The Impact of Slip Condition, Divider Properties and Warmth Exchange on Peristaltic Siphoning of Casson Liquid in a Tube

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Abstract

Warmth exchange and peristaltic wave engendering of a Casson liquid in a no uniform tube with divider properties has been explored under long wavelength and low Reynolds number suppositions. The articulations for speed, stream capacity and temperature are gotten systematically. The outcomes for speed, stream capacity and temperature acquired in the examination have been assessed numerically and talked about quickly. The impacts of yield pressure, versatility, slip and non-consistency parameters on the peristaltic siphoning are seen through diagrams. The results got for the peristaltic stream qualities uncover many energizing practices that warrant further investigation of the impacts of divider properties on the stream of non-Newtonian liquids in a tube.

Keywords: Peristaltic wave; wall properties; Casson fluid.

1. Introduction

Peristaltic wave engendering is a strategy of transporting liquid from lower weight to higher weight. This peristaltic wave proliferation discovers its applications in science, medicinal and designing field. Further dependent on this guideline, modern peristaltic siphons are likewise structured. Numerous examinations on peristaltic stream wonder have been performed in cylinder and channels. The greater part of examinations in industry and science demonstrates that the liquid conduct is non-Newtonian. Henceforth a few scientists are focusing on the peristaltic stream of non-Newtonian liquids in cylinders and channels [1-3]. Casson [4] watched the wellness of Casson liquid for displaying blood and expressed that at low shear rates the yield worry for blood is nonzero. Srivastava and Srivastava [5] saw that the peristaltic transport of blood by thinking about blood as a two liquid framework (Casson and Newtonian liquids).

Considering versatile property of the divider when the liquid is moving with the impact of peristalsis is fascinating reality to be taken note. Numerous examinations have been conveyed with stream of non-Newtonian liquids in versatile cylinders [6&7]. As the peristalsis happens by constriction and extension of divider, the divider must be of flexible in nature. Peristaltic transport of Hershel Bulkley liquid by considering the versatile properties of the channel [8] and the cylinder [9] have been watched. Vajravelu et. al. [10] considered on the peristaltic stream of casson liquid in a flexible cylinder.

Peristaltic stream with warmth exchange has a few applications, for example, bio-heat conduction in tissues, oxygenation and hemodialysis, heat exchangers and sun powered vitality and so on. In perspective on these applications, numerous scientists researched on peristaltic stream with warmth exchange. In physiological liquid stream issues, it is exceptionally fundamental to think about versatile properties of the divider. Radhakrishnamacharya et. al [11] watched the impact of warmth exchange and divider properties on the peristaltic stream of Newtonian liquid. Later peristaltic stream of intensity law liquid with warmth exchange and divider impacts was examined by Hayat et. al [12]. Also, most extreme of the physiological organs like corridors, natural channels, throat, digestive system, cervical waterway are observed to be non-uniform. In perspective on these viable applications Lakshminarayana et. al [13] focused on the peristaltic stream of Bingham liquid in a non-uniform channel with divider properties and warmth exchange. Nabil et. al [14] examined divider properties impact on the peristaltic movement of a couple pressure liquid with warmth and mass exchange through a permeable medium. As of late Lakshminarayana et. al [15] contemplated because of Slip, Divider Properties on Peristaltic Transport of a Directing Bingham liquid with warmth exchange. Impacts of warmth exchange on MHD peristaltic transport of dusty liquid in an adaptable channel was researched by Hayat and Javed [16].

1

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The reason for the present paper is to examine the impact of slip and warmth exchange on the peristaltic stream of a Casson liquid in a non-uniform channel with adaptable dividers. Articulations for speed, stream capacity and temperature has been determined logically with long wave length and low Reynolds number suspicions. The liquid stream relies upon numerous physical articulations like divider properties, slip parameter, non-consistency parameter rand yield pressure. These impacts of parameters are talked about in detail through diagrams. Since Casson show intently depicts blood stream in physiological frameworks, the outcomes acquired have vital applications in cardiovascular framework.

