STUDY OF MULTIDIMENSIONAL FRACTIONAL OPERATORS INVOLVING GENERAL POLYNOMIALS AND A MULTIVARIABLE H-FUNCTION

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ABSTRACT:In this paper, we study multidimensional fractional integral operators whose kernels involve the product of a generalized polynomials$S_{N_1\dots N_s}^{M_1\dots N_s}[x_1,\dots,x_s]$and the multivariable H-function. here, we obtainthe images of certain useful functions in our operators of study. The fractional integral operators studied by us are most general in nature and may be considered as generalizations of a number of uni and multidimensional fractional operators studied from time to time by several authors. For the sake of illustration, we give here exact references of the results obtained by Goyal et al. [8], Rajni [7], Gupta et al. [6] The importance of the present study lies in the fact that it unifies and extends the results of a large number of authors.

Multidimensional fractional integral operators :

We shall study in this paper the multidimensional fractional integral operators defined by means of the following equations:

$I_x[f(t_1,\dots,t_s)] = I_{x\,;\,y;E}^{\rho,\sigma;\,h\,;\,u,v}[f(t_1,\dots,t_s);x_1,\dots,x_s]$(I transformation)

$$= \prod_{j=1}^{s} x_j^{-\rho_j-\sigma_j}\int_0^{x_1}\cdots\int_0^{x_s}\left(\prod_{j=1}^{s} t_j^{\rho_j}(x_j-t_j)^{\sigma_j-1}\right)$$

$$S_{N_1,\dots,N_s}^{M_1,\dots,M_s}\left[y_1\left(\frac{t_1}{x_1}\right)^{g_1}\left(1-\frac{t_1}{x_1}\right)^{h_1},\cdots,y_s\left(\frac{t_s}{x_s}\right)^{g_s}\left(1-\frac{t_s}{x_s}\right)^{h_s}\right]$$

$$H\begin{bmatrix} E_1\left(\frac{t_1}{x_1}\right)^{u_1}\left(1-\frac{t_1}{x_1}\right)^{v_1} \\ \vdots \\ E_s\left(\frac{t_s}{x_s}\right)^{u_s}\left(1-\frac{t_s}{x_s}\right)^{v_s}\end{bmatrix} f(t_1,\dots,t_s)\,dt_1\cdots dt_s \qquad (1.1)$$

The multivariable H - function occurring in the equations (1.1) and throughout the present paper is defined and represented *as* follows :

$$H\begin{bmatrix} z_1 \\ \vdots \\ z_s\end{bmatrix} = H_{p,q:p_1,q_1,\dots,p_s,q_s}^{o,n:m_1,n_1,\dots,m_s,n_s}\begin{bmatrix} z_1 \\ \vdots \\ z_s\end{bmatrix}$$

$$= H_{p,q:p_1,q_1,\dots,p_s,q_s}^{o,n:m_1,n_1,\dots,m_s,n_s}\left[\begin{matrix} z_1 \\ \vdots \\ z_s\end{matrix}\,\middle|\,\begin{matrix}\left(a_j,\alpha_j^{(1)},\dots,\alpha_j^{(s)}\right)_{1,p}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\dots;\left(c_j^{(s)},\gamma_j^{(s)}\right)_{1,p_s} \\ \left(b_j,\beta_j^{(1)},\dots,\beta_j^{(s)}\right)_{1,q}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\dots;\left(d_j^{(s)},\delta_j^{(s)}\right)_{1,q_s}\end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\phi_1(\xi_1)\cdots\phi_s(\xi_s)\psi(\xi_1,\cdots,\xi_s)z_1^{\xi_1}\cdots z_s^{\xi_s}\,d\xi_1\cdots d\xi_s. \qquad (1.2)$$

For the convergence, existence conditions and other details of the above multivariable H-function we refer to the book by Srivastave et al. [(2.pp. 251-253 eqns. (C.2)-(C.8)].

Again $S_{N_1\dots N_s}^{M_1\dots N_s}[x_1,\dots,x_s]$occurring in the definitions of our operators of study stands for the multivariable polynomials given by Srivastava [1, p . 185 , eqn. (7)) which will be defined and represented in the present paper in the following slightly modified form.

$$S_{N_1\ldots N_s}^{M_1\ldots N_s}[x_1,\ldots,x_s]=\sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1k_1}}{k_1!}\cdots\frac{(-N_s)_{M_sk_s}}{k_s!}A[N_1,k_1;\ldots,N_s,k_s]x_1^{k_1}\cdots x_s^{k_s}, \tag{1.3}$$

where $N\,i = 0, 1, 2, \ldots, s$; $M_i \neq 0[i = 1, \ldots, s]M_i$ is an arbitrary positive integer. The coefficients $A[N_i, k_i; \ldots; N_s, k_s]$ being arbitrary constants, real or complex.

If we take $s = 1$ in the equation (1.3) and denote $A\,[N, k]$ thus obtained by $A_{N,k}$, we arrive at the well know general class of polynomials introduced ,by Srivastava [3, p.158, eqn. (1.1)].

