for analysis of organizational performance. It has a wide range of applications in measuring comparative efficiency of multiple inputs and outputs of a homogeneous set of decision making units (DMUs).

The inverse DEA problem can be described as: if we increase certain input to a particular unit, among the group of decision making unit and assume that the decision making unit (DMU) maintain its current efficiency with respect to other units then how much more outputs could the unit produce? Or if we want to increase the output to a certain level in such a way that the efficiency of the unit remains same then how much more inputs should be provided to the unit? Using the inverse DEA models the DEA model become more flexible and applicable to handle the practical situations. These types of problem are reported by Amin [6], Amin and Emrounejad [7] and Wei et al. [8].

6. Time series and Forecasting

Time series analysis and forecasting have various applications in the field of research such as data mining, business, economics, engineering, medicine, politics and many others. Application of inverse optimization in time series analysis and forecasting are reported by Amin and Emrounejad [9]. Inverse forecasting model can be used to estimate the forecasting parameters much accurately then the parameters estimated by ordinary least square based method.

7. Image Segmentation and Error correction

In transmission or communication, an image can be degraded by noise. There is an underlying assumption that must be obeyed by a correct image and the assumption is that a correct image tends to have area of uniform colors. Using this assumption, we can reset or modify the values of colors of the pixels so that the sum of deviation from the given colors (in the degraded image) plus the penalty for different adjacent colors is minimum. This is an example of inverse problem addressed by Hochbaum [38].

8. Political Gerrymandering

Gerrymandering is concern with the rearrangement of the boundary of the constituency to influence the outcome of election. In political gerrymandering [38], the goal is to modify the certain outcome while taking into account population numbers segmented as per various political options, and limitations on the geometry of the boundaries.

9. Portfolio Optimization

An investor wants to make decision about an investment on n pre-selected risky assets, for instance, n stocks in a stock market. Let $r \sim N(\mu, G)$ denote the random return vector of the n risky assets that are normally distributed with $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T$ as the expected return vector of the n assets and $G = (\sigma_{ij}) \in R^{n\times m}$ as the covariance matrix among the asset returns, where the matrix R is at least positive semi-definite (usually positive definite), σ_{ij}, $i, j = 1,2, \ldots, n$, are covariance between returns of assets i and j, and $\sigma_{ii} = {\sigma_i}^2$ the variance of i^{th} asset's return. Let x_i denote the investment proportion on risky asset i for $i = 1,2, \ldots, n$ then the portfolio selection problem for one single period is to find the values x_i's so as to maximize the expected return from the investment and minimize the risk of investor. This portfolio optimization problem is formulated as different mathematical programming problems by Iyenger and Kang [10], Zhang and Xu [11].

Let $(\hat{\mu}, \hat{G})$ be the current estimate of (μ, G), $\hat{x}$ is the optimal solution of the portfolio optimization problem. Suppose new observations of daily return on these stocks becomes available and μ_1 and G_1 are the new estimates of μ and G then the investor face the question that the portfolio $\hat{x}$ is still regarded as efficient or some adjustments should be made on the portfolio $\hat{x}$. Markets typically have transaction costs, therefore, re-balancing from $\hat{x}$ to a new efficient portfolio would result in a loss. On the other hand, a new efficient portfolio is likely to have a higher return. Thus the investor needs to balance these two factors. This type of situation can be tackled with the help of inverse optimization [10, 11].

Applications of inverse optimization in choosing optimal portfolio and in other problems in finance are also reported by Zhang and Xu [11], Carr and Lovejoy [12], Dembo and Rosen [13], Hartley and Bakshi [14] and Bertsimas et al. [15].

10. Quality Control in Production System

A manufacturer wants to maximize the net profit of the production system with a limitation in the cost of maintaining the desired quality level. However, in practice, the adjustment in the quality level is more difficult to adjust the net profit per unit in some systems. If the quality levels at each period can be determined by the estimation of market demand accordingly, then the problem face by the manufacturer is to adjust the net profit of each period as little as possible to make the given quality levels optimal, which is an example of inverse problem reported by Zhang and Xu [11].

11. Production Capacity Planning

An important application of inverse optimization in production capacity planning problem is reported in [11]. A production capacity planning problem is to determine optimal server and machine allocation to optimize a chosen performance measure of the system. Two important types of production capacity planning problems are balancing and targeting. A balancing problem of optimal reallocation of capacity and minimization of work in progress is given by the following convex separable program:

Min $\sum_{i=1}^{n} c_i\, f_i(x_i)$

s.t. $\sum_{i=1}^{n} x_i = b_i$

$x_i \geq u_i,\ i = 1,2, \dots, n.$ (3)

Where, x_i denote the capacity available at station i, $b > 0$ is the total capacity available to allocate, c_i is the estimated average value of work-in-progress associated with each job at station i, $f_i(x_i)$ the mean number of jobs at station i, u_i the net rate of jobs arriving at station i and $u_i > 0$ for all i.

If $\hat{x}_i$, i = 1,2,…,n are optimal solution obtained from the previous estimates of c_i (say c_i') and $\sum_{i=1}^{n} \hat{x}_i = b_i$ i.e. the total capacity available to allocate is same as previous capacity. Now the problem is to re- distribute the capacity at each station according to the new estimations d_i which is differ from c_i' as little as possible. However, changing the capacities among stations needs to pay some cost, and on the other hand the total work in progress can be reduced due to re allocation of capacity. Now the new decision should be use or not, it's depend on the trade-off between the reallocation cost and reduction of work in progress value. This kind of situation can be handled by solving the inverse optimization problem of (3) in the similar way as done by Zhang and Xu [11].

12. Isotonic Regression

An important application of inverse optimization arises from an important problem of regression called isotonic regression. An isotonic regression problem is defined as follows: for a given $a \in R^n$, find $x \in R^n$ that minimize $\|x - a\|$ subject to constraints $x_1 \leq x_2 \leq \cdots \leq x_n$(isotonicity), where $\|.\|$ is some specified norm. Ahuja Oriln [3] have shown through an example that a priori knowledge of a system, where the observations are in non-increasing order, can be used together with the observations ($a_1, a_2,\dots,a_n$) to estimate the model parameters ($x_1, x_2, \dots, x_n$), with the help of isotonic regression. They also reported the applications of isotonic regression in inverse sorting problem and job shop scheduling.

13 Transportation and logistics

There are many transportation models based on inverse optimization reported by researchers. Applications of these models have been observed in improving route recommendation[20,35] and Path routing problems[21].

14 Healthcare Systems

Many important applications of inverse optimization is been observed in the field of healthcare systems. Chan et al. [22] propose an inverse optimization approach to develop such a metric, which was applied to stage III colon cancer patients. Ayer [23] proposed an inverse optimization based model for Breast cancer screening. Moreover, Some other applications on radiation therapy treatment planning [13,24-27] and liver transplantation[16-19] are also been reported in the litrature.

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15. Miscellaneous Applications

There are many more applications of inverse optimization in a variety of real life situations. We are presenting some of them very briefly.

- Day et al. [39] shown that the inverse shortest path problem can be applied in the management of railroad impedance for shortest path based routing.
- Hochbaum [38] has also reported some applications of inverse problems in Medical prognosis, Geophysical analysis, Protein Synthesis and just in time scheduling.
- Zhou et al. [29] Proposed an inverse optimization based model to increase investment into renewable energy.
- In the recent years, researchers has shown their interest in inverse optimization and its applications in power systems and electricity markets [30-34].

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