2. Mathematical Formulation

Consider the peristaltic stream of a Casson liquid in a two-dimensional non-uniform channel limited by adaptable dividers (see physical model for subtleties) on which sinusoidal floods of moderate plentifulness are forced. The dividers are taken like extended layers. The geometry of the channel divider is given by

$$y = h(x,t) = d(x) + a \sin \frac{2\pi}{\lambda} (X - ct)$$
(1)

a is the amplitude, λ is the wave length, *d* is the mean half width of the channel, *m*' is the dimensional non-uniformity of the channel.

The equations governing the motion for the present problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
(2)

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} \right)$$
(3)

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \mu \left(\frac{\partial^2 v}{\partial x^2} \right)$$
(4)

$$\zeta \left(\frac{\partial T}{\partial t} + u \left(\frac{\partial T}{\partial x} \right) + v \frac{\partial T}{\partial y} \right) = \frac{k}{\rho} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$
(5)
The governing equations of motion of the flexible wall may be expressed as

 $L^*(h) = p - p_0 \tag{6}$

where L^* is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that

$$L^* = -\tau \frac{\partial^2}{\partial x^2} + m_1 \frac{\partial^2}{\partial t^2} + c_1 \frac{\partial}{\partial t}$$
(7)

Here τ is the elastic tension in the membrane, m_1 is the mass per unit area, c_1 is the coefficient of viscous damping forces, p_0 is the pressure on the outside surface of the wall due to the tension in the muscles.

Continuity of stress at y = h and using x – momentum equation yields

$$\frac{\partial}{\partial x}L^{*}(h) = \frac{\partial p}{\partial x} = \mu \left(\frac{\partial^{2} u}{\partial x^{2}}\right) + \frac{\partial}{\partial y} \left(\tau_{0}^{\frac{1}{2}} + \left(-\mu \frac{\partial u}{\partial y}\right)^{\frac{1}{2}}\right)^{2}$$
(8)

$$u = -h_1 \frac{\partial u}{\partial y} \quad at \qquad y = h = d + m' x + a \sin \frac{2\pi}{\lambda} (x - ct) \tag{9}$$

(10)

$$\frac{\partial I}{\partial y} = 0 \quad on \quad y = y_0, \quad T = T_1 \quad on \quad y = h$$

where
$$L^* = -\tau \frac{\partial^2}{\partial x^2} + m_1 \frac{\partial^2}{\partial t^2} + c_1 \frac{\partial}{\partial t}$$

For simplicity, we assume that $p_0 = 0$

$$x' = \frac{x}{\lambda}, y' = \frac{y}{d}, \psi' = \frac{\psi}{cd}, t' = \frac{ct}{\lambda}, h' = \frac{h}{d}, p' = \frac{d^2}{c\lambda\mu}, k' = \frac{k}{d^2}, \tau_0' = \frac{d}{\mu c}, \theta = \frac{T - T_0}{T_1 - T_0}$$
(11)

The non-dimensional governing equations after dropping primes, we get

$$R\delta\left(\frac{\partial^2\psi}{\partial t\partial y} + \frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right) = -\frac{\partial p}{\partial x} + \delta^2\left(\frac{\partial^3\psi}{\partial x^2\partial y}\right)$$
(12)

$$R\delta\left(\frac{\partial^2\psi}{\partial t\partial x} + \frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^4\left(\frac{\partial^3\psi}{\partial x^3}\right) + \delta^2\left(\frac{\partial^3\psi}{\partial x\partial y^2}\right)$$
(13)

$$R\delta\left(\frac{\partial\theta}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \frac{1}{\Pr}\left(\delta^{2}\frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}}\right)\theta + E\left(4\delta^{2}\left(\frac{\partial^{2}\psi}{\partial x\partial y}\right)^{2} + \left(\frac{\partial^{2}\psi}{\partial y^{2}} - \delta^{2}\frac{\partial^{2}\psi}{\partial x^{2}}\right)^{2}\right)(14)$$
$$\frac{\partial\psi}{\partial y} = -\beta\frac{\partial^{2}\psi}{\partial y^{2}} \quad at \qquad y = h = 1 + mx + \varepsilon \sin 2\pi(x-t)$$
(15)