We shall assume in this paper that

$$f(t_1,\ldots,t_s)=\begin{cases}0\prod_{j=1}^{s}|t_j|^{U_j}max\{|t_j|\}\to 0\\ 0\prod_{j=1}^{\infty}\left(|t_j|^{-R_j}e^{-w_j|t_j|}\right)min\{|t_j|\}\to\infty\end{cases} j=1,\ldots,s \tag{1.4}$$

The operators defined by (1.1) exist if

(i) $\min \mathrm{Re}\,[1+\rho_j+U_j+u_j(\frac{d_k^{(j)}}{\delta_k^{(j)}})]>0, \min \mathrm{Re}\,[\sigma_j+v_j(\frac{d_k^{(j)}}{\delta_k^{(j)}})]>0$

$(k=1,\ldots,m_j, j=1,\ldots,s)$.

(ii) $(g_j, h_j, u_j, v_j) \geq ,0 (j = 1, \ldots\ldots s)$ (not all zero simultaneously)

Result 1:

In this section we shall obtain the following images in our operators of study

$$I_x\left[\prod_{j=1}^{s}t_j^{\lambda_j}(x_j-t_j)^{\eta_j}\right]=\left(\prod_{j=1}^{s}x_j^{\lambda_j+\eta_j}\right)\sum_{K_1=0}^{N_1/M_1}\cdots\sum_{K_s=0}^{N_s/M_s}\left(\prod_{j=1}^{s}y_j^{k_j}\frac{(-N_j)_{M_jK_j}}{K_j!}\right)$$

$$A[N_1,k_1;\ldots,N_s,k_s]H_{p,q:p_1,q_1+1;\ldots,p_s,q_s+1}^{o,n:m_1,n_1+2;\ldots,m_s,n_s+2}\begin{bmatrix}E_1\\ \vdots\\ E_s\end{bmatrix}\begin{matrix}\left(a_j,\alpha_j^{(1)},\ldots,\alpha_j^{(s)}\right)_{1,p}:\\ \left(b_j,\beta_j^{(1)},\ldots,\beta_j^{(s)}\right)_{1,q}:\end{matrix}$$

$$(-\rho_1-\lambda_1-g_1k_1,u_1)(1-\sigma_1-\eta_1-h_1k_1,v_1)\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\ldots;$$

$$\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1}(-\rho_1-\sigma_1-\lambda_1-\eta_1-(g_1+h_1)k_1,u_1+v_1);\ldots;$$

$$\left.\begin{matrix}(-\rho_s-\lambda_s-g_sk_s,u_s)(1-\sigma_s-\eta_s-h_sk_s,v_s)\left(c_j^{(s)},\gamma_j^{(s)}\right)_{1,p_s}\\ \left(d_j^{(s)},\delta_j^{(s)}\right)_{1,q_s}(-\rho_s-\sigma_s-\lambda_s-\eta_s-(g_s+h_s)k_s,u_s+v_s)\end{matrix}\right] \tag{2.1}$$

Provided that

$$\min \mathrm{Re}\left[1+\rho_j+\lambda_j+u_j\left(\frac{d_k^{(j)}}{\delta_k^{(j)}}\right)\right]>0; \min \mathrm{Re}\,[\sigma_j+\eta_j+v_j(\frac{d_k^{(j)}}{\delta_k^{(j)}})]>0$$

$$[j=1,\ldots,s; k=1,\ldots,m_j].$$

Proof: To prove (2.1), first of all we express the I-operators involved in its left hand side in the integral form with the help of equation (1.1). Next, we $S_{N_1...N_s}^{M_1...N_s}[x_1,...,x_s]$ polynomials in terms of series with the help of equation (1.3), now we interchange the order of series and t_j-integrals (j=1,...,s) and express the multivariable H-function in terms of the Mellin Barnes type contour integrals with the help (1.2). Now we change the order of ξ_j- and t_j-integrals (j=1,...,s) (which is permissible under the conditions stated). Finally, on evaluating the t_j-integrals and reinterpreting the result thus obtained in termns of the multivariable H-function, we easily arrive the desired result after a little simplification.

Special Cases:

If we take $s = 1, v_1 = 1, g_1 = u_1 = 0$ and $n = p = q = 0$ in our operators of study given by (1.1) and reduce the H-function thus obtained to generalized hyper geometric function $_pF_q$[2,p.18, eqn. (2.6.3)], we at once arrive at the operators, which are in essence same as the operator studied by Goyal et al. [(8) p.121, eqns. (1.3)] and our result reduce to the corresponding results obtained by them [(8) p.125, eqns. (3.6)].

Again, reducing the H-function of s-variables to the Kampé de Fériet function with the help of a known result (5, p.272, eqn. (4.7)] and this latter function to generalized hypergeometric function [(4), p.39, eqn.(30)] in our operators of study given by (1.1) we easily arrive at the fractional operators studied by Gupta et al. [(6) p.56, eqns. (1.4)] and our results 1, reduce to the corresponding results obtained by them[6, p.60-62, eqns. (3.2),]

If we take $u_j = 1$, $E_j = 1$, $v_j = 0$ and $h_j=0$ (j=1,…,s) and n=p=q=0 in our operators of study given by (1.1) and reduce the product of s-Fox's H-function thus obtained to product of s-Gauss hypergeometric functions [(2, p.19, eqn.(2.6.8)], we at once arrive at the operators which are in essence same as the operator studied by Rajni [(7) p.2, eqns. (1.2.1)] and our result 1, reduce to the corresponding theorems obtained by her [(7) , p.7 eqns. (1.3.1)].

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