$$\delta^{2} \left(\frac{\partial^{3} \psi}{\partial x^{2} \partial y} \right) + \frac{\partial}{\partial y} \left(\tau_{0}^{\frac{1}{2}} + \left(-\frac{\partial^{2} \psi}{\partial y^{2}} \right)^{\frac{1}{2}} \right)^{2} - R \delta \left(\frac{\partial^{2} \psi}{\partial t \partial y} + \frac{\partial \psi}{\partial y} \frac{\partial^{2} \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^{2} \psi}{\partial y^{2}} \right)$$
$$= \left(E_{1} \frac{\partial^{3}}{\partial x^{3}} + E_{2} \frac{\partial^{3}}{\partial x \partial t^{2}} + E_{3} \frac{\partial^{2}}{\partial x \partial t} \right) h$$

Non-dimensional boundary conditions are

$$\psi_{p} = 0 \quad at \quad y = 0 \qquad \psi_{yy} = \tau_{0} \quad at \quad y = 0 \quad \psi = \psi_{p} \quad at \quad y = y_{0}$$
(17)
$$\frac{\partial \theta}{\partial y} = 0 \quad at \quad y = y_{0} \qquad \theta = 1 \quad at \quad y = h$$
(18)

where $\varepsilon = \frac{a}{d}$, $\delta = \frac{d}{\lambda}$ are geometric parameters, $R = \frac{cd\rho}{\mu}$ is the Reynolds number, $E_1 = -\frac{\tau d^3}{\lambda^3 \mu c}$, $E_2 = \frac{m_1 cd^3}{\lambda^3 \mu}$, $E_3 = \frac{cd^3}{\lambda^2 \mu}$ are the non-dimensional elasticity parameters, $m = \frac{\lambda m'}{d}$ is the non-uniform parameter. $\Pr = \frac{\rho v \zeta}{k}$ is the Prandtl number, $Ec = \frac{c^2}{\zeta (T_1 - T_0)}$ is the Eckert number, $m = \frac{\lambda m'}{d}$ is the non-uniform parameter, β is the Knudsen number (slip parameter) ∂y

3. Solution of the Problem

Using the long wavelength and low Reynolds number approximations, one can find from equations (12) to (16) that

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(\tau_0^{\frac{1}{2}} + \left(-\frac{\partial^2 \psi}{\partial y^2} \right)^{\frac{1}{2}} \right)^2$$
(19)
$$0 = -\frac{\partial p}{\partial x}$$
(20)

Equation (20) shows that p is not a function of y

$$0 = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} + Ec \left(\frac{\partial^2 \psi}{\partial y^2}\right)^2$$
(21)

On differentiating Eq. (19) with respect to y, we get

$$\frac{\partial^2}{\partial y^2} \left(\tau_0^{\frac{1}{2}} + \left(-\frac{\partial^2 \psi}{\partial y^2} \right)^{\frac{1}{2}} \right)^2 = 0$$
From Eq. (16) we get
$$\frac{\partial}{\partial y} \left(\tau_0^{\frac{1}{2}} + \left(-\frac{\partial^2 \psi}{\partial y^2} \right)^{\frac{1}{2}} \right)^2 = \left(E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t} \right) h$$
(22)

The closed form solution for equation (22) using the boundary conditions (15), (17) and (23) can be obtained as

$$u = \frac{A}{2}(h^{2} - y^{2}) + (B^{2} + \tau_{0})(h - y + \beta) - \frac{4}{3}\frac{\tau_{0}^{\frac{1}{2}}}{A}((Ah + B^{2})^{\frac{3}{2}} - (Ay + B^{2})^{\frac{3}{2}}) + A\beta h - 2\beta\tau_{0}^{\frac{1}{2}}(Ah + B^{2})^{\frac{1}{2}}$$

$$y_{0} \le y \le h$$
(24)

We find the upper limit of plug flow region using the boundary condition that $\psi_{yy} = 0$ at $y = y_0$. It is given by

$$y_0 = \frac{\tau_0 - B^2}{A}$$
(25)

Taking $y = y_0$ in equation (24) and using the relation (25), we get the velocity in the plug flow region as

$$u_{p} = \frac{A}{2}h^{2} + h(A\beta + 2B^{2}) + B^{2}(2\beta + \frac{4B^{2}}{3A}) - \frac{1}{6}Ay_{0}^{2} + (Ah + \frac{2}{3}B^{2} + \beta A)y_{0} - (Ay_{0} + B^{2})^{\frac{1}{2}} \left(2\beta + \frac{4}{3A}(Ah + B^{2})^{\frac{3}{2}}\right), \quad 0 \le y \le y_{0}$$
(26)

By using Equations (24) and (26), we get

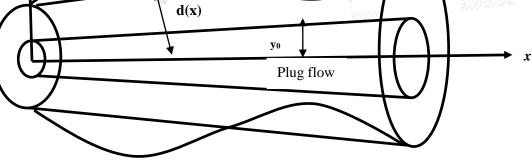
$$\begin{split} \psi &= \frac{Ah^{2}y}{2} - \frac{Ay^{3}}{3} + (B^{2} + \tau_{0})hy - (B^{2} + \tau_{0})\left(\frac{y^{2}}{2} - \frac{y_{0}^{2}}{2}\right) + A\beta hy + \beta(B^{2} + \tau_{0})y \\ &- \frac{4\tau_{0}^{\frac{1}{2}}}{3A} \left(y\left(Ah + B^{2}\right)^{\frac{3}{2}} - \frac{2}{5A}\left(Ay + B^{2}\right)^{\frac{5}{2}}\right) - 2\beta\tau_{0}^{\frac{1}{2}}\left(Ah + B^{2}\right)^{\frac{1}{2}}y - \frac{A}{6}y_{0}^{3} \qquad (27) \\ &+ \frac{4}{3A}\tau_{0}^{2}y_{0} - \frac{8}{15A^{2}}\tau_{0}^{3} \\ \psi_{p} &= y \left(\frac{A}{2}h^{2} + h(A\beta + 2B^{2}) + B^{2}(2\beta + \frac{4B^{2}}{3A}) - \frac{1}{6}Ay_{0}^{2} \\ &+ (Ah + \frac{2}{3}B^{2} + \beta A)y_{0} - (Ay_{0} + B^{2})^{\frac{1}{2}}\left(2\beta + \frac{4}{3A}(Ah + B^{2})^{\frac{3}{2}}\right) \right) \end{split}$$

$$\tag{28}$$

Where $A = -8\varepsilon\pi \left[(E_1 + E_2)\cos 2\pi (x-t) - \frac{E_3}{2\pi}\sin 2\pi (x-t) \right], \quad B = \tau_0^{\frac{1}{2}} + (-\tau_0)^{\frac{1}{2}}$ (29)

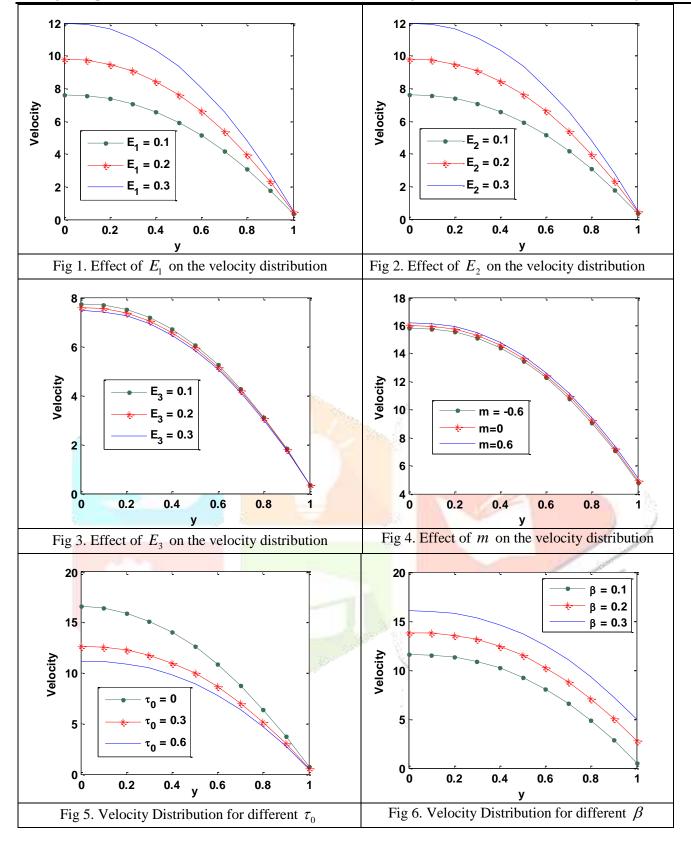
By solving (21) with the help of equation (27) and (18) the temperature field is obtained as

$$\theta = 1 + Br \left\{ \frac{A^{2}}{3} \left(y_{0}^{3} y - \frac{y^{4}}{4} - y_{0}^{3} h + \frac{h^{4}}{4} \right) + \left((B^{2} + \tau_{0})^{2} + 4B^{2} \tau_{0} \right) \left(y_{0} y - \frac{y^{2}}{2} - y_{0} h + \frac{h^{2}}{2} \right) \right. \\ \left. + \left(B^{2} + 3\tau_{0} \right) A \left(y_{0}^{2} y - \frac{y^{3}}{3} - y_{0}^{2} h + \frac{h^{3}}{3} \right) - \frac{4\tau_{0}^{\frac{1}{2}}}{105A^{2}} \left(\left(Ay + B^{2} \right)^{\frac{7}{2}} - \left(Ah + B^{2} \right)^{\frac{7}{2}} \right) \right. \\ \left. - \frac{8\tau_{0}^{\frac{1}{2}}}{3A} \left(y - h \right) \left(Ay_{0} + B^{2} \right)^{\frac{3}{2}} \left(Ay_{0} + B^{2} + \tau_{0} \right) + \frac{16\tau_{0}^{\frac{1}{2}}}{15A^{2}} \left[y \left(Ay + B^{2} \right)^{\frac{5}{2}} - h \left(Ah + B^{2} \right)^{\frac{5}{2}} \right] \\ \left. + \left(y - h \right) \left(Ay_{0} + B^{2} \right)^{\frac{5}{2}} \right] + \frac{16\tau_{0}^{\frac{1}{2}}}{15A^{2}} \left(B^{2} + \tau_{0} \right) \left[\left(Ay + B^{2} \right)^{\frac{5}{2}} - \left(Ah + B^{2} \right)^{\frac{5}{2}} \right] \right\}$$

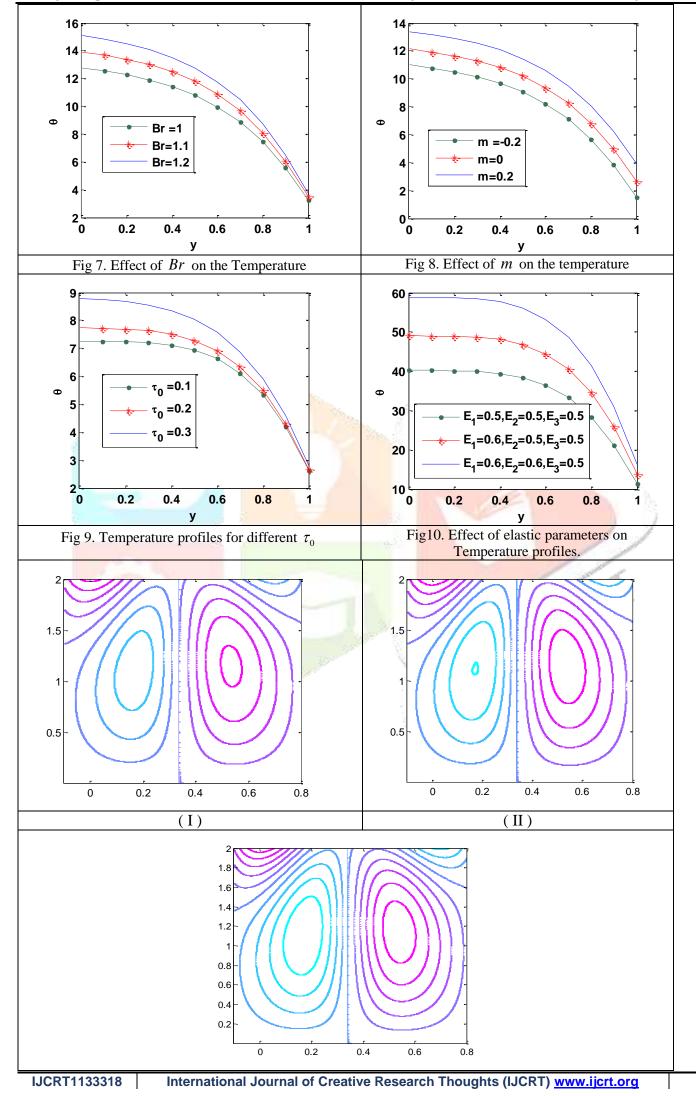


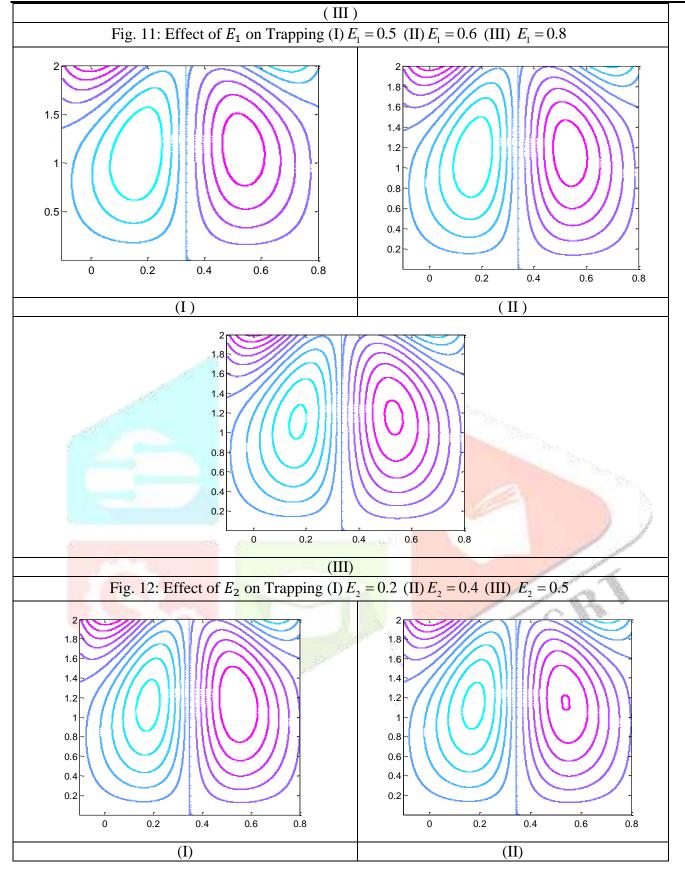
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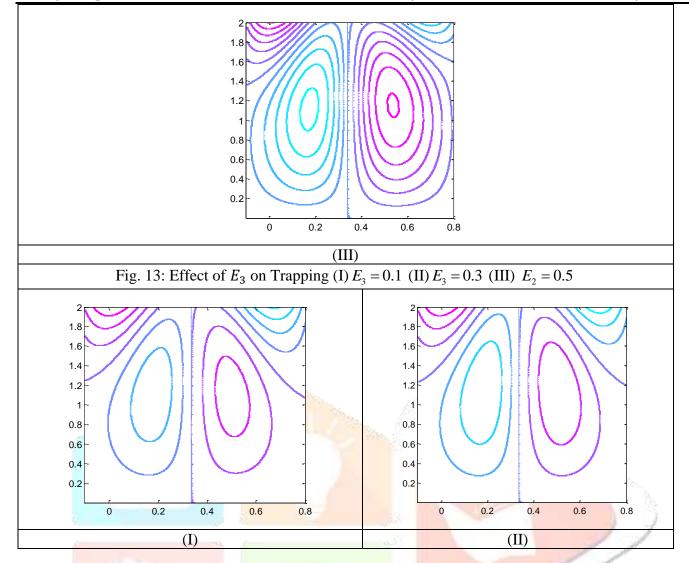
Physical model



7







4. Discussions and Results

The velocity, stream function and temperature for the present problem depends on the following important physical parameters, the wall parameters which characterize the viscoelastic behavior of the flexible walls. The non-uniform parameter determines the non-uniformity of the channel. The yield stress . The effect of these parameters are discussed graphically from Fig (1) to Fig (15)

Figures (1),(2) and (3) depict respectively, the behaviors of the velocity u versus y with changes in elastic parameters, namely, . The effect of an increase in the rigidity parameter gives rise to an increase in the velocity . The same behaviour is observed for the other elastic parameter . While the parameter is concerned an opposite behaviour is observed. Figure (4) shows the variation of velocity with for different values of non-uniform parameter [namely convergent channel , uniform channel and divergent channel]. From the graphical representation, it is observed that the velocity in the case of convergent channel is less than that in the uniform channel and this is less than the divergent channel. The variation of velocity with is calculated for different values of yield stress and is depicted in Figure (5). It can be seen from Figure that the velocity depends on yield stress and it decreases with increasing yield stress . The variation of slip parameter with velocity is observed in Figure (6). As the slip increases the velocity is increasing.

The effect of various parameters on temperature are illustrated in Figures (7) - (10). From Figure (7), we observe that increment in enhance the temperature field. Further it is noted from figure (8) that temperature increase for large values of . Figure (11) depicts that as the yield stress increases the temperature is increasing. We see from Figure (10) that the temperature increases with increasing and whereas it decreases with increasing .

4.1 Trapping Phenomenon

Another interesting phenomena of peristalsis is trapping, the formation of an internally circulating bolus of fluid which moves along with the wave. Fig (11) shows that for higher rigidity E1, the size of the trapped bolus increases. From Fig.(12) we observe that more trapped bolus appear with increase in stiffness parameter. Further as viscous damping force increases, the size of the trapped bolus also increases as seen in Fig (13.

Thus, we have seen that associated parameters play an important role in the growth and decay of the trapping bolus. These qualitative results may have some significance in understanding the transport of blood in the small blood vessels.

5. Conclusions

The present paper finds its application to the peristaltic flow of a non-Newtonian fluid with non-zero yield stress namely casson fluid to study the changes in the blood flow pattern when a catheter is inserted into a channel with flexible walls. In this application, we assume that the plug flow region represents catheter and the remaining non-plug flow region represents the fluid.

The present study also deals with the Peristaltic motion of an yield stress fluid (Casson fluid which closely describes blood flow) in a two dimensional channel under the influence of wall properties. The governing equations have been linearized under long wavelength approximation and analytical expressions for velocity and stream function have been derived. The fluid model considers the yield stress parameter along with wall slope parameter, thus giving useful information about the blood flow characteristics. The results are analyzed for different values of pertinent parameters namely rigidity , stiffness , viscous damping force , non-uniform parameter and yield stress . Some of the interesting findings are :

- > The velocity increases with increasing values of the elastic parameters.
- The velocity decreases with an increase in the yield stress in the plug flow as well as in the non-plug flow regions.
- Streamline pattern shows that the size and number of the trapped bolus increases with increase in rigidity, stiffness, viscous damping force of the wall.
- > The size of the trapped bolus decreases with increasing values of the yield stress.

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